E Methods for Streambed Mobility/ Stability Analysis

Appendix E—Methods for Streambed Mobility/Stability Analysis

This appendix provides background on the use and limitations of several sediment entrainment equations that are the most practical for stream-simulation applications. These equations are used in stream-simulation design to verify whether the sediment sizes to be used in the simulation (sizes based on reference reach data) are as mobile or as stable as intended. Specifically, the purpose of these equations in stream-simulation design is to ensure that:

- Similar particle sizes move at similar flows in both the reference-reach channel and stream-simulation design channel.
- Key pieces (permanent features) in the stream-simulation design channel are stable for the high bed-design flow.

The equations are one set of tools that help the designer modify the simulation-bed width, the bed-material size, and/or the design slope to compensate for a difference between the stream-simulation channel and the reference reach. The difference might be a flow constriction (as in a wide **flood plain** that is blocked by the road fill), or a steeper slope. Within limits, designers can use these equations to change the design parameters so that a given-size particle moves at the same flow as in the reference reach.

The equations do not apply to all stream types and flow conditions. For example, they are not relevant to channels with cohesive soils making up their bed and banks. They do apply to alluvial channels composed of granular material where erosion occurs by entrainment of individual particles; however, each equation is applicable only in conditions similar to those for which it was developed. We strongly recommend that you understand the source, derivation, and limitations of these equations before you use them. It is always wise to compare results from more than one equation, and check those results for reasonableness in the field. Knowledgeable designers may elect to use other equations for specific applications, but the ones described here are a good starting point for many stream-simulation design situations.

E.1 FLOW HYDRAULICS: SHEAR STRESS AND UNIT DISCHARGE

A particle on the streambed begins to move when drag and lift forces exerted by the flow on the particle exceed the forces resisting motion. Resisting forces include the submerged weight of the particle and intergranular friction between particles (figure E.1). The flow at which the particle just begins to move is called the critical flow or the critical entrainment flow.

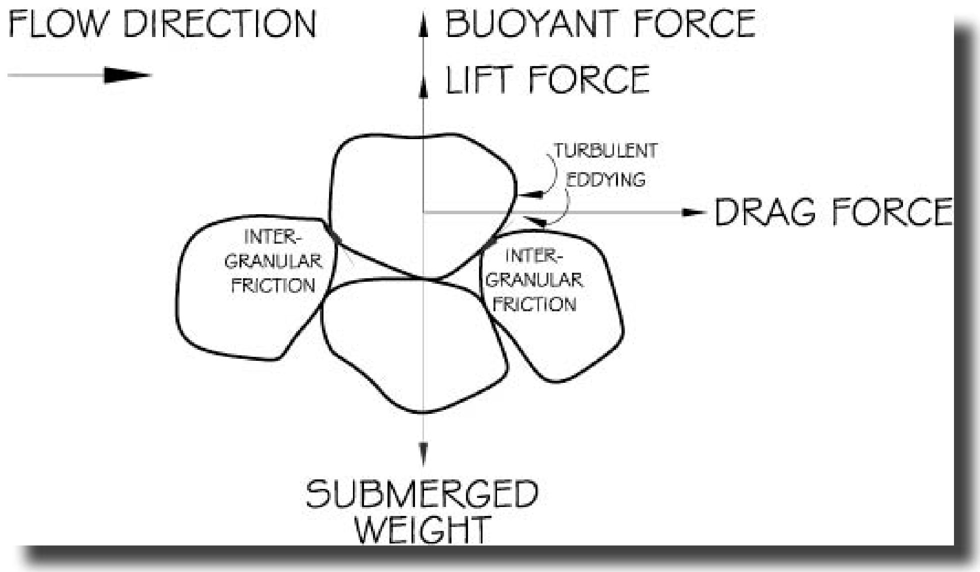

Figure E.1—Schematic diagram illustrating the interaction between drag and lift forces, the buoyant force, and resisting forces (submerged weight of the particle and intergranular friction between particles). Diagram is modified from Carling 1992; Julian 1995; and Knighton 1998.

There are two common approaches to quantifying the driving forces acting on a particle during any specific flow: average boundary shear stress and unit discharge.

The average boundary shear stress exerted by flowing water on its boundary is:

Equation E.1

$$\tau = \gamma RS$$

where:

τ = average boundary shear stress (lb/ft^2)

γ = specific weight of water (62.4 lb/ft^3)

R = **hydraulic radius** (ft)

S = energy slope or bed slope (ft/ft).

Hydraulic radius is average flow depth, determined by dividing the cross-section flow area by the wetted perimeter. Because we are most interested in the mobility or stability of particles on the channel bed, boundary shear stress is calculated for flows within the active stream bed width or

bankfull width (figure E.2). Use active streambed width for streams with gently sloping or vegetated banks where that part of the cross section is subject to substantially lower shear stresses than the rest of the bed and there is less evidence of sediment transport. Where bankfull width is substantially the same as active streambed width, as in rectangular channels, either can represent active-channel width. Flows outside of those boundaries (i.e., flood-plain flow) should not be included in the calculations because they will underestimate the boundary shear stress being exerted on the channel bed.

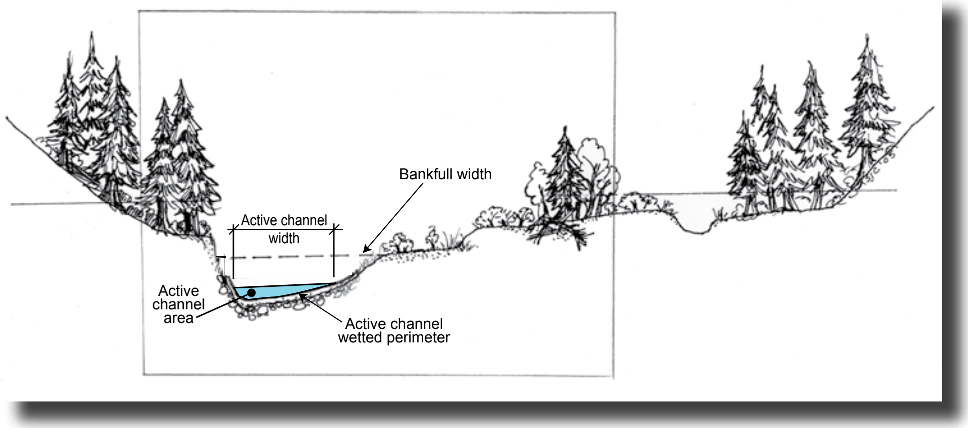

Figure E.2—Active channel width and hydraulic radius.

For channels with gradients greater than 1 percent, and where the flow depth is shallow with respect to the channel-bed particle size (relative submergence, R/D_{50}, values less than 10), Bathurst (1987) suggested using discharge-per-unit width instead of average boundary shear stress for determining particle mobility. The reason is that water depth in such channels can be highly variable and is more difficult to measure accurately than discharge (Bathurst 1987). The following equation defines unit discharge:

Equation E.2

$$q = Q/w$$

where:

q is the unit discharge (cfs/ft or ft2/s; cms or m2/s)

Q is discharge (cfs or cms)

w is the active channel width for bedload transport (ft or m) at a given cross section.

Although there are no existing guidelines for defining the active-channel width for bedload transport, we suggest using the bed width between the lower banks to represent active-channel width because it is typically the zone of active bedload transport. The unit discharge should be determined for the portion of the total flow that occurs over the active channel bed (figure E.2). Calculating unit discharge using the total discharge instead of the portion of discharge occurring over the active-channel width would overestimate the flow actually exerting force on the active-channel bed. This overestimation of unit discharge would be magnified when floodwaters inundate a wide flood plain.

E.1.1 Models for Calculating Flow Hydraulics

The hydraulic parameters in both equations 1 and 2 are calculated for a range of discharges and require the use of hydraulic models such as a cross section analyzer (e.g., WinXSPRO, http://www.stream.fs.fed.us/publications/winxspro.html) or step-**backwater** model (e.g., HEC-RAS, http://www.hec.usace.army.mil/software/hec-ras/). WinXSPRO uses a resistance-equation approach (e.g., Manning equation) and basic continuity to calculate channel geometry, flow hydraulics, and sediment transport potential at a single cross section (Hardy et al. 2005). Flow is assumed to be relatively uniform; that is, width, depth, and flow area are relatively constant along the channel, and the bed slope, water-surface slope, and energy slope are essentially parallel. WinXSPRO is also valid for gradually varied flow that is more typical of natural channels, so long as energy losses are primarily due to boundary friction (see section A.3.6). The program allows the user to subdivide the channel cross section so that overbank areas, mid-channel islands, and high-water overflow channels may be analyzed separately. The reliability of the WinXSPRO output data depends on the reliability of the cross section and bed slope data collected in the field for input into the program and the selection of channel boundary roughness or Manning's **roughness** coefficient (n). Please refer to Hardy et al. (2005) for guidelines on collecting cross section and slope data, and the various methods available in the WinXSPRO program for determining channel roughness. WinXSPRO cannot model flow hydraulics through a culvert. We recommend modeling the stream-simulation design channel inside a culvert as an open channel, but with vertical walls having low roughness values as a surrogate for the culvert. Typically, stream-simulation culverts are of sufficient width and capacity that the culvert shape has a negligible effect on flow hydraulics. However, if overbank flood-plain flows are being funneled through the culvert or the crossing is prone to **debris** jams, culvert shape could potentially affect flow hydraulics.

HEC-RAS utilizes the step-backwater method to calculate a one-dimensional, energy-balanced, water-surface profile that is a function of discharge, channel/flood-plain boundary roughness, and channel geometry (USACE 2006). For a specified discharge and assumed friction and form energy losses (channel/flood-plain boundary roughness, flow expansion/contraction) the step-backwater method iteratively calculates an energy-balanced, water-surface elevation between the surveyed cross-sections. When applying the step-backwater method to natural channels, the basic assumptions are that (1) flow is relatively steady or constant along the surveyed reach, (2) flow is gradually varied between successive cross sections, (3) flow is one dimensional, (4) slopes are less than 10 percent, and (5) the energy slope between successive cross sections is constant across the cross section. Based on the modeling results for a given discharge, various hydraulic parameters can be calculated at each cross section for both the total cross section and for sections of a subdivided cross section (e.g., channel, flood plain, channel banks, active bed width). The reliability of the HEC-RAS modeling results and subsequent hydraulic calculations depend on the accuracy with which surveyed channel/valley dimensions represent actual topography. The accuracy of estimating energy losses due to channel/flood-plain boundary roughness and to channel expansion/contraction also directly affects the reliability of model results. Please refer to USACE (2006) for guidelines on using the HEC-RAS step-backwater model. HEC-RAS can model flow through the stream-simulation design channel by using the “Lid” option. The cross-section data are entered as the bottom half of the structure and the “lid” data are entered as the top half of the culvert. Any culvert shape can be modeled, but the actual culvert shape the model uses will depend on the number of points the user inputs to define the pipe shape. Several cross sections with “lids” can be used to represent the length of the culvert. Stream-simulation culverts are almost always of sufficient width and capacity that flow is not pressurized; however, this can and should be verified in HEC-RAS at individual sites. Flow could become pressurized if a substantial volume of **overbank flow** is funneled through the culvert, or if a debris jam reduces the opening area and causes water to submerge the inlet.

E.1.2 What Flows to Analyze

Your choice of analysis flows for sediment entrainment will depend on the question you are trying to answer. The most common questions in stream simulation are:

At what flow (e.g., bankfull, 10-year flood, 50-year flood, 100-year flood, etc.) are the D_{84} and/or D_{95} particle sizes of the channel bed mobilized? The same flow should mobilize the D_{84} and D_{95} particle sizes in the reference-reach channel and the stream-simulation channel.

Are key pieces stable for the high bed-design flow (e.g., 10-year flood, 50-year flood, 100-year flood, etc.)? Rocks used as permanent features such as banks or roughness elements should not be mobilized by the high bed-design flow.

The following sections show how to answer these questions.

E.2 PARTICLE ENTRAINMENT IN NATURAL CHANNELS

Many readers will be familiar with the Shields equation, which predicts critical shear stress for particle entrainment based on particle size. The Shields equation is most applicable in well-sorted streambeds composed of particles of a narrow range of sizes. For these streambeds, the relationship of forces driving and resisting particle movement at the moment of entrainment (figure E.1) can be expressed as a dimensionless ratio known as the Shields parameter:

Equation E.3

$$\tau^* = \tau_c / (\gamma_s - \gamma) D$$

where:

τ^* is the Shields parameter

τ_c is the critical average boundary shear stress at which the sediment particle begins to move (lb/ft^2)

γ_s is the specific weight of the sediment particle (lb/ft^3)

γ is the specific weight of the fluid (lb/ft^3)

D is the median size particle diameter of the channel bed (ft)

The Shields parameter—the dimensionless ratio of hydrodynamic forces acting on the bed to the submerged weight of the particles—has been determined experimentally for a wide range of particle sizes (table E.1). The parameter increases nonlinearly as the particle size increases from medium size sands to very coarse gravels (ranging from 0.029 to 0.050). For cobbles and boulders, the Shields parameter approaches a constant value of 0.054. However, Shvidchenko and Pender (2000) demonstrated that channel slope and flow depth influence the Shields parameter; the Shields parameter increases as slope increases and as flow depth with respect to particle size decreases.

Table E.1—Shield's parameter for different particle sizes. Modified from Julien 1995

Particle size classification	Particle size, D (mm)	Angle of repose, ϕ (degrees)	Shield's parameter, τ^*	Critical shear stress, τ_c (lb/ft^2)
very large boulders	> 2,048	42	0.054	37.37
large boulders	1,024-2,048	42	0.054	18.68
medium boulders	512-1,024	42	0.054	9.34
small boulders	256-512	42	0.054	4.67
large cobbles	128-256	42	0.054	2.34
small cobbles	64-128	41	0.052	1.13
very coarse gravels	32-64	40	0.050	0.54
coarse gravels	16-32	38	0.047	0.25
medium gravels	8-16	36	0.044	0.12
fine gravels	4-8	35	0.042	0.057
very fine gravels	2-4	33	0.039	0.026

The equation used to determine the Shields parameter for gravels, cobbles, and boulders is $\tau^* = 0.06 \tan\phi$ *.*

The Shield's parameter and critical shear stress values are for the smallest number in the particle-size interval.

Assuming $\gamma_s = 165$ lb/ft^3 and $\gamma = 62.4$ lb/ft^3, equation 4 can be rearranged and simplified to calculate the critical shear stress to entrain a particle:

Equation E.4 $\tau_c = \tau^* (102.6 D)$

All equations can be used with either metric or English units, as long as the units are kept consistent. In metric units, equation 4 is $\tau_c = \tau^*$ (16170 D). The metric unit of shear stress is newtons/square meter (N/m^2).

Most channels where bed mobility requires analysis for stream-simulation design are poorly sorted, that is they are made up of a wide variety of different particle sizes. In poorly sorted streambeds, the calculated values of τ^* and τ_c do not accurately predict sediment entrainment in the channel. To account for the variability of particle sizes in gravel- and cobble-bed channels, τ^* is often assigned a constant value of 0.045 in the unmodified Shields equation. However, Buffington and Montgomery (1997), in a thorough review of past entrainment studies, found that the assumption of a constant value of 0.045 is not always appropriate; reference based and visually based values of τ^* ranged from 0.052-0.086 and 0.030-0.073, respectively.

Subsequently, the Shields equation was modified for poorly sorted channels to account for the influence of adjacent particles on the stability of a given particle (Andrews 1983; Wiberg and Smith 1987; Komar 1987; Bathurst 1987). Because larger particles shield smaller ones, stronger flows are needed in poorly sorted streambeds for entraining the small particles when compared to streambeds composed of uniformly sized particles. Similarly, in poorly sorted streambeds, larger particles are entrained at weaker flows, because the larger particles project into the flow. This increased exposure enhances their entrainment despite their greater weight (figure E.3). In addition, the larger particles surrounded by smaller particles have smaller **pivoting angles**, causing them to rotate more easily from their resting position on the bed (Komar and Li 1986; Wiberg and Smith 1987).

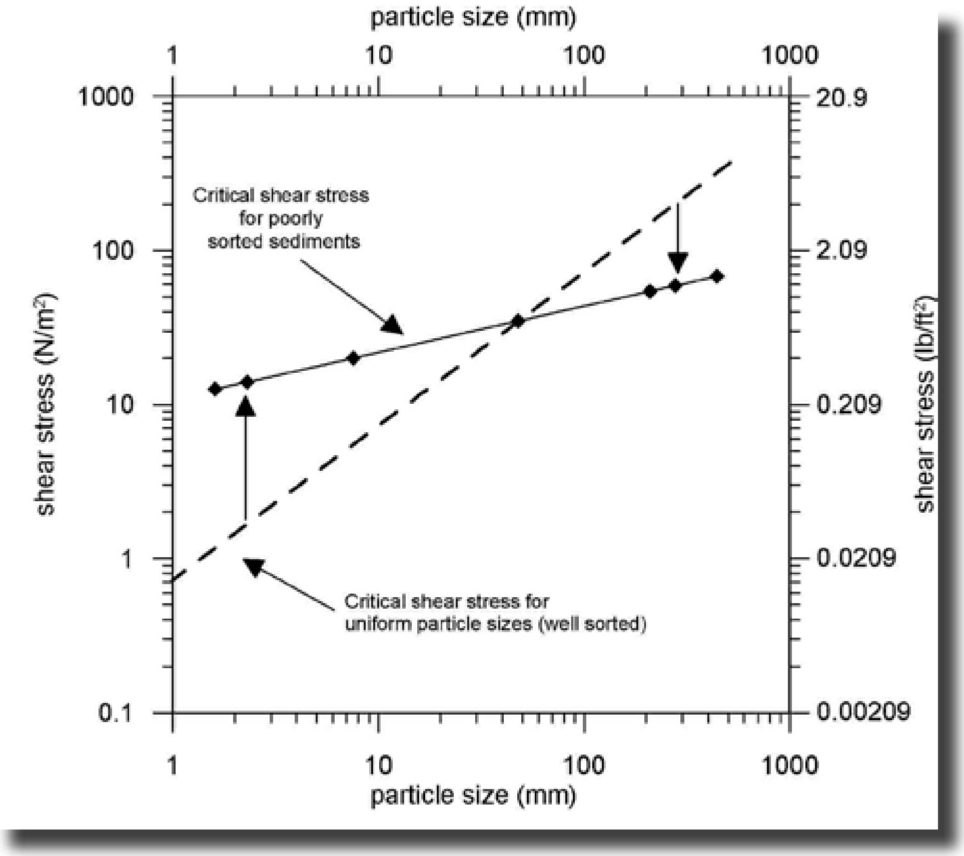

Figure E.3—Critical shear stress for well-sorted sediments compared to critical shear stress for poorly sorted sediments. Higher critical shear stress is needed to entrain smaller particles in poorly-sorted sediments because they are shielded by the larger particles. Lower critical shear stress is needed to entrain larger particles because their protrusion into the flow causes them to experience greater hydrodynamic forces. The critical shear stress line for the poorly-sorted sediments crosses the critical shear stress line for well-sorted sediment at the reference particle size (D_{50}).

In the sections that follow, we examine two approaches for evaluating the stability of particles in poorly sorted channel beds: (1) modified critical shear stress and (2) critical unit discharge.

E.2.1 Modified Critical Shear Stress Approach

The modified critical shear stress equation is based on the relationship between the particle size of interest (D_i) and D_{50}, which is assumed to be unaffected by the shielding/exposure effect (Andrews 1983; Bathurst 1987; Komar 1987, 1996; Komar and Carling 1991). For stream simulation, the particle size of interest, D_i, is usually D_{84} and/or D_{95}, because key **grade controls** are in this size range. When these particles begin to move, much of the streambed is in motion and the structure of the channel bed will change.

The modified critical shear stress equation (Komar 1987, 1997; Komar and Carling 1991) is as follows:

Equation E.5

$$\tau_{ci} = \tau_{D50} (\gamma_s - \gamma) D_i^{0.3} D_{50}^{0.7}$$

where:

τ_{ci} is the critical shear stress at which the sediment particle of interest begins to move (lb/ft^2 or N/m^2).

τ_{D50} is the dimensionless Shields parameter for D_{50} particle size (this value can either be obtained from table E.1, or the value 0.045 can be used for a poorly sorted channel bed).

D_{50} is the diameter (ft or m) of the median or 50th percentile particle size of the channel bed .

D_i is the diameter (ft or m) of the particle size of interest. For stream simulation the particle size of interest is typically D_{84} and/or D_{95}.

Assuming γ_s = 165 lb/ft^3 and γ = 62.4 lb/ft^3, equation 5 can be simplified to:

Equation E.6

$$\tau_{ci} = 102.6\, \tau_{D50}\, D_i^{0.3} D_{50}^{0.7}$$

The modified critical shear stress equation is appropriate for assessing particle stability in riffles and plane-bed channels (i.e., where flow is relatively uniform or gradually varied between cross sections) with channel-bed gradients less than 0.05 (5 percent) and D_{84} particles ranging between 10 and 250 mm (2.5 to 10 inches). Because of the uncertainty and variability of determining τ_{D50} (see earlier discussion), multiple values should be used to assess how it influences the results. If the reference reach bed is tightly packed and/or **imbricated**, the Shields parameter and critical shear stress will be higher than in the newly constructed stream-simulation bed. The stream-simulation bed may need to have larger material to offset the difference in bed stability. Likewise, if angular material must be used for the stream-simulation bed, the Shields parameter and critical shear will be higher than for rounded river rock.

When applying the critical shear stress equation, be sure that the diameter for the particle size of interest (e.g., D_{84} or D_{95}) is not larger than 20 to 30 times the D_{50} particle diameter. For D_i/D_{50} ratios greater than 30, equation E.6 is not accurate because a large particle will roll easily over surrounding smaller sediments (Komar 1987, 1996; Carling 1992). D_{84}/D_{50} or D_{95}/D_{50} ratios are typically less than 5 in natural channels. However, where a design uses larger rock to create permanent, stable features such as banks or roughness elements, check that those rock diameters do not exceed 20 to 30 times D_{50}.

To determine critical entrainment flow at a given cross section using the modified shear stress approach, use the following process:

From equation E.6, find the critical shear stress (τ_{ci}) for the particle size of interest (e.g., D_{84}) at a given cross section. Assume τ_{D50}= 0.045 or use table E.1 to determine τ_{D50} for the D_{50} particle size.

Calculate the boundary shear stress (equation E.1) within the active channel for a range of discharges using a hydraulic model such as WinXSPRO or HEC-RAS.

To determine whether the particle will move, compare the active-channel boundary shear stress for a particular flow to the critical shear stress for the particle size of interest. If the critical shear stress (τ_{ci}) of a given particle is less than the active-channel boundary shear stress (τ) being exerted on the particle by the flow, the particle will be entrained. If the critical shear stress (τ_{ci}) is greater than the active-channel boundary shear stress (τ) being exerted on the particle by the flow, the particle will not be entrained. See the sidebar in section E.2.3 for an example calculation.

E.2.2 Critical Unit Discharge Approach

For channels steeper than 1 percent ($S = 0.01$) where the flow depth is shallow with respect to the channel bed particle sizes ($R/D_{50} < 10$), water depth can be quite variable because large rocks or **wood** pieces on or near the surface influence depth (Bathurst 1987). For such channels, Bathurst et al. (1987) used flume data to construct the following equation, which predicts the critical unit discharge for entraining the D_{50} particle size in well-sorted sediments:

Equation E.7

$$q_{c\text{-}D50} = \frac{0.15\, g^{0.5}\, D_{50}^{1.5}}{S^{1.12}}$$

where:

$q_{c\text{-}D50}$ is the critical unit discharge to entrain the D_{50} particle size (cfs/ft or ft^2/s, cms/m or m^2/s)

D_{50} is the median or 50th percentile particle size (ft or m)

g is gravitational acceleration (32.2 ft/s^2 or 9.8 m/s^2)

S is bed slope (ft/ft or m/m)

In the flume studies, particle sizes ranged between 3 and 44 mm (0.1 and 1.7 inches), the experimental bed materials were uniform (i.e., well-sorted), slopes ranged between 0.0025 and 0.20, and ratios of water depth to particle size approached 1 (Bathurst 1987).

Bathurst (1987) used equation E.7 to predict the entrainment of particles in poorly sorted channel beds, by comparing the particle size of interest (e.g., D_{84} or D_{95}) to a reference particle size. The reference particle size is the D_{50} particle size, which is assumed to move at the same flow as in a well-sorted channel. The critical unit discharge for entraining a particle size of interest is determined by:

Equation E.8

$$q_{ci} = q_{c\text{-}D50} (D_i/D_{50})^b$$

where:

q_{ci} is the critical unit discharge to entrain the particle size of interest (cfs/ft or ft^2/s, cms/m or m^2/s)

D_i is the particle size of interest (mm)

D_{50} is the median or 50th percentile particle size (mm)

The exponent b is a measure of the range of particle sizes that make up the channel bed. It quantifies the effects on particle entrainment of smaller particles being hidden and of larger particles being exposed to flow. Calculate the exponent from:

Equation E.9

$$b = 1.5(D_{84}/D_{16})^{-1}$$

where:

D_{84} is the 84th percentile particle size (mm)

D_{16} is the 16th percentile particle size (mm)

Equations E.8 and E.9 were derived from limited data and are most appropriate for assessing particle stability in riffles and plane-bed channels (i.e., where flow is relatively uniform or gradually varied between cross sections) with slopes ranging between 0.0360 and 0.0523, widths ranging between 20 and 36 feet, D_{16} particle sizes between 32 and 58 millimeters (1.3 and 0.67 inches), D_{50} particle sizes between 72 and 140 millimeters (2.8 and 5.5 inches), and D_{84} particle sizes between 156 and 251 millimeters (6 and 10 inches).

To determine the critical entrainment flow for a given particle size, use the following process (See the sidebar in section E.2.3 for an illustration):

1) Using equation E.7, calculate the critical unit discharge ($q_{c\text{-}D50}$) needed to entrain the D_{50} particle size at any given cross section.

2) Using equation E.9, calculate the exponent (b) based on the ratio between the D_{84} particle size and D_{16} particle size.

3) Using equation E.8, calculate the critical unit discharge (q_{ci}) needed to entrain the particle size of interest at any given cross section (e.g., D_{84} or D_{95}).

4) Using equation E.2, calculate the unit discharge within the active channel for a range of discharges using a hydraulic model such as WinXSPRO or HEC-RAS.

To determine whether the particle will move at any given discharge, compare the unit discharge for that flow to the critical unit discharge for the particle size of interest. If the critical unit discharge (q_{ci}) of a given particle is less than the unit discharge (q) being exerted on the particle by the flow, the particle will be entrained. If the critical unit discharge (q_{ci}) is greater than the unit discharge (q) being exerted on the particle by the flow, the particle will not be entrained.

E.2.3 Uncertainty in Predicting Particle Entrainment

The modified critical shear stress equations (equations E.5 and E.6) and critical unit discharge equation (equation E.8) improved on the original critical shear stress equations (equations E.3 and E.4) by incorporating the effects of shielding and exposure on the entrainment of sediments. However, the modified critical shear stress and critical unit discharge equations do not account for other factors, such as:

Fluctuating flows

Fluctuating flows can cause temporary increases in near-bed, instantaneous stresses that cause particles to be entrained at lower values than predicted by average shear stress values acting on the bed (Nelson et al. 1995; Knighton 1998). Depending on channel and flow conditions, instantaneous shear-stress values near the bed can be 2 to 3 times greater than the average boundary shear stress (Richardson et al. 1990; Knighton 1998).

Particle shape

Angular particles require higher shear stresses to move than spherical particles of similar size, because of a greater **pivot angle** associated with angular particles (Reid and Frostick 1994). Flat, disc-shaped particles are usually well imbricated, making the particles more resistant to entrainment (Carling 1992).

Channel-bed structure

For channel beds composed of particles coarser than 8 millimeters (0.3 inch), Church (1978) demonstrated that the Shields parameter can vary by a factor of two, depending on whether the channel bed is loosely consolidated or tightly packed. Recently deposited sediments can be poorly packed, making it easier for those particles to be entrained than if they were tightly packed or highly consolidated (Church 1978; Reid and Frostick 1994). With time after a large flood, the bed consolidates as low flows slightly rearrange particles so that they are more tightly packed and more difficult to entrain (Church 1978). Reid et al. (1985) demonstrated that the shear stress needed to entrain particles could be up to three times higher than the average when the flood occurred after an extended period of no bed disturbance.

Although table E.1 and equations E.4, E.6, and E.8 suggest a distinct threshold at which a particle is entrained, the previous discussion makes clear that the entrainment of a particle does not occur at a distinct critical shear stress value, but instead may occur over a range of critical shear stresses. Nevertheless, the equations provide insights on the relative mobility of channel-bed sediments for a range of flows. If the flows that mobilize sediment in the reference reach are different than those in the stream-simulation design channel, these equations can help you assess whether the difference is significant, and whether the stream-simulation design channel needs to be adjusted so that its particle mobility is similar to that of the reference reach.

Using the equations to determine if D_{84} moves at bankfull flow in the reference reach

The reference reach selected for a stream-simulation culvert on Example Creek is a pool-riffle reach with a gravel-cobble bed (figure E.4).

Figure E.4—Downstream view of Example Creek, a pool-riffle channel composed primarily of gravels and cobbles and local inputs of wood.

Bankfull flow, 106 cubic feet per second, was estimated using the HEC-RAS step-backwater model to generate a water-surface profile that matched bankfull elevations identified in the field using geomorphic indicators. The portion of flow over the active bed width was 102 cubic feet per second. Channel data for one of the cross sections are listed below:

Bankfull width (W_{bf}) = 18.7 feet
Active bed width (w_a) = 15.3 feet
Slope (s) = 0.0142 feet/feet
Hydraulic radius for the active channel during bankfull flow (R_{bf}) = 1 foot
D_{84} = 120 mm (0.39 feet)
D_{50} = 52 mm (0.17 feet)
D_{16} = 27 mm (0.089 feet)

Determine whether the D_{84} particle moves at bankfull flow at this cross section.

Modified critical shear stress equation

Find critical shear stress for D_{84} using equation E.6:

τ_{D50} = 0.050 (from table E.1) for 52 mm particles

$$\tau_{ci} = 102.6\ \tau_{D50}\ D_i^{0.3}\ D_{50}^{0.7} \qquad \text{(equation E.6)}$$

$$\tau_{c\text{-}D84} = 102.6(0.050)(0.39\ ft)^{0.3}(0.17\ ft)^{0.7} = 1.12\ lb/ft^2$$

Find the average boundary shear stress in the reference reach at bankfull flow (τ_{bf}) using equation E.1:

$$\tau = \gamma RS \qquad \text{(equation E.1)}$$

$$\tau_{bf} = (62.4\ lb/ft^2)(1\ ft)(0.0142) = 0.90\ lb/ft^2$$

The D_{84} particle is stable at bankfull flow because because $\tau_{c\text{-}D84}$ (1.12 lb/ft²) is greater than τ_{bf} (0.90 lb/ft²)

The D_{84} particle size is stable at bankfull flow.

How well does the modified critical shear stress equation apply here?

- $D_{84}/D_{50} = 2.3$, which is much less than 30.
- Slope < 5 percent.
- Channel unit is a riffle.
- D_{84} particle size of 120 millimeter is between the range of 10 and 250 millimeters.

Conclusion: The modified critical shear stress equation (equation E.6) is applicable to this stream.

Critical unit discharge equation

Find the critical unit discharge for D_{50} ($q_{c\text{-}D50}$) using equation E.7:

$$q_{c\text{-}D50} = \frac{0.15\, g^{0.5}\, D_{50}^{1.5}}{S^{1.12}} \qquad \text{(equation E.7)}$$

$$q_{c\text{-}D50} = \frac{(0.15)(32.2\ \text{ft/s}^2)^{0.5}\,(0.17\ \text{ft})^{1.5}}{0.0142^{1.12}} = 7.0\ \text{cfs/ft}$$

Calculate b (which quantifies the range in particle sizes) using equation E.9:

$$b = 1.5(D_{84}/D_{16})^{-1} \qquad \text{(equation E.9)}$$

$$b = 1.5(0.39\ \text{ft}/0.089\ \text{ft})^{-1} = 0.34$$

Find critical unit discharge for D_{84} ($q_{c\text{-}D84}$) using equation E.8:

$$q_{ci} = q_{c\text{-}D50}\,(D_i/D_{50})^{b} \qquad \text{(equation E.8)}$$

$$q_{c\text{-}D84} = 7\ \text{cfs/ft}(0.39\ \text{ft}/0.17\ \text{ft})^{0.342} = 9.3\ \text{cfs/ft}$$

Calculate unit discharge in the reference reach active channel at bankfull flow using equation 2:

$$q = Q/w_a \qquad \text{(equation E.2)}$$

$$q = Q_{bf\text{-}active\ ch}/w_a = 102\ \text{cfs}/15.3\ \text{ft} = 6.7\ \text{cfs/ft}$$

The D_{84} particle is stable at bankfull flow because $q_{c\text{-}D84}$ (9.3 cfs/ft) is greater than $q_{bf\text{-}active\ ch}$ (6.7 cfs/ft). The results for critical unit discharge agree with those of the modified critical shear stress equation.

Is the critical unit discharge equation (equation E.8) appropriate for this stream?

- Slope > 1 percent.
- Channel unit is a riffle.
- D_{16}, D_{50}, and D_{84} are smaller than the particle sizes used to develop the critical unit discharge equation for D_{84}.
- $R_{bf}/D_{50} = 5.9$, which is < 10 (low relative submergence).

Figure E.4 shows there are some scattered large roughness elements (logs and rocks) that appear to be higher than 1-foot (R_{bf}) above the streambed. Conclusion: The critical unit discharge equation (equation E.8) should be used with caution since particle sizes are outside the range of particle sizes used to develop the equation.

E.3 SEDIMENT MOBILITY/STABILITY ANALYSIS EXAMPLE: SCHAFER CREEK TRIBUTARY

This example applies the modified critical shear stress and critical unit discharge approaches to a stream-simulation design on a tributary of Schafer Creek in the Olympic National Forest, Washington. The purpose is to (1) evaluate whether the stream-simulation design bed would have similar mobility/stability as the reference-reach channel and (2) adjust the stream-simulation design so that it has similar mobility and stability as the reference-reach channel. At this site, sediment mobility was actually evaluated for several reaches within the **project profile** to determine the range of sediment mobility; however, this example limits the discussion to comparing sediment mobility/stability between the stream-simulation design bed and the reference-reach channel.

E.3.1 Channel and Road-stream Crossing Background Information

The channel upstream and downstream from the road crossing has a plane-bed to pool-riffle morphology and is slightly to moderately confined with greater channel confinement downstream from the crossing. The channel upstream and downstream from the road-stream crossing has bankfull widths ranging between 5.5 and 7.6 meters (18 to 25 feet), pool residual depths ranging between 0.3 and 0.5 meters (1.0 to 1.6 feet), and channel gradients ranging between 1 and 2 percent (figure E.5). The channel-bed surface is composed primarily of gravel- and cobble-sized sediment. The channel bed is moderately to well armored; the subarmor layer consists of a poorly sorted mixture of cobbles, gravels, and sands. Channel **bed structures** in the riffles and plane-bed channel segments consist primarily of **transverse bars** or rock clusters composed of cobbles and small boulders.

The existing culvert at the crossing is undersized, in a deteriorated condition, and is a partial **barrier** to **anadromous** fish at various life stages and flows (figure E.6). The culvert is a round corrugated pipe with a diameter of 1.52 meters (5 feet) diameter and a length of 30.5 meters (100 feet). There is a 0.4-meter (1.3-foot) drop at the culvert outlet and the associated plunge pool has a residual pool depth of 2.1 meters (6.9 feet) which is four times deeper than other pools along the channel (figure E.5). Sediment accumulation at the culvert inlet extends about 25-meters (82 feet) upstream from the culvert (figure E.5). Based on evidence such as increased bank heights, undercut banks, and localized bank failures, the channel downstream from the road-stream crossing has incised about 0.5 to 1.0 meter (1.6 to 3.3 feet) (figure E.5).

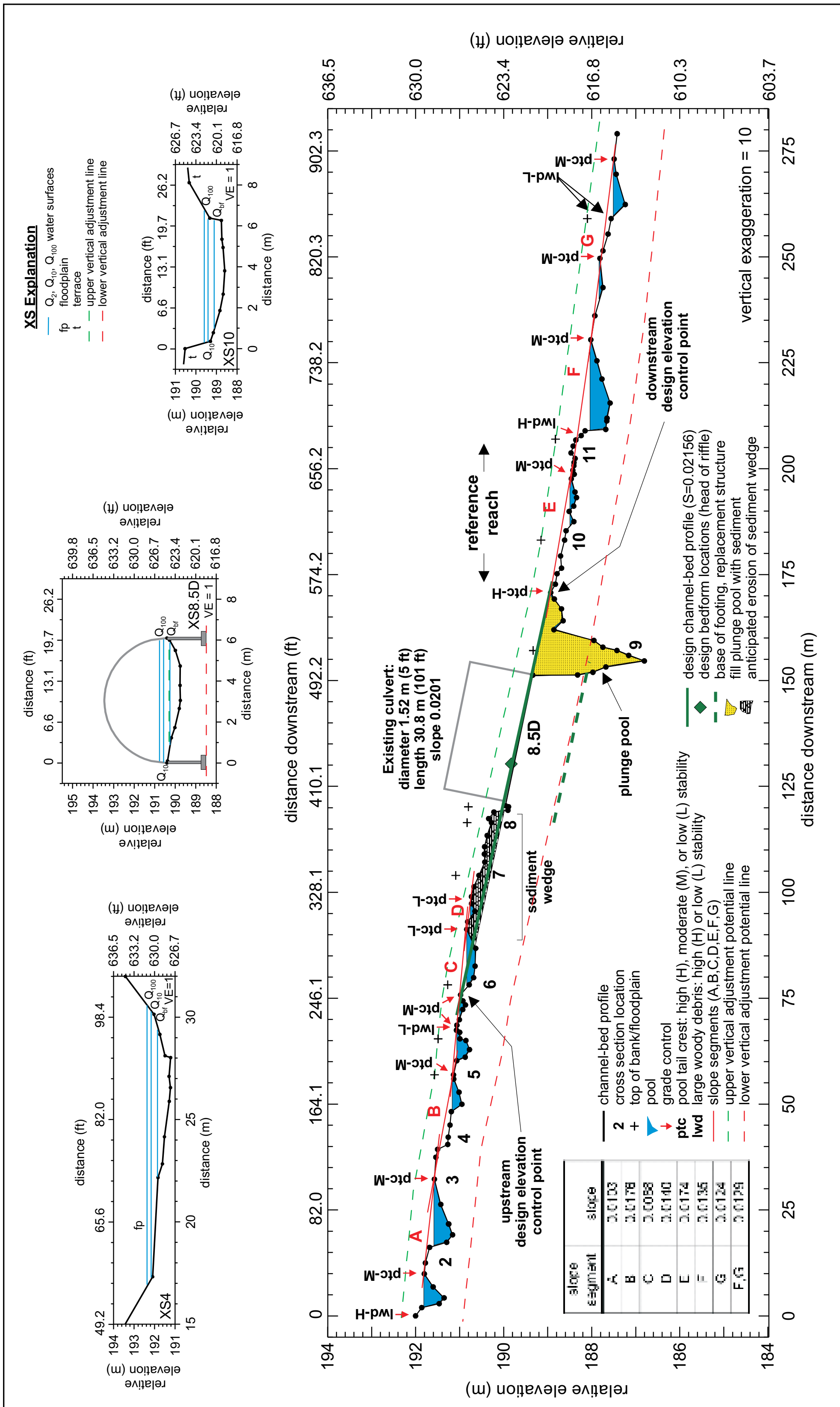

Figure E.5—A longitudinal profile and selected cross sections showing channel characteristics and dimensions along the Schafer Creek tributary.

Figure E.6—The culvert outlet and the upper segment of the reference-reach channel at the Schafer Creek tributary. Note the perch of the culvert outlet and its size relative to the channel.

The Schafer Creek tributary site is an example of a common situation where an undersized culvert has created a moderate plunge at its outfall and the downstream channel has incised. In essence, the existing undersized culvert is acting as a **nickpoint** preventing **channel incision** from continuing upstream. In situations such as this, reestablishing geomorphic continuity between the upstream and downstream channels requires either a stream-simulation channel that is slightly steeper than the reference reach or extensive restoration of the downstream channel to raise the channel bed.

A downstream reference reach (XS10) was chosen because the stream is slightly steeper, more confined, and the bed is somewhat coarser than other segments of the channel; these characteristics are needed inside the replacement structure. The channel geometry and channel-bed sediment characteristics from XS10 were used to develop a preliminary design for the stream-simulation channel bed. The replacement structure for this crossing was a 6.1-meter (20-feet)-wide open-bottom arch; this width is similar to the bankfull width at XS10. The preliminary stream-simulation channel-bed design slope is 0.0216, which is 24-percent steeper than the reference-reach channel slope (figure E.5). The difference in slopes makes it important to check for similar mobility.

Both the modified critical shear stress and critical unit discharge approaches are applicable at this site based on channel characteristics and both were used in the sediment mobility/stability analyses for comparison (tables E.2 and E.3). The step-backwater model HEC-RAS was used to model flow hydraulics for a range of discharges between just below bankfull to just above the Q_{100} discharge. Using geomorphic indicators for bankfull and the HEC-RAS model, bankfull discharge was estimated to be 3 cubic meters per second (m^3/s). Regional regression equations were used to predict the discharges for the Q_2, Q_{10}, Q_{50}, and Q_{100} floods. Selected hydraulic parameters from the model used in the sediment mobility/stability analyses are summarized in tables E.2 and E.3.

E.3.2 Modified Critical Shear Stress Approach

Results from the modified critical shear stress analysis show that the D_{84} particle size of 160 millimeters in both the preliminary stream-simulation design channel and reference-reach channel has a critical shear stress ($\tau_{c\text{-}D84}$) of 81 N/m^2 (table E.2A-B). In the reference-reach channel the D_{84} particle size is mobilized at a lower discharge of about 5 m^3/s when the active channel boundary shear stress is 82 N/m^2 (table E.2A and figure E.7). In contrast, the D_{84} particle size in the design channel is mobilized at a discharge of about 4 m^3/s when the active channel boundary shear stress is 85 N/m^2 (table E.2B and figure E.7). The steeper slope in the preliminary design channel is the primary reason for the higher shear stresses when compared to the reference-reach channel for a given discharge. At this site, the project team decided not to reconfigure the channel to reduce the slope through the crossing. Instead they opted to increase the D_{50} and D_{84} particle sizes to achieve entrainment at the same discharge in both the design channel and reference-reach channel. The D_{50} and D_{84} particles were increased in size by 20 percent from 95 to 114 millimeters and 160 to 192 millimeters, respectively (table E.2C and figure E.7). These sediment-size adjustments increased the critical shear stress for the D_{84} particle ($\tau_{c\text{-}D84}$) in the stream-simulation design channel from 81 N/m^2 to 97 N/m^2. The changes are within the acceptable range of 25-percent difference in slope and particle size between the stream simulation and reference channels.

Table E.2—Summary of flow hydraulics and particle mobility/stability for the reference-reach channel and design channel using the modified critical shear stress approach

Hydraulics												Particle Mobility/Stability					
Recurrence Interval	Discharge, Q (m^3/s)	Floodplain n value	Channel n value	Channel slope S_c	Energy slope S_e	Total flow width W_t (m)	Channel flow width W_c (m)	Total hydraulic radius R_t (m)	Channel hydraulic radius R_c (m)	Total boundary shear stress τ_t (N/m^2)[a]	Channel boundary shear stress τ_c (N/m^2)[b]	D_{50} (mm)	D_{84} (mm)	Angle of repose ϕ	Shield's entrainment for D_{50} τ_{D50}	Critical shear stress to entrain D_{84} particle size $\tau_{c\text{-}D84}$ (N/m^2) [d]	D_{84} Particle mobile (yes /no)
A. Reference reach cross section: XS10																	
	2.00	0.150	0.050	0.0174	0.0174	5.19	5.19	0.30	0.30	51	51	95	160	42	0.045	81	no
	3.00	0.150	0.050	0.0174	0.0174	5.55	5.55	0.37	0.37	63	63	95	160	42	0.045	81	no
	4.00	0.150	0.050	0.0174	0.0174	5.81	5.81	0.43	0.43	73	74	95	160	42	0.045	81	no
2	4.20	0.150	0.050	0.0174	0.0174	5.86	5.86	0.44	0.44	75	75	95	160	42	0.045	81	no
	5.00	0.150	0.050	0.0174	0.0174	6.01	6.01	0.47	0.48	80	82	95	160	42	0.045	81	yes
	6.00	0.150	0.050	0.0174	0.0174	6.10	6.01	0.52	0.53	89	91	95	160	42	0.045	81	yes
10	6.70	0.150	0.050	0.0174	0.0174	6.18	6.01	0.54	0.57	92	97	95	160	42	0.045	81	yes
	7.00	0.150	0.050	0.0174	0.0174	6.22	6.01	0.55	0.58	94	99	95	160	42	0.045	81	yes
25	7.80	0.150	0.050	0.0174	0.0174	6.30	6.01	0.58	0.62	99	106	95	160	42	0.045	81	yes
	8.00	0.150	0.050	0.0174	0.0174	6.32	6.01	0.59	0.63	101	108	95	160	42	0.045	81	yes
50	8.80	0.150	0.050	0.0174	0.0174	6.40	6.01	0.61	0.67	104	114	95	160	42	0.045	81	yes
	9.00	0.150	0.050	0.0174	0.0174	6.42	6.01	0.62	0.67	106	114	95	160	42	0.045	81	yes
100	9.90	0.150	0.050	0.0174	0.0174	6.51	6.01	0.65	0.71	111	121	95	160	42	0.045	81	yes
	10.00	0.150	0.050	0.0174	0.0174	6.52	6.01	0.65	0.72	111	123	95	160	42	0.045	81	yes
	11.00	0.150	0.050	0.0174	0.0174	6.61	6.01	0.68	0.76	116	130	95	160	42	0.045	81	yes

[a] $\tau_t = \gamma\ R_t\ S_e$ (similar to equation E.1).

[b] $\tau_c = \gamma\ R_c\ S_e$ (similar to equation E.1).

[c] τ_{D50} was assumed to be 0.045. Alternative values could be obtained from table E.1.

[d] $\tau_{c\text{-}D84} = 16170\ (\tau^*_{D50})\ (D_{84})^{0.3}\ (D_{50})^{0.7}$ (similar to equation E.5).

Table E.2—Summary of flow hydraulics and particle mobility/stability for the reference-reach channel and design channel using the modified critical shear stress approach (continued)

Hydraulics												Particle Mobility/Stability					
Recurrence Interval	Discharge, Q (m^3/s)	Floodplain n value	Channel n value	Channel slope S_c	Energy slope S_e	Total flow width W_t (m)	Channel flow width W_c (m)	Total hydraulic radius R_t (m)	Channel hydraulic radius R_c (m)	Total boundary shear stress τ_t (N/m^2)[a]	Channel boundary shear stress τ_c (N/m^2)[b]	D_{50} (mm)	D_{84} (mm)	Angle of repose ϕ	Shield's entrainment for D_{50} τ_{D50}	Critical shear stress to entrain D_{84} particle size $\tau_{c\text{-}D84}$ (N/m^2)[d]	D_{84} Particle mobile (yes /no)
B. Preliminary stream simulation design channel: XS8.5D																	
	2.00	0.028	0.050	0.0216	0.0212	4.67	4.67	0.31	0.31	66	64	95	160	42	0.045	81	no
	3.00	0.028	0.050	0.0216	0.0211	5.49	5.47	0.36	0.36	76	75	95	160	42	0.045	81	no
	4.00	0.028	0.050	0.0216	0.0212	6.02	5.90	0.40	0.41	85	85	95	160	42	0.045	81	yes
2	4.20	0.028	0.050	0.0216	0.0212	6.07	5.90	0.41	0.42	87	87	95	160	42	0.045	81	yes
	5.00	0.028	0.050	0.0216	0.0211	6.09	5.90	0.45	0.47	95	97	95	160	42	0.045	81	yes
	6.00	0.028	0.050	0.0216	0.0211	6.07	5.90	0.49	0.52	104	107	95	160	42	0.045	81	yes
10	6.70	0.028	0.050	0.0216	0.0210	6.06	5.90	0.52	0.56	110	115	95	160	42	0.045	81	yes
	7.00	0.028	0.050	0.0216	0.0210	6.06	5.90	0.53	0.57	112	117	95	160	42	0.045	81	yes
25	7.80	0.028	0.050	0.0216	0.0209	6.05	5.90	0.56	0.61	119	125	95	160	42	0.045	81	yes
	8.00	0.028	0.050	0.0216	0.0209	6.05	5.90	0.57	0.62	121	127	95	160	42	0.045	81	yes
50	8.80	0.028	0.050	0.0216	0.0209	6.04	5.90	0.59	0.66	125	135	95	160	42	0.045	81	yes
	9.00	0.028	0.050	0.0216	0.0209	6.04	5.90	0.60	0.67	127	137	95	160	42	0.045	81	yes
100	9.90	0.028	0.050	0.0216	0.0208	6.02	5.90	0.63	0.71	133	145	95	160	42	0.045	81	yes
	10.00	0.028	0.050	0.0216	0.0208	6.02	5.90	0.63	0.71	133	145	95	160	42	0.045	81	yes
	11.00	0.028	0.050	0.0216	0.0208	6.01	5.90	0.66	0.75	140	153	95	160	42	0.045	81	yes

[a] $\tau_t = \gamma\ R_t\ S_e$ (similar to equation E.1).

[b] $\tau_c = \gamma\ R_c\ S_e$ (similar to equation E.1).

[c] τ_{D50} was assumed to be 0.045. Alternative values could be obtained from table E.1.

[d] $\tau_{c\text{-}D84} = 16170\ (\tau^*_{D50})\ (D_{84})^{0.3}\ (D_{50})^{0.7}$ (similar to equation E.5).

Table E.2—Summary of flow hydraulics and particle mobility/stability for the reference-reach channel and design channel using the modified critical shear stress approach (continued)

Hydraulics												Particle Mobility/Stability					
Recurrence Interval	Discharge, Q (m^3/s)	Floodplain n value	Channel n value	Channel slope S_c	Energy slope S_e	Total flow width W_t (m)	Channel flow width W_c (m)	Total hydraulic radius R_t (m)	Channel hydraulic radius R_c (m)	Total boundary shear stress τ_t (N/m^2)[a]	Channel boundary shear stress τ_c (N/m^2)[b]	D_{50} (mm)	D_{84} (mm)	Angle of repose ϕ	Shield's entrainment for D_{50} τ_{D50}	Critical shear stress to entrain D_{84} particle size $\tau_{c\text{-}D84}$ (N/m^2)[d]	D_{84} Particle mobile (yes/no)
C. Adjusted stream simulation design channel: XS8.5D																	
	2.00	0.028	0.050	0.0216	0.0212	4.67	4.67	0.31	0.31	66	64	115	190	42	0.045	97	no
	3.00	0.028	0.050	0.0216	0.0211	5.49	5.47	0.36	0.36	76	75	115	190	42	0.045	97	no
	4.00	0.028	0.050	0.0216	0.0212	6.02	5.90	0.40	0.41	85	85	115	190	42	0.045	97	no
2	4.20	0.028	0.050	0.0216	0.0212	6.07	5.90	0.41	0.42	87	87	115	190	42	0.045	97	no
	5.00	0.028	0.050	0.0216	0.0211	6.09	5.90	0.45	0.47	95	97	115	190	42	0.045	97	yes
	6.00	0.028	0.050	0.0216	0.0211	6.07	5.90	0.49	0.52	104	107	115	190	42	0.045	97	yes
10	6.70	0.028	0.050	0.0216	0.0210	6.06	5.90	0.52	0.56	110	115	115	190	42	0.045	97	yes
	7.00	0.028	0.050	0.0216	0.0210	6.06	5.90	0.53	0.57	112	117	115	190	42	0.045	97	yes
25	7.80	0.028	0.050	0.0216	0.0209	6.05	5.90	0.56	0.61	119	125	115	190	42	0.045	97	yes
	8.00	0.028	0.050	0.0216	0.0209	6.05	5.90	0.57	0.62	121	127	115	190	42	0.045	97	yes
50	8.80	0.028	0.050	0.0216	0.0209	6.04	5.90	0.59	0.66	125	135	115	190	42	0.045	97	yes
	9.00	0.028	0.050	0.0216	0.0209	6.04	5.90	0.60	0.67	127	137	115	190	42	0.045	97	yes
100	9.90	0.028	0.050	0.0216	0.0208	6.02	5.90	0.63	0.71	133	145	115	190	42	0.045	97	yes
	10.00	0.028	0.050	0.0216	0.0208	6.02	5.90	0.63	0.71	133	145	115	190	42	0.045	97	yes
	11.00	0.028	0.050	0.0216	0.0208	6.01	5.90	0.66	0.75	140	153	115	190	42	0.045	97	yes

[a]. $\tau_t = \gamma\ R_t\ S_e$ (similar to equation E.1)

[b]. $\tau_c = \gamma\ R_c\ S_e$ (similar to equation E.1)

[c]. τ_{D50} was assumed to be 0.045. Alternative values could be obtained from table E.1.

[d]. $\tau_{c\text{-}D84} = 16170\ (\tau^*_{D50})\ (D_{84})^{0.3}\ (D_{50})^{0.7}$ (similar to equation E.5).

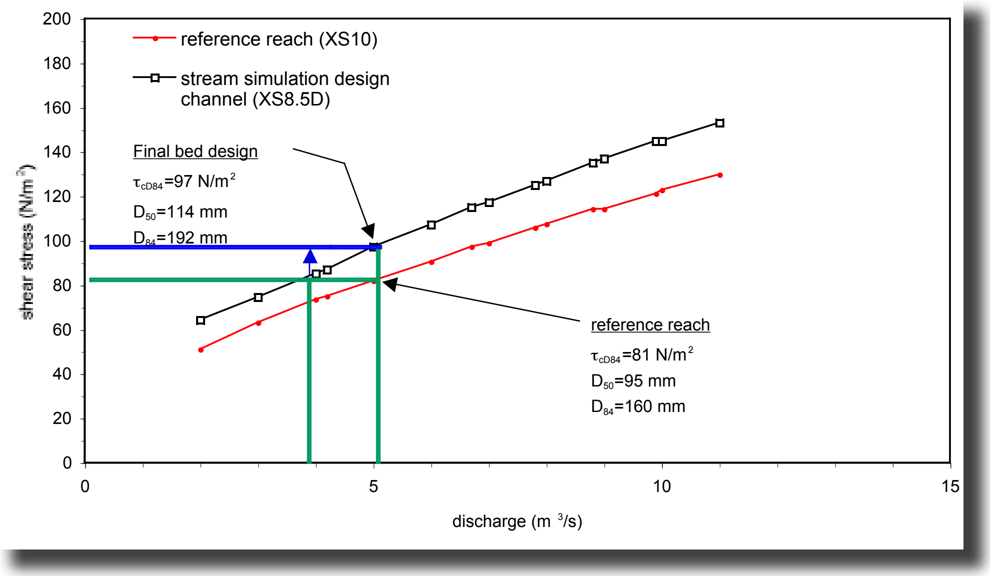

Figure E.7—Plot of shear stress versus discharge showing when the D_{84} particle is mobilized in the reference-reach channel and the preliminary stream-simulation design channel. To achieve similar D_{84} particle mobility in the stream-simulation design channel at the same flow as the reference-reach channel, the D_{50} and D_{84} particle sizes were increased in size by 20 percent to 114 mm and 192 mm, respectively.

E.3.3 Critical Unit Discharge Approach

Results from the critical unit discharge analysis show that the D_{84} particle size of 160 millimeters has a critical unit discharge ($q_{c\text{-}D84}$) of 1.59 m^2/s in the reference-reach channel, whereas it is 1.25 m^2/s in the preliminary stream-simulation design channel (table E.3A-B). The lower critical unit discharge in the design channel indicates the D_{84} particle size will mobilize at lower discharges when compared to the reference-reach channel because of the steeper slope (figure E.8). The D_{84} particle size is mobilized at a discharge of about 7.8 m^3/s in the reference-reach channel, whereas it is mobilized at a discharge of 5.6 m^3/s in the preliminary design channel. The unit discharges for the various discharges are similar between the reference-reach channel and the design-channel reach because they have similar active channel widths and entrenchment ratios (table E.3 and figure E.8). To achieve similar sediment mobility in the stream-simulation design channel at the same discharge as the reference-reach channel, the D_{16}, D_{50}, and D_{84} particle sizes were increased from 43 to 50 millimeters, 95 to 112 millimeters, and 160 to 188 millimeters, respectively (table E.3C and figure E.8). These sediment size adjustments increased the critical unit discharge for the D_{84} particle ($q_{c\text{-}D84}$) in the stream-simulation design channel from 1.25 m^2/s to 1.59 m^2/s.

Table E.3—Summary of flow hydraulics and particle mobility/stability for the reference-reach channel and design channel using the critical unit discharge approach

Hydraulics										Particle Mobility/Stability						
Recur-rence Interval	Dis-charge, Q (m^3/s)	Active Channel width discharge Q_a(m^3/s)	Flood plain n value	Channel n value	Total flow width W_t (m)	Active channel width W_a (m)	Total unit discharge q_t(m^2/s)[a]	Active channel unit discharge q_a(m^2/s)[b]	Channel slope S_c	D_{16} (mm)	D_{50} (mm)	D_{84} (mm)	Particle size range measure b[c]	Critical unit discharge for D_{50} $q_{c\text{-}D50}$ (m^2/s)[d]	Critical unit discharge to entrain D_{84} particle size $q_{c\text{-}D84}$ (m^2/s)[e]	D_{84} particle mobile (yes/no)
A. Reference reach cross section: XS10																
	2.00	1.96	0.15	0.05	5.14	4.40	0.39	0.45	0.0174	43	95	160	0.40	1.29	1.59	no
	3.00	2.89	0.15	0.05	5.46	4.40	0.55	0.66	0.0174	43	95	160	0.40	1.29	1.59	no
	4.00	3.80	0.15	0.05	5.72	4.40	0.70	0.86	0.0174	43	95	160	0.40	1.29	1.59	no
2	4.20	3.98	0.15	0.05	5.76	4.40	0.73	0.90	0.0174	43	95	160	0.40	1.29	1.59	no
	5.00	4.69	0.15	0.05	5.93	4.40	0.84	1.06	0.0174	43	95	160	0.40	1.29	1.59	no
	6.00	5.54	0.15	0.05	6.04	4.40	0.99	1.26	0.0174	43	95	160	0.40	1.29	1.59	no
10	6.70	6.12	0.15	0.05	6.12	4.40	1.09	1.39	0.0174	43	95	160	0.40	1.29	1.59	no
	7.00	6.37	0.15	0.05	6.15	4.40	1.14	1.45	0.0174	43	95	160	0.40	1.29	1.59	no
25	7.80	7.03	0.15	0.05	6.24	4.40	1.25	1.60	0.0174	43	95	160	0.40	1.29	1.59	yes
	8.00	7.19	0.15	0.05	6.26	4.40	1.28	1.63	0.0174	43	95	160	0.40	1.29	1.59	yes
50	8.80	7.85	0.15	0.05	6.34	4.40	1.39	1.78	0.0174	43	95	160	0.40	1.29	1.59	yes
	9.00	8.01	0.15	0.05	6.36	4.40	1.42	1.82	0.0174	43	95	160	0.40	1.29	1.59	yes
100	9.90	8.74	0.15	0.05	6.44	4.40	1.54	1.99	0.0174	43	95	160	0.40	1.29	1.59	yes
	10.00	8.82	0.15	0.05	6.45	4.40	1.55	2.00	0.0174	43	95	160	0.40	1.29	1.59	yes
	11.00	9.62	0.15	0.05	6.55	4.40	1.68	2.19	0.0174	43	95	160	0.40	1.29	1.59	yes

[a] $q_t = Q / W_t$ (similar to equation E.2).

[b] $q_a = Q_a / W_a$ (similar to equation E.2).

[c] $b = 1.5(D_{84}/D_{16})^{-1}$ (equation E.9).

[d] $q_{c\text{-}D50} = 0.15\ (g)^{0.5}\ (D_{50})^{1.5}\ (S)^{-1.12}$ (equation E.7).

[e] $q_{c\text{-}D84} = q_{c\text{-}D50}\ (D_{84}/D_{50})_b$ (similar to equation E.8).

Table E.3—Summary of flow hydraulics and particle mobility/stability for the reference-reach channel and design channel using the critical unit discharge approach (continued)

Hydraulics									Particle Mobility/Stability							
Recurrence Interval	Discharge, Q (m^3/s)	Active Channel width discharge Q_a (m^3/s)	Flood plain n value	Channel n value	Total flow width W_t (m)	Active channel width W_a (m)	Total unit discharge q_t (m^2/s)[a]	Active channel unit discharge q_a (m^2/s)[b]	Channel slope S_c	D_{16} (mm)	D_{50} (mm)	D_{84} (mm)	Particle size range measure b[c]	Critical unit discharge for D_{50} $q_{c\text{-}D50}$ (m^2/s)[d]	Critical unit discharge to entrain D_{84} particle size $q_{c\text{-}D84}$ (m^2/s)[e]	D_{84} particle mobile (yes/no)
B. Stream simulation preliminary design: XS8.5D																
	2.00	1.99	0.15	0.05	4.35	4.11	0.46	0.48	0.0216	43	95	160	0.40	1.01	1.25	no
	3.00	2.96	0.15	0.05	4.90	4.27	0.61	0.69	0.0216	43	95	160	0.40	1.01	1.25	no
	4.00	3.89	0.15	0.05	5.54	4.27	0.72	0.91	0.0216	43	95	160	0.40	1.01	1.25	no
2	4.20	4.08	0.15	0.05	5.62	4.27	0.75	0.95	0.0216	43	95	160	0.40	1.01	1.25	no
	5.00	4.77	0.15	0.05	6.03	4.27	0.83	1.12	0.0216	43	95	160	0.40	1.01	1.25	no
	6.00	5.61	0.15	0.05	6.09	4.27	0.99	1.31	0.0216	43	95	160	0.40	1.01	1.25	yes
10	6.70	6.18	0.15	0.05	6.08	4.27	1.10	1.45	0.0216	43	95	160	0.40	1.01	1.25	yes
	7.00	6.43	0.15	0.05	6.07	4.27	1.15	1.51	0.0216	43	95	160	0.40	1.01	1.25	yes
25	7.80	7.09	0.15	0.05	6.06	4.27	1.29	1.66	0.0216	43	95	160	0.40	1.01	1.25	yes
	8.00	7.25	0.15	0.05	6.06	4.27	1.32	1.70	0.0216	43	95	160	0.40	1.01	1.25	yes
50	8.80	7.89	0.15	0.05	6.05	4.27	1.45	1.85	0.0216	43	95	160	0.40	1.01	1.25	yes
	9.00	8.06	0.15	0.05	6.05	4.27	1.49	1.89	0.0216	43	95	160	0.40	1.01	1.25	yes
100	9.90	8.78	0.15	0.05	6.04	4.27	1.64	2.06	0.0216	43	95	160	0.40	1.01	1.25	yes
	10.00	8.86	0.15	0.05	6.04	4.27	1.66	2.07	0.0216	43	95	160	0.40	1.01	1.25	yes
	11.00	9.65	0.15	0.05	6.03	4.27	1.82	2.26	0.0216	43	95	160	0.40	1.01	1.25	yes

[a] $q_t = Q / W_t$ (similar to equation E.2).

[b] $q_a = Q_a / W_a$ (similar to equation E.2).

[c] $b = 1.5(D_{84}/D_{16})^{-1}$ (equation E.9).

[d] $q_{c\text{-}D50} = 0.15\,(g)^{0.5}\,(D_{50})^{1.5}\,(S)^{-1.12}$ (equation E.7).

[e] $q_{c\text{-}D84} = q_{c\text{-}D50}\,(D_{84}/D_{50})_b$ (similar to equation E.8).

Table E.3—Summary of flow hydraulics and particle mobility/stability for the reference-reach channel and design channel using the critical unit discharge approach (continued)

Hydraulics								Particle Mobility/Stability							
Recurrence Interval	Discharge, Q (m^3/s)	Active Channel width discharge Q_a (m^3/s)	Flood plain n value	Channel n value	Total flow width W_t (m)	Active channel width W_a (m)	Total unit discharge q_t (m^2/s)[a]	Active channel unit discharge q_a (m^2/s)[b]	Channel slope S_c	D_{16} (mm)	D_{50} (mm)	D_{84} (mm)	Particle size range measure b[c]	Critical unit discharge for D_{50} $q_{c\text{-}D50}$ (m^2/s)[d]	Critical unit discharge to entrain D_{84} particle size $q_{c\text{-}D84}$ (m^2/s)[e]	D_{84} particle mobile (yes/no)
C. Adjusted stream simulation design channel: XS8.5D																
	2.00	1.99	0.15	0.05	4.35	4.11	4.11	0.48	0.0216	50	112	188	0.40	1.29	1.59	no
	3.00	2.96	0.15	0.05	4.90	4.27	4.27	0.69	0.0216	50	112	188	0.40	1.29	1.59	no
	4.00	3.89	0.15	0.05	5.54	4.27	4.27	0.91	0.0216	50	112	188	0.40	1.29	1.59	no
2	4.20	4.08	0.15	0.05	5.62	4.27	4.27	0.95	0.0216	50	112	188	0.40	1.29	1.59	no
	5.00	4.77	0.15	0.05	6.03	4.27	4.27	1.12	0.0216	50	112	188	0.40	1.29	1.59	no
	6.00	5.61	0.15	0.05	6.09	4.27	4.27	1.31	0.0216	50	112	188	0.40	1.29	1.59	no
10	6.70	6.18	0.15	0.05	6.08	4.27	4.27	1.45	0.0216	50	112	188	0.40	1.29	1.59	no
	7.00	6.43	0.15	0.05	6.07	4.27	4.27	1.51	0.0216	50	112	188	0.40	1.29	1.59	no
25	7.80	7.09	0.15	0.05	6.06	4.27	4.27	1.66	0.0216	50	112	188	0.40	1.29	1.59	yes
	8.00	7.25	0.15	0.05	6.06	4.27	4.27	1.70	0.0216	50	112	188	0.40	1.29	1.59	yes
50	8.80	7.89	0.15	0.05	6.05	4.27	4.27	1.85	0.0216	50	112	188	0.40	1.29	1.59	yes
	9.00	8.06	0.15	0.05	6.05	4.27	4.27	1.89	0.0216	50	112	188	0.40	1.29	1.59	yes
100	9.90	8.78	0.15	0.05	6.04	4.27	4.27	2.06	0.0216	50	112	188	0.40	1.29	1.59	yes
	10.00	8.86	0.15	0.05	6.04	4.27	4.27	2.07	0.0216	50	112	188	0.40	1.29	1.59	yes
	11.00	9.65	0.15	0.05	6.03	4.27	4.27	2.26	0.0216	50	112	188	0.40	1.29	1.59	yes

[a] $q_t = Q / W_t$ (similar to equation E.2).

[b] $q_a = Q_a / W_a$ (similar to equation E.2).

[c] $b = 1.5(D_{84}/D_{16})^{-1}$ (equation E.9).

[d] $q_{c\text{-}D50} = 0.15\,(g)^{0.5}\,(D_{50})^{1.5}\,(S)^{-1.12}$ (equation E.7).

[e] $q_{c\text{-}D84} = q_{c\text{-}D50}\,(D_{84}/D_{50})_b$ (similar to equation E.8).

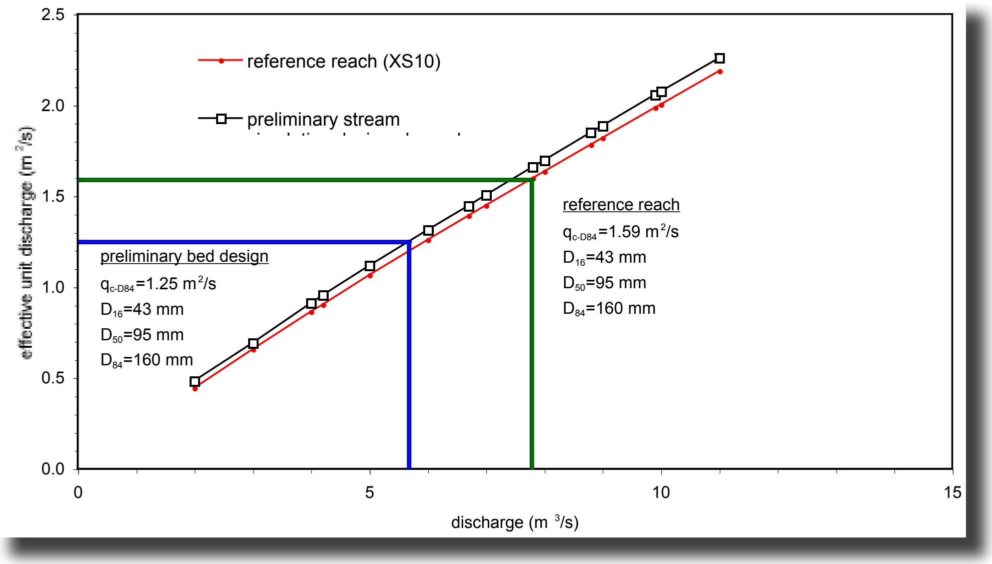

Figure E.8—Plot of unit discharge versus discharge showing when the D_{84} particle is mobilized in the reference-reach channel and the preliminary stream-simulation design channel. To achieve similar D_{84} particle mobility in the stream simulation design channel at the same flow as the reference-reach channel, the D_{16}, D_{50}, and D_{84} particle sizes were increased in size by 17.5 percent to 50 mm, 112 mm, and 188 mm, respectively.

E.3.4 Summary

The modified critical-shear stress and the critical unit-discharge analyses resulted in similar increases in the D_{50} and D_{84} particle sizes in the design channel to achieve similar mobility to the reference-reach channel. Although the discharge at which sediment mobility occurs is different between the two approaches, the direct comparison between the design channel and reference reach channel minimizes any differences in the end result (sizing of sediments) because the assumptions that go into the flow models and sediment mobility/stability analyses are the same between the design channel and reference-reach channel. Thus any errors in our assumptions used in the analyses cancel each other out between the design channel and the reference channel.

E.4 SIZING IMMOBILE KEY PIECES

To size rocks intended to remain in place permanently **(banklines** and some **key features**), start from the size of rocks that appear to be immobile in the reference reach. Use all applicable equations to determine whether that size will move at the **high bed-design flow**. These rocks are often much larger than the rest of the stream-simulation material in which they are embedded, and accurately estimating critical entrainment flow is difficult because most equations do not account for such large size differences. Therefore, use several equations, compare their results, and size the key pieces accordingly.

One analysis procedure has been developed specifically for determining when individual large rocks move (by sliding or rolling) (Fischenich and Seal 2000). The analysis applies to boulders on a flat bed or on a sloped bank, whether embedded or resting on the surface.

Most equations for sizing large permanent rock material in streams are for designing riprap blankets, where rocks are embedded in a layer of other large rocks. These equations are not directly applicable to individual rocks or clusters, but they provide alternative estimates of stable rock sizes for comparison. One standard method is included in HEC-11 (Brown 1989). Rather than individual rock sizes, the method yields the median rock size for a stable riprap gradation in which D_{max} is about 1.5 to 2 times D_{50}. This method uses either shear stress or water velocity to represent the driving forces for entrainment.

Two other riprap models developed by the U.S. Army Corps of Engineers (USACE) may be useful. They were developed from laboratory and analytical work for designing riprap bank protection and rock chutes such as spillways. For full descriptions of the two models, see EM 1110-2-1601 (USACE 1994). The manual is available at http://www.usace.army.mil/publications/eng-manuals/em1110-2-1601.

Both of the USACE riprap models are intended for the design of stable riprap banks and beds with angular rock. Angular rock locks and wedges together thereby resisting rolling and sliding. If using round rock, increase rock size to achieve the level of stability of an angular rock. Abt et al. (1988) studied the difference in stability of angular and rounded rock at slopes from 1 to 20 percent. Although the data set of Abt et al. (1998) is not large, the data indicated the round rock was stable when D_{50} was 40-percent greater than the angular rock.

Both models also assume a riprap gradation that is relatively well sorted compared to a natural streambed. Most graded riprap mixtures have D_{84}/D_{16} ratios less than 3. Although it does not present supporting data, the USACE suggests that the equations do apply to mixtures with D_{84}/D_{16} ratios up to 7. D_{84}/D_{16} in natural cobble/gravel channels can be much higher than this, but until better models are developed, these are useful for estimating stable rock sizes.

Unless supported or buttressed by other rocks of a similar size, individual large rocks may move when the smaller bed mixture around them is scoured. When that happens, the large rock can roll or sink into the bed. If you size rocks using these riprap equations, you will need similar-size rocks to support them. Be sure to buttress individual key pieces, including bank line rocks, with buried footer rocks.