Distance-limited perpendicular distance sampling for coarse woody debris: theory and field results

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Coarse woody debris (CWD) has been identified as an important component in many forest ecosystem processes. Perpendicular distance sampling (PDS) is one of the several efficient new methods that have been proposed for CWD inventory. One drawback of PDS is that the maximum search distance can be very large, especially if CWD diameters are large or the volume factor chosen is small. Here, we propose a modified method, which we call distance-limited PDS, that addresses this limitation. We describe the field procedures, develop new a new estimator, and provide a proof of unbiasedness of that estimator. The new method is compared with line intersect sampling (LIS) and PDS in a field trial in the northeastern United States. The results of the field trial show that this new method requires as little as $\frac{1}{10}$ the sampling effort of LIS in order to achieve comparable variance of the estimates.

Introduction

The past decade has seen substantial interest in efficient methods for collecting information on coarse woody debris (CWD) in forest ecosystems.[1,2] Here, we consider CWD as downed dead wood, not including standing dead trees or fine fuels. CWD is of current interest because it serves as habitat for many plant and animal species,[3] as a carbon and nutrient sink[4] and as a fuel source for forest fires.[5]

Williams and Gove[6] proposed a new method, referred to as perpendicular distance sampling (PDS), for assessing CWD volume. Theoretical and simulation results showed that the PDS estimator was design unbiased and generally had low variance compared with other sampling strategies. Williams *et al.*[7] extended PDS to the assessment of surface area of CWD with similar results, while Williams *et al.*[8] provide additional results for handling practical problems such as slope corrections and multi-stemmed logs. PDS is very efficiently implemented because measurements are required only on CWD pieces whose diameter is borderline for inclusion in the sample. Thus many pieces can be included or excluded from the sample with only a visual inspection.

Survey design and data analysis for PDS uses a 'volume factor' (or a 'surface area factor'), which is a user-defined constant that is directly analogous to the 'basal area factor' familiar from 'variable radius' or horizontal point sampling.[9] A design-unbiased estimator of volume (or surface area) is obtained simply by multiplying the number of logs sampled at each sample point by the appropriate factor. In contrast, with methods such as fixed-area plot sampling, accurate log volumes are required and can be difficult to obtain for irregularly shaped logs.[10] Even for 'regular' logs, volume or surface area estimation requires the use of an approximating formula, such as Smalian's or Newton's formula. In other methods, such as point relascope sampling (PRS),[11] extensions have been developed to yield design-unbiased estimators of volume with little extra effort,[12] where early approximation methods were often tedious.[13] In line intersect sampling (LIS[14,15]) diameter measurements are required on each log to obtain estimates of volume. No approximating formula is needed, but either sample lines or logs must be oriented randomly, or additional measurements will be required for design unbiasedness. In PDS, the measurement of diameter (or cross-sectional area) is required at only one point, and only for those CWD pieces that are judged to be 'borderline' (i.e. not clearly in, or out of, the sample). No approximating formulas or assumptions about log orientation or shape are required.

PDS suffers from two major shortcomings. The first is that, in its original conception, it could not be used to assess the number of logs, nor any other attribute such as length. To address this issue, Williams *et al.*[7] suggested using a circular fixed-area plot co-located at each sample point. Ducey *et al.*[16] suggest an alternative approach based on importance sampling. The other drawback of PDS is that the maximum search distance for large

logs can be great. For example, when using PDS to estimate the volume of logs per hectare, the maximum search distance is proportional to the squared diameter of the largest logs that might be encountered.[6] As we show below, this can force an awkward design decision in the choice of volume factor, in which the risk of failing to detect large logs (and obtaining a biased estimate) must be balanced against the risk of sampling very few logs, yielding unacceptably high variance.

This paper presents a modification of PDS, which we call *distance-limited perpendicular distance sampling*, or DLPDS. As the name implies, this technique allows the user to specify a maximum search distance in advance. One drawback is that a diameter measurement is required on a subset of the sampled logs, although the number of measurements is still far fewer than other existing methods except PDS and, perhaps, critical PRS.[12]
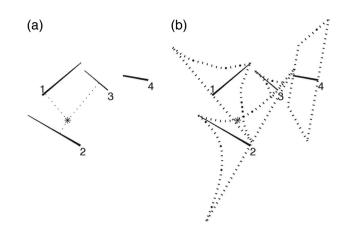
This paper is organized as follows:

- We briefly review PDS.
- We illustrate the conditions under which the search distances for PDS can become unreasonably large, and discuss the challenges to survey design that can arise.
- We give modifications to implement DLPDS, and proofs of design unbiasedness for estimating volume.
- We present results from an initial field trial comparing DLPDS, PDS and LIS for estimating CWD volume.

PDS overview

Suppose we are interested in estimating the total volume or surface area of CWD on a tract of land with area A, ha. (The tract need not be contiguous, but it must be recognizable and its area must be known.) Denote the total CWD volume on the tract as V, m^3. We would like to estimate either V or, equivalently, V/A. There are N pieces of fallen CWD within the boundaries of the tract; each log i contributes v_i to the total V. The total N and the characteristics of the logs (size, shape, position, and orientation) are unknown, and we make no assumptions about them. To estimate $V = \sum_N v_i$, we will locate one or more sample points uniformly at random within the tract. To begin with, we will consider the case of a single sample point.

In PDS, the observer scans from a sample point to the 'perpendicular point' of each piece in the vicinity, if the perpendicular point exists (Figure 1a). The perpendicular point is the point on the piece (or, for a crooked or forked piece, the point on the line or 'needle' connecting the two ends of the piece;[7]) where the piece is perpendicular to the observer's line of sight. The observer then compares the cross-sectional area of the piece at the perpendicular point, $g(h)$, to the distance D from the sample point to the perpendicular point, using a simple multiplication to transform the cross-sectional area to a 'limiting distance' D_L. (Replacing cross-sectional area with circumference leads to a protocol for estimating CWD surface area .[8]) If $D \leq D_L$, the piece is sampled (Figure 1b). In a field application, a visual estimate of D and $g(h)$ (or, for approximately circular pieces, diameter), aided by a chart of D_L against $g(h)$ and diameter for quick comparison, will often allow the observer to discern whether a piece should or should not be sampled without any actual measurement. If

Figure 1 Sampling pieces with PDS. A sample point (*) falls amid a group of pieces, numbered 1-4. The observer sights on each piece at its perpendicular point (see panel *a*). The sample point is past the end of piece number 4; that piece has no perpendicular point and is not sampled. The inclusion zone for each piece is shown in panel *b*. The inclusion zone is the zone within which the perpendicular distance to the piece is less than the critical distance, and the piece is sampled. In this case, pieces 1 and 2 will be sampled, while pieces 3 and 4 will not.

and only if that decision is not clear, the piece is 'borderline' and measurements of D and $g(h)$ are required.

This sampling protocol for PDS defines an 'inclusion zone' for each of the N pieces, where the inclusion zone defines an area about each piece. The piece will be included in the sample whenever the point falls within the inclusion zone. But what is the shape and size of this inclusion zone? Each piece has a central axis of length H. The width of the inclusion zone at any point h ($0 \leq h \leq H$) along H is twice the limiting distance D_L based on piece size at that point, because the piece will be included in the sample from both sides. But recall that $D_L = Kg(h)$, where K is a constant of proportionality that one would choose in advance as part of the survey design. (As we show below, choosing K is analogous to choosing the width of an angle gauge to determine the basal area factor in horizontal point sampling.) The land area covered by the inclusion zone is therefore

$$a = 2K \int_0^H g(h) \, \mathrm{d}h = 2KV.$$

This inclusion area is proportional to the volume of the piece (V) because, as a basic result from calculus, $V = \int_0^H g(h) \, \mathrm{d}h$ is the true volume of the piece.

The probability that a randomly located sample point will fall in the inclusion zone for piece i is $p_i = a_i/(10\,000A)$ (recalling that a_i is in units of m^2 and A is in ha). If n pieces are sampled at a point, design-unbiased estimators of V and V/A are

$$\hat{V} = \sum_{i \in n} \frac{v_i}{p_i}$$

$$= 10\,000A \sum_{i \in n} \frac{v_i}{a_i}$$

$$= 10\,000A \sum_{i \in n} \frac{v_i}{2Kv_i}$$

$$= n \frac{10\,000A}{2K}$$

and

$$\frac{1}{A}\hat{V} = n \frac{10\,000}{2K}.$$

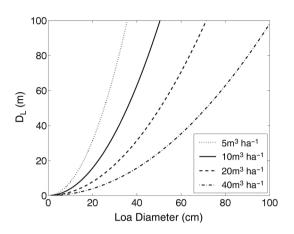Design unbiasedness follows from the fact that $\hat{V}$ is a Horvitz–Thompson estimator.[6]

The estimators of volume and of volume per unit area require only the number of pieces sampled; no diameter or length measurements are needed (except any that are required to accurately determine the inclusion of borderline pieces). The fraction $F = 10\,000/(2K)$ serves as the *volume factor* for PDS.[6] This factor directly converts the number of pieces sampled to an estimate of total volume per hectare, and is analogous to the basal area factor in horizontal point sampling. Because there is a one-to-one relationship between F and K, one may choose either and solve for the other as convenient to the purpose at hand.

The estimators for volume and surface area will be biased downward if portions of the inclusion zones for some pieces fall outside the tract, and no corrective action is taken. Williams and Gove[6] and Ducey *et al.*[17] discuss boundary correction techniques for PDS based on the walkthrough method. Techniques for dealing with sloped terrain, inclined or crooked pieces, and forked pieces are discussed by Williams *et al.*[8]

If a single sample point yields an unbiased estimator $\hat{V}$, then the mean of the estimators from m sample points $\bar{V} = (1/m)\sum_{j=1}^{m}\hat{V}_j$ will also be unbiased. If points are located by simple random sampling, then the usual formulae for variance, standard error and confidence limits of a sample apply directly.

Motivations for DLPDS

The primary design choices in PDS are the selection of a volume factor F (and hence a value for K), selection of an overall sampling regime (e.g. simple or stratified random sampling) and total sample size (i.e. number of sample points). A successful choice of volume factor should satisfy two major criteria. First, it should provide a reasonable number of sampled pieces per point, so that the variance between points would be relatively small and the total sample size could be kept manageable. (The average number of pieces sampled per point will be $(10\,000V/A)/F$, and this may be estimated in advance if one has some *a priori* knowledge about V/A.) Second, a successful choice of F would keep K reasonably small, so that the largest (and hence farthest) pieces to be sampled can be detected reliably, and any measurements required for borderline pieces can be done quickly and accurately. These two criteria are in direct conflict: a small value of F increases the average number of pieces sampled (and will typically reduce the between-point variance in the estimates), but will also increase K. A comparable situation exists in horizontal point sampling of standing trees and related techniques for standing dead wood, and has received considerable study.[9,18–20].

Figure 2 Relationship between piece diameter and limiting distance D_L when sampling for volume per hectare using PDS with four different volume factors.

Figure 2 illustrates this conflict, which creates a significant potential weakness for PDS. The challenge is particularly acute when using PDS to sample for volume (as opposed to surface area), because D_L is linear in the cross-sectional area $g(h)$, and therefore quadratic in diameter. Although the exact relationship between D_L when sampling for volume versus surface area would depend on the constants of proportionality (or volume factor and surface area factor) used in the two schemes, and there is no *a priori* reason why these should be identical, the nonlinearity in D_L when sampling for volume suggests the potential for 'runaway' limiting distances as piece sizes become large.

In principle, it should be simple to choose a volume factor based on advance estimates of the maximum piece diameter that might be encountered, and a reasonable bound on the search distance based on expected visibility through the understorey. For example, if one were to conduct an inventory in a very open forest type with a grassy understorey (such as some forest types in western North America), with clear visibility out to 40 m and maximum piece diameters of approximately 40 cm, one might consult Figure 2 and select a volume factor of 20 m³/ha. However, suppose one were sampling in dense, mesic, secondary forests with a heavy understorey. In these conditions, maximum visibility might be 20 m or less, but maximum piece diameters could be 40 cm or larger. Under these circumstances, a volume factor of 40 m³/ha would be an absolute minimum. But V/A will also be quite low, so the expected number of sampled pieces per point will be quite small, and the resulting variance (and therefore required number of sample points) could prove unacceptably large. Decreasing the volume factor might reduce the variance, but with an increasing probability that large pieces will be missed in the field (unless crews spend an inordinate amount of time carefully searching very large areas). The result would likely be a downward bias, caused not by any theoretical failure but by the field error driven by a poor design choice.

This problem will be exacerbated when planning a single inventory protocol over large forest areas with a wide range of stand types. If some stand types contain large-diameter CWD, and some have poor visibility (with the worst case being both

situations simultaneously), no single choice of F is likely to prove satisfactory over all conditions.

In horizontal point sampling, one solution might be to switch to a fixed-radius plot for trees of very large diameter. This effectively 'caps' the search distance. Similarly, in PDS one solution might be to 'cap' the search distance, so that even very large pieces (or portions of pieces) have a D_L that is reasonable given the forest environment and working conditions. However, there is a substantial difference between these solutions for horizontal point sampling and PDS. In the horizontal point sampling case, the inclusion zone remains a simple circle for all trees; the radius of the circle varies with diameter for small trees and is fixed for large trees, but having measured the diameter of a given tree the area of its inclusion zone can be calculated directly, no matter its diameter. In sampling CWD with PDS, some portions of any given log might have small cross section while others have large cross section. The inclusion zone (the area of which is not even calculable from the typical field measurements in PDS) will then depart from any simple relationship with volume, in a potentially complex way that depends on the taper of the piece. As we show in the next section, this requires modification of both the field procedures and estimators for PDS, but unbiased estimation of volume is still possible with limited measurements.

Procedure and estimators for DLPDS

When PDS is conducted with an upper limit or cap on D_L, we call this as DLPDS. We will describe the field procedures first, then give the associated estimators, and finally prove the design unbiasedness of the estimators.
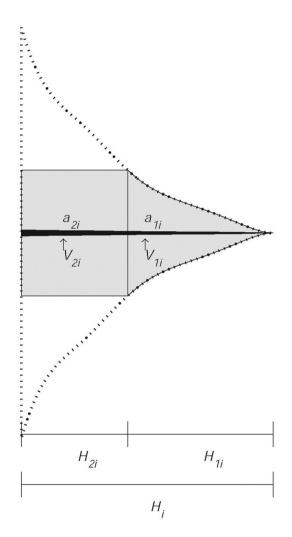
In general, all the assumptions and the description of field implementation are identical between PDS and DLPDS, with one exception: the decision on whether or not to tally the piece involves two possible cases, depending on the cross-sectional area $g(h)$. Let D_{max} be the maximum search distance, and let $g_{max} = D_{max}/K$. In other words, g_{max} is the cross-sectional area at which, in ordinary PDS, $D_L = D_{max}$. The two cases for the sampling decision are as follows:

Case 1. If $g(h) \leq g_{max}$, then $D_L = Kg(h)$ as in ordinary PDS. If $D \leq D_L$, the piece is sampled. In other words, in Case 1 we proceed exactly as for ordinary PDS. Field procedures, including the use of ocular estimation where appropriate and checking of borderline logs, proceed in an identical fashion to PDS as outlined above.

Case 2. If $g(h) > g_{max}$, then $D_L = D_{max}$. The piece is sampled if $D \leq D_{max}$, and ignored if $D > D_{max}$. However, even if the piece is much closer than D_{max}, $g(h)$ must still be measured: $g(h)$ is *required* to determine the contribution of the piece to $\hat{V}$.

The effect of D_{max} on the inclusion zone in DLPDS is illustrated in Figure 3. Having sampled pieces according to the procedure outlined above, a new method of forming estimates of V (or V/A) is required. Let n_1 be the number of pieces sampled in Case 1, and let n_2 be the number of pieces sampled in Case 2. Then, recalling that A is in hectares,

$$\hat{V}_{DLPDS} = n_1 \frac{10\,000A}{2K} + \frac{10\,000A}{2D_{max}} \sum_{i \in n_2} g_i(h)$$

Figure 3 Inclusion zone for volume enlarged to show division of the inclusion zone.

and

$$\frac{1}{A}\hat{V}_{DLPDS} = \frac{10\,000n_1}{2K} + \frac{10\,000}{2D_{max}} \sum_{i \in n_2} g_i(h)$$

provide design-unbiased estimators of V and V/A.

To prove the unbiasedness of $\hat{V}_{DLPDS}$, it is useful to divide each piece and its inclusion zone according to the two cases in the protocol above (Figure 3). Let H_{1i} be the part of H_i that falls in case 1, and let H_{2i} be the part that falls in case 2. Although Figure 3 shows H_{1i} and H_{2i} each as contiguous, that is not strictly required; the two parts may be interspersed on an irregularly tapering piece. Let $v_{1i} = \int_{H_{1i}} g(h)\,dh$ be the portion of v_i found in H_{1i}, and $v_{2i} = \int_{H_{2i}} g(h)\,dh$ be the portion of v_i found in H_{2i}. Likewise, let $a_{1i} = \int_{H_{1i}} 2Kg(h)\,dh = \int_{H_{1i}} 2D_L\,dh$ be the portion of a_i associated with H_{1i}, and $a_{2i} = \int_{H_{2i}} 2Kg_{max}\,dh = 2H_{2i}D_{max}$ be the portion of a_i associated with H_{2i}. Note that $H_i = H_{1i} + H_{2i}$, $v_i = v_{1i} + v_{2i}$, and $a_i = a_{1i} + a_{2i}$.

Let the contribution of piece i to $\hat{V}_{DLPDS}$ be $\hat{v}_i$, such that $\hat{V}_{DLPDS} = \sum_N \hat{v}_i$. Now, piece i fails to appear in the sample count or tally with probability $(1 - p_i) = (A - a_i)/A$, and in that case contributes nothing to $\hat{V}_{DLPDS}$, so $\hat{v}_i = 0$. Log i occurs as a Case 1 tally with probability $p_{1i} = a_{1i}/A$, and contributes $\hat{v}_i = A/2K$ if that occurs. Finally, piece i occurs as a Case 2 tally with probability $p_{2i} = a_{2i}/A$, and in that case contributes $\hat{v} = Ag_i(h)/2D_{max}$. We may write the expected value of $\hat{v}_i$, $E\hat{v}_i$, as

$$E\hat{v}_i = (1 - p_i) \times 0 + p_{1i} \times \frac{A}{2K} + p_{2i} \times E_{h \in H_{2i}} \left[\frac{Ag(h)}{2D_{max}} \right].$$

This simplifies immediately to

$$E\hat{v}_i = \frac{a_{1i}}{2K} + \frac{a_{2i}}{2D_{max}} \times E_{h \in H_{2i}} g(h).$$

Recall that $a_{1i} = \int_{H_{1i}} Kg_i(h) \, dh = 2Kv_{1i}$, so

$$E\hat{v}_i = v_{1i} + \frac{a_{2i}}{2D_{max}} \times E_{h \in H_{2i}} g(h).$$

Now, $a_{2i} = 2H_{2i}D_{max}$, so $a_{2i}/2D_{max} = H_{2i}$. In addition, the probability of sampling $g(h) \in H_{2i}$ is equal to the probability of sampling the corresponding length $h \in H_{2i} = 1/H_{2i}$. Therefore,

$$E_{h \in H_{2i}} g(h) = \frac{1}{H_{2i}} \int_{H_{2i}} g(h) \, dh$$
$$= \frac{1}{H_{2i}} v_{2i}$$

so that

$$H_{2i} E_{h \in H_{2i}} g(h) = v_{2i}$$

Thus,

$$E\hat{v}_i = v_{1i} + v_{2i} = v_i$$

and by the basic properties of expectations,

$$E\hat{V}_{DLPDS} = \sum_{i=1}^{N} E\hat{v}_i = \sum_{i=1}^{N} v_i = V$$

and we see that $\hat{V}_{DLPDS}$ is design unbiased.

Field trial description

To provide a preliminary evaluation of the practicality of PDS and DLPDS, a field trial was conducted comparing these two methods with LIS for volume estimation in seven forest stands. LIS was selected for comparison because, like PDS and DLPDS, a design-unbiased estimator exists for volume per hectare, without resorting to an approximating formula for piece volume. LIS also provides a useful standard because the sample line crosses all pieces to be sampled; thus

there is little opportunity for non-detection bias.[21] All seven study stands were located in southeastern New Hampshire. Portions of one stand (Mendum's Pond) had recently been harvested; this stand was analyzed both in its entirety, and with the sample points divided between the harvested and unharvested portions. Characteristics of the study stands are given in Table 1.

LIS was conducted using a 40 m sample line centred on each sample point, oriented in a random direction. PDS was conducted using volume factors of 42 m³/ha (600 ft³/ac) and 70 m³/ha (1000 ft³/ac). DLPDS used $D_{max} = 20.1$ m (66 ft), with K corresponding to nominal volume factors of 14 m³/ha (200 ft³/ac) and 42 m³/ha (600 ft³/ac). These volume factors and distance limits were selected based on crude initial estimates – both 'optimistic' and 'pessimistic' – of likely maximum piece sizes, of likely visibility (20 m), and an anticipation that V/A would likely be considerably less than 100 m³/ha. The first method performed at a given point was chosen on a rotating basis, and timed with a stopwatch; these times were used to estimate 'missing' times with a regression on the number of pieces sampled. Timing began upon arrival at the sample point and ended at the conclusion of CWD measurement; times reported here do not include travel time between sample points. All methods were performed by a single-person crew at each point; points were divided equally between the two crew members. Both crew members had substantial experience with LIS but had been trained on PDS and DLPDS at only four points prior to beginning work in the study stands.

For the purposes of this survey, all material with $g > 7.5$ cm was defined as CWD. This definition is consistent with the typical definition of 1000-h fuels.

Key results are shown in Table 2 where it can be seen that the average amount of time to implement DLPDS and PDS was approximately 1 min per point, regardless of the volume factor. In comparison, it took between 3.5 to 4 min to implement the LIS sample, in part because the line must be established even if few or no pieces will be sampled.

Because estimates of volume were highly non-normal, we used a bootstrap percentile test (10 000 bootstrap samples) to test for differences between methods within each stand. In nearly all stands, differences between LIS and the PDS and DLPDS methods were not significant ($p \geq 0.10$ for all combinations of LIS with another method), indicating no detectable difference in realized bias. One exception occurred at the West Foss Farm site, where PDS with a 70 m³/ha volume factor showed an unexpected overestimate relative to LIS. This instance may simply be a 'false positive', as no correction is made here for multiple comparisons. The other exception was the harvested portion of the Mendum's Pond stand, where substantial downward bias was observed in both DLPDS and PDS. This bias was also severe enough to be evident for the combined harvested and unharvested Mendum's Pond area. The bias in PDS with a 70 m³/ha volume factor was not statistically significant in this stand, but this is almost certainly due to the large variance in estimates obtained with that method. It is clear that significant care must be exercised whenever logging slash (or substantial input from natural disturbance) is a major contributor both to down woody material and to reduced visibility. Further work may be needed to define successful PDS and/or DLPDS design choices in these difficult working conditions.

Table 1 Characteristics of the seven study stands. The Mendum's Pond stand was analysed both in its entirety, and split into harvested and unharvested portions

Stand name	Description	Live tree basal area (m^2/ha)	Live tree quadratic mean diameter (cm)	Dominant overstorey species
College Woods A	Unmanaged late-successional reserve	32	33	*Pinus strobus, Tsuga canadensis*
College Woods B	Late rotation managed forest	56	43	*Pinus strobus, Fagus grandifolia, Acer rubrum*
East Foss Farm	Young managed forest	27	24	*Pinus strobus, Acer rubrum, Quercus rubra*
West Foss Farm	Young managed forest	21	20	*Pinus strobus, Populus grandidentata, Acer rubrum*
McDonald Managed Area	Mature managed forest	31	32	*Quercus rubra, Pinus strobus, Carya ovata*
McDonald Natural Area	Unmanaged late-successional reserve	40	32	*Pinus strobus, Acer rubrum, Quercus rubra*
Mendums Pond	Two-cohort managed forest; portions recently harvested	28	18	*Tsuga canadensis, Pinus strobus, Quercus rubra, Acer rubrum*

We calculated relative efficiency as

$$E = \frac{\bar{t}s^2}{\bar{t}_{LIS}s^2_{LIS}}$$

where the sample variance (s^2) and average time ($\bar{t}$) in the numerator are for the method being compared with LIS. E can be interpreted as the sampling time required to obtain a desired sample variance for $\hat{V}$, as a fraction of the time required to obtain the same sample variance using LIS.[22,23] A low value of E indicates better efficiency. A brief derivation of E along with alternative measures that might be considered when travel time ought to be charged against each method is given in the Appendix. Table 2 suggests that, at least under these conditions, both PDS and DLPDS provide substantial efficiency gains relative to LIS. Because this efficiency measure does not incorporate bias, efficiency values are not shown whenever bias was statistically significant. The exceptionally low values of E for PDS with a 70 m^3/ha volume factor in the harvested and combined Mendums Pond areas may also be an anomaly.

We note that because our times do not include travel between sample points, E strictly measures the efficiency of CWD sampling *per se*. This implies that travel between sample points is largely a 'sunk cost' determined by logistics and by the objectives of a multiresource inventory. We also note that there is no fixed relationship between the number of sample points and total travel time, especially in the systematic sample layouts that dominate most practical inventories, and that arranging the LIS transects to fall along the routes between sample points and thus to minimize the travel time would violate the randomness assumption underpinning the design unbiasedness of the most commonly used LIS estimators (the 'unconditional estimator,'[15]). However, we also acknowledge that there are situations in which the travel time between points should be charged against the CWD inventory, and that would change the appropriate measure of efficiency. In such cases, a variety of strategies (such as clusters of DLPDS points) might be attractive for optimizing overall efficiency, but full exploration of those alternatives is beyond the scope of this investigation.

DLPDS allowed the use of a lower effective volume factor than would have been feasible in these stands with PDS. For $F = 14$ m^3/ha, the smaller factor employed with DLPDS, $K = 357.3$. Logs within the College Woods Natural Area commonly reach 60 cm in diameter at their basal end, and would have D_L up to 101 m, clearly an unreasonable search distance within a densely forested area. Within many of the other stands, pieces up to 30 cm in diameter are frequent, with D_L up to 25 m. Even this distance exceeds visibility in the younger stands, which often have a dense understorey of shade-tolerant trees and shrubs. By enforcing a maximum search distance of 20.1 m, DLPDS allowed the selection of an effective volume factor that yielded a sample variance quite competitive with LIS in most stands in this study.

Discussion and conclusions

Although the choice of volume or surface area factor can be a challenging problem for PDS under some circumstances, adding a maximum search distance in DLPDS provides an alternative that remains design unbiased. A preliminary field trial confirms that an appropriate value of D_{max} can allow a DLPDS design to have a sample variance competitive with LIS, using an effective volume factor that would not be achievable with ordinary PDS. This field trial, which also represents the first formal field trial for PDS, also shows that both PDS and DLPDS may provide dramatic improvements in efficiency relative to LIS, in part because of substantial reductions in time per sample point. Similarly, Valentine *et al.*[24] report rapid field work in a demonstration of PDS, with an average field time of 2 min or less per sample point.

Where a comprehensive description of CWD including the number of pieces per hectare is required, neither PDS alone nor DLPDS alone will suffice. A natural solution is to combine PDS or DLPDS for rapid estimation of volume per hectare, with a count

Table 2 Results of a field test of LIS and PDS in New Hampshire. Efficiency of LIS is 1.00 by definition; lower efficiency scores indicate improvements in efficiency. For example, an efficiency of 0.50 for a method would indicate that method requires only half the time required by LIS to achieve the same confidence limit widths. Values are indicated with a (*) and efficiency is shown as 'na' when results were significantly different from LIS, indicating probable field bias (bootstrap percentile test, $p < 0.10$)

	LIS (40 m line)	DLPDS 14 m³/ha	DLPDS 42 m³/ha	PDS 42 m³/ha	PDS 70 m³/ha
College Woods A ($n = 21$)					
Estimate (m³/ha)	23.1 ± 8.4	29.1 ± 6.7	31.9 ± 8.6	30.0 ± 8.3	26.7 ± 11.3
C.V. (%)	167	105	124	126	194
Avg. Time (s)	293	77	49	47	43
Efficiency	1.00	0.16	0.17	0.15	0.27
College Woods B ($n = 21$)					
Estimate (m³/ha)	23.5 ± 7.3	15.7 ± 5.3	19.1 ± 8.5	20.0 ± 8.0	26.7 ± 13.2
C.V. (%)	142	153	204	183	227
Avg. Time (s)	269	53	44	44	43
Efficiency	1.00	0.10	0.22	0.20	0.53
East Foss Farm ($n = 24$)					
Estimate (m³/ha)	16.2 ± 4.1	17.8 ± 8.9	22.2 ± 10.9	17.5 ± 7.1	17.5 ± 8.7
C.V. (%)	125	244	239	199	243
Avg. Time (s)	268	51	44	43	39
Efficiency	1.00	0.88	1.13	0.47	0.64
West Foss Farm ($n = 21$)					
Estimate (m³/ha)	20.1 ± 4.2	21.5 ± 5.5	31.2 ± 8.7	30.0 ± 8.3	36.7 ± 10.4*
C.V. (%)	95	117	128	126	130
Avg. Time (s)	299	62	49	47	48
Efficiency	1.00	0.36	0.71	0.62	na
McDonald Managed Area ($n = 34$)					
Estimate (m³/ha)	25.6 ± 5.1	22.5 ± 4.6	25.0 ± 6.7	23.5 ± 6.4	24.7 ± 9.3
C.V. (%)	115	118	157	160	219
Avg. Time (s)	312	67	46	45	42
Efficiency	1.00	0.17	0.26	0.23	0.45
McDonald Natural Area ($n = 17$)					
Estimate (m³/ha)	59.5 ± 13.4	57.3 ± 12.8	66.0 ± 15.8	59.3 ± 13.5	61.7 ± 14.6
C.V. (%)	93	92	99	94	97
Avg. Time (s)	343	94	59	57	59
Efficiency	1.00	0.25	0.24	0.17	0.20
Mendums Pond ($n = 44$)					
Estimate (m³/ha)	22.4 ± 5.1	13.9 ± 3.6*	10.6 ± 3.6*	9.5 ± 3.0*	14.3 ± 4.9
C.V. (%)	150	170	227	209	226
Avg. Time (s)	309	53	41	40	38
Efficiency	1.00	na	na	na	0.09
Mendums Pond Harvested Area ($n = 25$)					
Estimate (m³/ha)	30.3 ± 8.5	15.9 ± 5.8*	12.0 ± 5.7*	10.1 ± 4.4*	16.8 ± 7.3
C.V. (%)	140	182	237	218	218
Avg. Time (s)	332	54	41	40	39
Efficiency	1.00	na	na	na	0.11
Mendums Pond Unharvested ($n = 19$)					
Estimate (m³/ha)	12.1 ± 2.3	11.1 ± 2.8	8.8 ± 4.0	8.8 ± 4.0	11.0 ± 6.0
C.V. (%)	85	108	199	199	237
Avg. Time (s)	280	53	41	40	36
Efficiency	1.00	0.26	0.43	0.42	0.85

on a fixed-area plot for rapid estimation of the number of pieces per hectare, as suggested by Williams *et al.*[7] Preliminary field tests of this approach in montane forests of Colorado and British Columbia are promising. When volume of CWD is the sole survey objective, as in some fuel surveys, or is the major requirement for determining carbon or nutrient content of CWD, as in many biogeochemical studies, both PDS and DLPDS deserve further testing. Alternatively, the approach based on importance sampling suggested by Ducey *et al.*[16] could be used. The extension of DLPDS to the importance sampling framework is straightforward.[25]

Although we did not conduct a field test with the surface area as the target variable, the easy extension of PDS and DLPDS to that situation suggest that further evaluation would be useful. Knowledge of a maximum probable piece diameter, reasonable visibility to such a piece and approximate level of piece volume or surface area per hectare should allow the design of DLPDS sampling approaches that avoid non-detection bias and have low sampling variance. If the results of the field trial presented here prove typical, and the simulation results of Gove *et al.* [25] suggest that they might, DLPDS should provide another useful technique in addition to fixed area plots, LIS, PDS, and critical PRS, for estimation of volume in CWD.

Finally, we note that there is a third alternative to ordinary PDS (in which the limiting distance is a smooth function of piece size) and DLPDS (in which the limiting distance is a mixture of a smooth function over some ranges of the piece size, and a constant in others). The third option is to let the limiting distance be a constant irrespective of piece size. From the basic geometry suggested in Figure 3, one may deduce that this option would give rise to a probability-proportional-to-length sampling scheme for downed wood. The practical and sampling properties of this third option deserve further exploration.[26]

Conflict of interest statement

None declared.

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Appendix

Here, we present a brief derivation of the efficiency measure E, along with a discussion of its assumptions and some alternatives that might be considered when the travel time between sample points should factor into the evaluation of sample efficiency.

We follow Valentine *et al.*[27] in employing a Monte Carlo integration, design-based approach. Suppose we have alternative sampling methods, each of which is design unbiased. We conduct sampling in a region $\mathcal{A}$ of area A to determine the total volume of CWD V within $\mathcal{A}$, drawing sample locations x uniformly at random within $\mathcal{A}$. Let $\hat{v}_s(x)$ denote the estimate of V returned by sample method s when the sample location is x. In the design-based framework, the population of CWD is fixed, so the selection of a sampling method (and any associated parameters) fully determines the function $\hat{v}_s(x)$. Design unbiasedness of the method and its estimator implies

$$\int_{\mathcal{A}} \hat{v}_s(x)\, \mathrm{d}x = V$$

If n sample locations are chosen, then we use the mean of the $\hat{v}_s(x)$ as our estimate $\bar{V}_s$, and design unbiasedness follows immediately,[27,28]

It also follows from elementary sampling theory[28] that the variance of $\bar{V}_s$ is

$$\sigma^2_{\bar{V}_s} = \frac{\sigma^2_{\hat{v}_s}}{n}$$

irrespective of the distribution of $\hat{v}_s$ (provided its variance exists); we make no normality assumption here. Note further that under the design-based framework, we make no assumption about the spatial independence of $\hat{v}_s$; it is the sampling locations $x_i, i = 1\ldots n$ that are random and not the realized values $\hat{v}_s$.[29] For a desired value of $\sigma^2_{\bar{V}_s}$ and fixed $\sigma^2_{\hat{v}_s}$, the relationship can be solved

to provide the usual sample size equation

$$n_s = \frac{\sigma^2_{\hat{v}_s}}{\sigma^2_{\bar{V}_s}}$$

Where the specification is not in terms of $\sigma^2_{\bar{V}_s}$ but of the width of the normal approximation confidence limits, an additional term appears but as this term cancels in our derivation of E we can ignore it. The expected time required to meet the variance specification is simply $\bar{t}_s n_s$, where $\bar{t}_s$ is the mean time required per sample point. For now we assume that this only includes setup and measurement time, and not travel between points; we shall return to this issue below.

Now suppose we wish to compare the time requirement of two methods – setting one as the reference – when those methods must provide equal sampling variance. By direct substitution into the above equations, we may obtain the ratio of the sample times as

$$\frac{\bar{t}_s n_s}{\bar{t}_{ref} n_{ref}} = \frac{\bar{t}_s \sigma^2_{\hat{v}_s}/\sigma^2_{\bar{V}}}{\bar{t}_{ref} \sigma^2_{\hat{v}_{ref}}/\sigma^2_{\bar{V}}}$$

$$= \frac{\bar{t}_s \sigma^2_{\hat{v}_s}}{\bar{t}_{ref} \sigma^2_{\hat{v}_{ref}}}$$

Substituting the empirical variance s^2 for the unknown population variance σ^2 yields the definition of efficiency used in this study, with LIS as the reference method. Essentially identical efficiency measures have been used in a forestry context by Jordan *et al.*[23], Affleck,[22] and Rubinstein and Kroese[30] call this measure 'time efficiency' in a computational context.

With the definition of $\bar{t}_s$ above, E does not incorporate the travel time between points. In our view, this is most appropriate when the overall sampling design (number and location of sample points, and hence travel requirements) is being driven by sampling considerations for other variables, as would most often be the case in a multiresource inventory. In such a case, the travel costs are essentially 'sunk costs' and should not be charged against the CWD measurement. In other situations, it may be appropriate to consider travel costs as part of the cost of the CWD inventory, but there are a great many possibilities. We touch on two main situations here.

In inventories of relatively small parcels, common professional practice includes systematic line plot or grid surveys. Strictly speaking, the variance relationships given above for simple random sampling are inaccurate for systematic surveys, but they are often taken as sufficiently accurate that they are used for both specification of survey quality and reporting of survey results.[31] In these cases, total travel cost can be made completely insensitive to the number of sample points n. For example, suppose in the reference case that we would employ 40 sample points in an inventory of a 100 ha tract, installing a sample point every 125 m along transects spaced 200 m apart. If an alternative method required installation of 80 sample points, those sample points could be installed every 62.5 m along the same transects spaced 200 m part, with essentially no change in travel time requirements. In this case, ranking methods according to E yields the same ranking as an alternative criterion that incorporates travel cost per point, because the travel cost increment is constant for each method. E loses its interpretation as the ratio of total inventory cost, though its interpretation as a ratio of actual sampling time remains valid.

We note that arranging LIS transects so that they fall along travel routes between points could lead to time savings, but would violate the random orientation assumption associated with design unbiasedness of the conventional LIS estimator and its sample variance.[15]

At the other extreme, suppose each new sample point requires a fixed travel time investment, no matter how many sample points have already been installed. In other words, travel to each point is independent of other points and of the method employed. This situation probably corresponds well to large-scale inventories in which only a single sample location can be visited in a single sortie from an office or other base of operations. In these cases, a revised efficiency criterion is computable by incrementing the sample time per point of each method by the same fixed amount, and it is possible to solve this criterion for the break-even travel time at which $E=1$. We note, however, that in these cases it may be more efficient to use a cluster of samples at each sample point no matter what method is employed. The field efficiency of clusters of DLPDS points relative to LIS samples (or clusters of LIS transects, such as those employed by the national inventory program of the United States [32]) is beyond the scope of this preliminary study.