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Modeling Relations between Compacted and Uncompacted Crown Ratio for the Northern United States

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Tree crown ratio is useful in various applications such as prediction of tree mortality probabilities, growth potential, and fire behavior. Crown ratio is commonly assessed in two ways: (1) compacted crown ratio (CCR—lower branches visually moved upwards to fill missing foliage gaps) and (2) uncompacted crown ratio (UNCR—no missing foliage adjustment). The national forest inventory of the United States measures CCR on all trees, whereas only a subset of trees also are assessed for UNCR. Models for 27 species groups are presented to predict UNCR for the northern United States. The model formulation is consistent with those developed for other US regions while also accounting for the presence of repeated measurements and heterogeneous variance in a mixed-model framework. Ignoring random-effects parameters, the fit index values ranged from 0.43 to 0.78, and root mean squared error spanned 0.08–0.15; considerable improvements in both goodness-of-fit statistics were realized via inclusion of the random effects. Comparison of UNCR predictions with models developed for the southern United States exhibited close agreement, whereas comparisons with models used in Forest Vegetation Simulator variants indicated poor association. The models provide additional analytical flexibility for using the breadth of northern region data in applications where UNCR is the appropriate crown characteristic.

Keywords: forest inventory, mixed models, heterogeneous variance, growth and yield, fire behavior

Tree crown size and condition are of considerable interest to foresters for a variety of reasons. Crown information is a key indicator of tree vitality and thus often serves as a primary predictor variable in tree growth and mortality models (Hilt and Teck 1989, Burkhart et al. 2008, Vospernik et al. 2010). Crown condition is also critical for prediction of forest-fire behavior (Ex et al. 2015, Hevia et al. 2018) and also plays a role in postdisturbance evaluations of weather, insect, and disease phenomena (Morin et al. 2015, Chen et al. 2017). The amount of ecosystem services, e.g., carbon sequestration, air pollution removal, and energy efficiency, provided by a tree is directly correlated with crown characteristics (McPherson and Simpson 1999, Nowak et al. 2014). However, the crown measurement protocol has a direct bearing on the usefulness of the information for a specific application. CCR assessments entail visual "movement" of lower branches (as needed) to fill branch and foliage gaps farther up in the crown. Thus, CCR accounts for missing foliage and thus is a better indicator of total amount of leaf area. The UNCR is based on the actual position of the lowest live branch, which describes the true vertical crown presence. CCR measurements tend to be more suitable for applications such as growth models (Hann and Hanus 2002), whereas UNCR is better aligned to applications requiring the true height to crown-base ratio, such as fire behavior prediction (Alvarez et al. 2012) and tree taper modeling (Valentine and Gregoire 2001). Ideally, tree measurements would include both CCR and UNCR; however, that is rarely the case because of measurement time, seasonality of foliage, and costs for fieldwork.

The lack of both CCR and UNCR measurements on each tree petitions for the ability to convert from one measure to another. It is possible to directly predict either of these attributes from models using tree and stand attributes as predictor variables (Temesgen et al. 2005, Ducey 2009, Leites et al. 2009, Popoola and Adesoye 2012). However, no assurance of consistency is obtained, i.e., UNCR ≥ CCR. Thus, it is preferable to develop methods that provide more direct relations between the two measures to help ensure compatibility. In the United States, models

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have been developed to predict UNCR from CCR for the Pacific Northwest (Monleon et al. 2004), Interior West (Toney and Reeves 2009), and southern (Randolph 2010) regions. This is due, in large part, to CCR being measured on all trees in the forest inventory of the U.S, whereas only a small subset has the additional UNCR measurement (Schomaker et al. 2007). As Randolph (2010) suggested, there is a need to produce similar models for the northern region to fully encompass the conterminous United States. Thus, the objectives of this study are to (1) develop models for the prediction of UNCR using CCR as a predictor variable, (2) account for heterogeneous error variance and lack of independence among observations during model calibration, (3) assess how the model predictions compare to those of Forest Vegetation Simulator (FVS) variants covering the northern region, and (4) compare southern and northern region model predictions for species common to both areas.

Methods and Material

Data

The data used in this study were collected by the Forest Inventory and Analysis (FIA) program within the US Forest Service. The data were collected in the 24 states served by the Northern Research Station FIA unit (Figure 1), which represents the geographic area currently lacking models for conversion from compacted to CCR. The annualized forest inventory implemented by FIA is conducted by measuring a portion of the plots each year with all plots being measured over a period of 5–7 years. This process is repeated temporally, such that each plot gets remeasured every 5–7 years. FIA uses a quasisystematic sampling design having a sampling intensity of approximately 1 plot per 5,937 acres (2,428 hectares) (Reams et al. 2005). The field data were collected using a 0.166-acre (0.067-hectare) cluster plot design, where each plot is composed of four circular subplots having a 24-ft (7.32-m) radius with one subplot located at plot center and the remaining three subplots centered at azimuths 120, 240, and 360° and distance of 120 ft (36.58 m) from the plot center (Bechtold and Scott 2005). Each subplot contains a circular microplot with 6.8-ft (2.07-m) radius established 12 ft (3.66 m) at azimuth 90° from the subplot center. In forested areas, trees having a diameter at breast height (dbh) of ≥5.0 in. (12.70 cm) are measured on subplots; trees with a dbh of ≥1.0 in. (2.54 cm) and <5.0 in. (12.70 cm) are measured on the microplots. A myriad of tree- and condition-based data is obtained on each plot (US Forest Service 2018a). Condition-level (similar to stand-level) variables include forest type, stand size class, regeneration status, land owner group, reserve status, land use, and tree density. Separate forest conditions within a plot are delineated when any one of these variables is not uniform throughout the plot area. Individual tree measurements include species, dbh, total height, and CCR. Additional tree measurements that include UNCR are taken on a subset of plots (US Forest Service 2018b), resulting in both CCR and UNCR data being available for these trees. This subset serves as the data for this study. To facilitate handling of the large number of tree species encountered across the study area, species groups containing species of similar characteristics were used. These groupings largely match those used by the northern FIA program (Burrill et al. 2017). A general summary of the data is given in Table 1.

Analysis

Based on findings from other research that suggest the logistic model performs well in crown ratio modeling (Soares and Tomé 2001, Fu et al. 2015) and to maintain consistency with the formulation for other regions of the United States, the general model form specified was

$$\widehat{\mathrm{UNCR}} = \frac{1}{1 + \exp\left(f\left(X\beta\right)\right)} \tag{1}$$

where $X\beta$ = linear combination of predictor variables and associated estimated model parameters.

Initial investigations of predictor variables included compacted crown ratio (CCR), natural logarithm of dbh (Ldbh), total tree height (HT), and basal area per acre of live trees (BALIVE). Although each of the parameter estimates corresponding to these variables was statistically significant (α = 0.05), evaluation of their predictive power indicated that removal of both HT and BALIVE resulted in an increase of approximately 0.005 in root mean squared error (RMSE) (defined in Equation 5) with a corresponding decrease of about 0.002 in the fit index (see Equation 7). This outcome suggests that little practical predictive power was afforded using HT and BALIVE information in addition to that provided by CCR and Ldbh. This is consistent with the findings of previous work (Monleon et al. 2004, Randolph 2010). Thus, in an attempt to be both parsimonious and consistent with the aforementioned models covering other regions of the United States, the model chosen was:

$$\widehat{\mathrm{UNCR}}_{hikj} = \frac{1}{1 + \exp(\beta_0 + \beta_1 \mathrm{CCR}_{hikj} + \beta_2 \mathrm{Ldbh}_{hikj}))} + \varepsilon_{hikj} \tag{2}$$

where subscripts h = measurement year for plot i, i = plot, k = condition nested in plot i, j = tree nested in condition k, β_0–β_2 are estimated parameters, and ε_{hikj} denotes the random error. In this analysis, conditions were treated as the primary areal unit, as it is common that a plot only has one condition, and when multiple conditions are present, they often each contribute new information because of dissimilar stand characteristics.

Management and Policy Implications

Management and policy decisions are often made in the context of expected outcomes derived from applications such as stand-development and fire-prediction models. The models developed in this study may facilitate better management and policy decisions in two ways: (1) allowing the use of uncompacted crown ratio (UNCR) for data where only compacted crown ratio (CCR) was measured, and (2) improving growth and fire projection systems with more accurate models. In regard to (1), if CCR is used in systems where UNCR is expected, predictions of stand development and the probability of crown fire will likely be erroneous, which may lead to poor decisions. The context of (2) is that the broad spatial scale of the data used in this study may provide more accurate predictions than existing models developed from a smaller range of species, stand conditions, and geographic extent. Thus, there may be opportunities to update systems with models that perform more reliably, although further evaluation would be necessary. As this study completes the availability of UNCR prediction models for the conterminous US, their implementation can be beneficial at a broad range of spatial domains, thus promoting better decisionmaking from local to national levels.

Table 1. Sample size (*n*) and summary statistics for dbh (inches), UNCR, and CCR for 27 species groups in the northern region.

	dbh				UNCR				CCR			
n	Min.	Mean	Max.	IQR	Min.	Mean	Max.	IQR	Min.	Mean	Max.	IQR
2,110	1.0	9.3	26.4	5.4	0.04	0.42	0.99	0.20	0.02	0.31	0.90	0.18
1,979	1.0	8.7	21.8	4.3	0.05	0.45	0.99	0.30	0.01	0.31	0.99	0.20
6,406	1.0	10.0	39.8	6.9	0.01	0.52	0.99	0.30	0.01	0.37	0.99	0.20
1,722	1.0	7.4	19.2	3.1	0.05	0.52	0.99	0.30	0.05	0.39	0.99	0.18
31,755	1.0	5.2	23.9	4.9	0.01	0.61	0.99	0.35	0.01	0.45	0.99	0.30
6,588	1.0	9.0	37.0	5.4	0.03	0.67	0.99	0.32	0.02	0.47	0.99	0.25
16,289	1.0	7.6	28.1	3.8	0.01	0.63	0.99	0.31	0.01	0.44	0.99	0.22
3,856	1.0	8.7	32.2	3.7	0.02	0.53	0.99	0.29	0.02	0.41	0.99	0.20
9,500	1.0	10.1	43.6	6.5	0.02	0.60	0.99	0.30	0.01	0.41	0.95	0.15
5,992	1.0	11.4	41.8	7.7	0.03	0.52	0.99	0.20	0.01	0.38	0.87	0.15
4,003	1.0	9.5	36.4	5.4	0.01	0.52	0.99	0.20	0.01	0.35	0.85	0.10
6,539	1.0	10.7	34.0	7.3	0.01	0.54	0.99	0.25	0.01	0.38	0.99	0.15
6,462	1.0	8.1	28.4	5.0	0.05	0.61	0.99	0.30	0.04	0.42	0.90	0.20
4,432	1.0	8.0	32.4	5.0	0.10	0.58	0.99	0.25	0.01	0.42	0.89	0.15
17,831	1.0	8.2	43.2	5.1	0.01	0.58	0.99	0.25	0.01	0.41	0.96	0.12
28,463	1.0	7.8	38.4	4.6	0.01	0.52	0.99	0.21	0.01	0.37	0.90	0.15
7,551	1.0	6.5	38.0	6.0	0.01	0.67	0.99	0.35	0.01	0.45	0.95	0.20
858	1.0	8.5	31.3	5.4	0.05	0.50	0.99	0.23	0.01	0.36	0.91	0.15
1,789	1.0	6.5	26.4	4.5	0.01	0.60	0.99	0.30	0.01	0.40	0.99	0.20
10,648	1.0	7.5	40.3	4.5	0.01	0.49	0.99	0.25	0.01	0.35	0.99	0.15
14,078	1.0	7.4	61.5	4.9	0.01	0.42	0.99	0.17	0.01	0.33	0.90	0.15
3,122	1.0	9.3	33.5	5.7	0.05	0.53	0.99	0.25	0.01	0.38	0.90	0.15
2,447	1.0	11.0	39.1	7.8	0.05	0.49	0.99	0.23	0.05	0.35	0.90	0.10
1,621	1.0	9.9	33.4	6.4	0.05	0.53	0.99	0.25	0.05	0.37	0.85	0.15
24,135	1.0	7.3	51.6	4.0	0.01	0.53	0.99	0.26	0.01	0.36	0.99	0.20
6,233	1.0	6.6	31.2	5.1	0.01	0.54	0.99	0.30	0.01	0.36	0.99	0.20
8,445	1.0	4.0	29.0	4.4	0.01	0.58	0.99	0.35	0.01	0.38	0.95	0.15

Note: CCR, compacted crown ratio; dbh, diameter at breast height; IQR, interquartile range; UNCR, uncompacted crown ratio. IQR is the difference between 75th and 25th percentiles of the distribution of values.

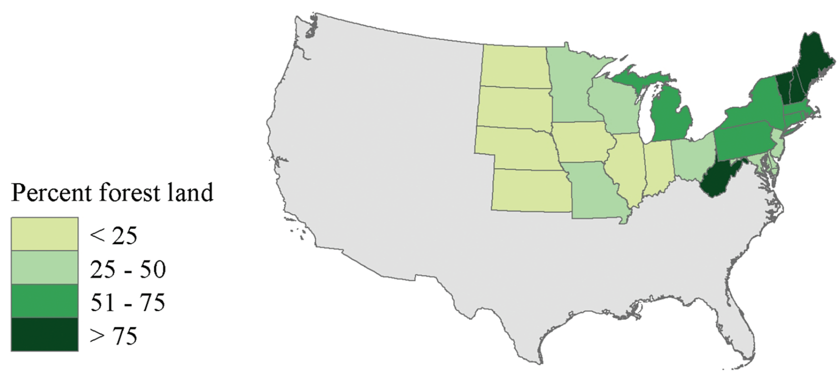

Figure 1. Twenty-four state study area in the northern region of the United States with percent of forest land category by state.

Having specified the model form, issues of heteroscedasticity and lack of independence among observations also need to be addressed. Examination of residuals resulting from fitting model (model 2) indicated a violation of the homogeneous variance assumption. Similar to the finding of Soares and Tomé (2001), the variance increased as predicted values decreased. Further investigation suggested a nonlinear increase in variance as CCR and Ldbh decreased. Considering the observed relations, the distribution of the random errors was specified as:

$$\varepsilon_{hikj} \approx N\left(0,\ \sigma_e^2\right) \approx N\left(0,\ \exp(\lambda_0 + \lambda_1 \mathrm{CCR}_{hikj} + \lambda_2 \mathrm{Ldbh}_{hikj})\right) \quad (3)$$

where $N(0, \sigma^2_e)$ indicates a normal distribution with mean 0 and variance σ^2_e, and λ_0–λ_2 are estimated parameters.

The data structure suggests tree attributes within a condition are correlated, particularly because of being in a common environment, e.g., crown closure influences sunlight penetration and the support of crown base height. As noted earlier, plots are revisited at regular intervals such that repeated measurements of the same conditions over time are also present in the data. Both circumstances indicate a lack of independence among observations. Proper statistical treatment of correlated observations is necessary to avoid bias in estimates of variance (West et al. 1984). In nonlinear regression applications, many researchers address correlated observations via specification of a mixed-effects model (Gregoire and Schabenberger 1996). Introduction of random-effects parameters has been successfully used in models of tree taper (Yang et al. 2009, Westfall and Scott 2010), tree height (Sharma and Parton 2007, Vargas-Larreta et al. 2009), and tree growth (Budhathoki et al. 2008, Rohner et al. 2018). Thus, a mixed model approach was used to address correlated observations in this study. Preliminary analyses indicated the best fit of the model to the data was obtained by associating the random-effect parameters with the intercept term (β_0). Saud et al. (2016) also found this placement to perform better than alternatives in mixed models for the crown ratio of shortleaf pine in the southern United States. The final model formulation used for analysis was:

$$\widehat{\mathrm{UNCR}}_{hikj} = \frac{1}{\begin{array}{c}1+\exp(\beta_0+\theta_{hik}+\phi_{ik}\\ +\beta_1\mathrm{CCR}_{hikj}+\beta_2\mathrm{Ldbh}_{hikj})\end{array}} + \varepsilon_{hikj} \tag{4}$$

$$\theta_{hik} \approx N\left(0, \sigma^2_\theta\right)$$

$$\phi_{ik} \approx N\left(0, \sigma^2{}_\phi\right)$$

where θ_{hik} and ϕ_{ik} are random-effect parameters assumed to be normally distributed having mean = 0 with variances σ^2_θ and $\sigma^2{}_\varphi$, respectively. θ_{hik} represents multiple measurements of the same condition, whereas ϕ_{ik} corresponds to similarities among trees within a condition.

Because of the large number of species and small sample sizes for rarer species, model 4 was fitted separately to 27 species groups (Table 1). Model goodness of fit was assessed via RMSE, mean absolute error (MAE), and a fit index corresponding to R^2 in linear regression (FI):

$$\mathrm{RMSE} = \sqrt{\frac{\sum\left(\mathrm{UNCR}_{hikj} - \widehat{\mathrm{UNCR}}_{hikj}\right)^2}{n}} \tag{5}$$

$$\mathrm{MAE} = \frac{\sum\left|\mathrm{UNCR}_{hikj} - \widehat{\mathrm{UNCR}}_{hikj}\right|}{n} \tag{6}$$

$$\mathrm{FI} = 1 - \frac{\sum\left(\mathrm{UNCR}_{hikj} - \widehat{\mathrm{UNCR}}_{hikj}\right)^2}{\sum\left(\mathrm{UNCR}_{hikj} - \overline{\mathrm{UNCR}}\right)^2} \tag{7}$$

where $\widehat{\mathrm{UNCR}}_{hikj}$ is the model prediction, UNCR_{hikj} is the observed UNCR, $\overline{\mathrm{UNCR}}$ is the mean observed UNCR, and n is the number of observations. The model fit statistics were calculated including the estimated random effects as well as assuming a likely operational scenario where the random effects were ignored (or equivalently assuming their expected value of zero).

Model validation exercises are often conducted to avoid overfitting and to assess the decline in predictive performance when applied to new observations. In this case, overfitting is unlikely because of the large sample sizes (Table 1) and the inclusion of only two predictor variables in the model. Further, some concerns have been expressed that model validation methods provide little new information and may sometimes provide misleading results (Kozak and Kozak 2003, Tedeschi 2006). Nonetheless, model performance was examined via Monte Carlo cross-validation (Shao 1993) by randomly dividing the data into 75 percent fitting/25 percent validation sets. For computational efficiency, regression analyses of model 2 were conducted using the fitting data, with RMSE and MAE assessed using the validation data. After repeating this process 100 times, the mean and standard deviation of RMSE and MAE were calculated. The final models given in Equation 4 were fitted to all available data as described in Table 1.

In addition to assessing the performance of the models developed for the northern region, it is also of interest to compare predictions of UNCR with those from other published sources. The Forest Vegetation Simulator (FVS) is a multifaceted growth and yield system applicable across the United States (Dixon 2002). Coverage of the northern region is accomplished primarily through three variants applicable to the northeast (Dixon and Keyser 2008a), lake states (Dixon and Keyser 2008b), and central states (Dixon and Keyser 2008c). Prediction of UNCR for all of these variants is based on the models from Holdaway (1986), which uses dbh and stand basal area (per acre) as predictors. The results reported by Holdaway are used for most species groups in FVS, but users should consult the aforementioned documentation for each variant to find the specific coefficients being implemented. The FVS methodology was applied to the data used in this study to compare predictions of UNCR.

Similarly, comparisons with the models developed for the southern region were desired for species common to both regions. The model coefficients and species group assignments presented in Randolph (2010) were used to obtain predicted values for the common species occurring in this study. Note that although the model formulation shown in model 2 is identical for the northern and southern applications, the data set on which the southern region models were built was limited to trees ≥5.0 in. dbh. The primary statistics of interest in these comparisons were the mean and standard deviation of the differences, as well as the assessment of statistical and practical significance of those differences.

Results and Discussion

Despite being applied across 27 species groups, the results of the regression analyses were consistent in that nearly all estimated parameters were statistically significant at the 95 percent confidence level (Table 2). The exceptions were λ_1 for jack pine (group 4) and $\sigma^2{}_\theta$ for tupelo and blackgum (group 19). For jack pine, the nonsignificant λ_1 suggests residual variation for this model is largely unaffected by the magnitude of CCR. The nonsignificant $\sigma^2{}_\theta$ for the tupelo and blackgum group signified little modification to the estimated fixed-effect parameter β_0 was needed to account for condition-to-condition variability. It should be noted that the $\sigma^2{}_\varphi$ parameter was statistically significant for all groups, indicating a nontrivial effect of the temporally repeated measurements of the same conditions. The only other anomaly in the estimated parameters occurred among red pine (group 8), where λ_2 was positively valued in comparison with being negatively valued for all other groups. Red pine differs from the other groups in that the residual variance increases with increasing Ldbh (Figure 2). This phenomenon may be partially due to most red pine in the region being in monoculture plantations, which may alter competition dynamics and resultant growth patterns as compared to naturally established forests.

The models provided a range of goodness of fit to the data across the species groups encountered in the northern United States (Table 3). With the random-effects parameters set to zero, RMSE spanned 0.08–0.15, with a mean across all species groups of 0.12; MAE varied between 0.06 and 0.12 with an overall mean of 0.09; FI exhibited a maximum of 0.78 for loblolly and shortleaf pines (group 1), a minimum of 0.43 for tupelo and blackgum (group 19), and a mean of 0.58. The model fit statistics from this study are

Table 2. Estimated parameters for model (4) with residual variance from (3) fitted to 27 species groups.

Group	Name	β_0	β_1	β_2	λ_0	λ_1	λ_2	$\sigma_{2\theta}$	$\sigma_{2\varphi}$
1	Loblolly and shortleaf pines	1.5425	–4.7368	0.1180	–3.6039	*–0.6926*	–0.7982	0.0192	0.0158
2	Other yellow pines	1.5031	–5.2376	0.1598	–2.5491	–1.3664	–0.8323	*0.0393*	0.0653
3	Eastern white pine	1.4942	–4.9138	0.0910	–3.6567	–1.5194	–0.3673	0.0448	0.0655
4	Jack pine	1.5125	–4.6564	*0.0927*	–4.6561	**–0.0789**	*–0.1951*	0.0859	0.0491
5	Spruce and balsam fir	1.3848	–4.9635	0.1195	–3.1131	–3.0714	–0.2244	0.0682	0.0660
6	Eastern hemlock	0.9869	–4.5940	0.1291	–2.5121	–3.7954	–0.2527	0.0673	0.0718
7	Other eastern softwoods	0.9622	–4.3375	0.1404	–2.4476	–3.3929	–0.4010	0.0862	0.0638
8	Red pine	1.4917	–5.2445	0.1846	–4.5910	–1.5932	*0.1414*	0.0788	0.0600
9	Select white oaks	0.8512	–4.2605	0.2092	–2.1939	–3.3562	–0.4895	0.0515	0.0675
10	Select red oaks	1.1379	–4.2314	0.1596	–2.5797	–2.3027	–0.7060	0.0228	0.0313
11	Other white oaks	0.9781	–4.4068	0.2235	–2.1834	–2.3846	–0.7511	0.0440	0.0410
12	Other red oaks	1.1222	–4.5126	0.1818	–2.3615	–2.8695	–0.5821	0.0290	0.0446
13	Hickory	0.8390	–4.3864	0.2558	–2.1709	–3.6006	–0.4735	0.0426	0.0459
14	Yellow birch	1.1200	–4.3840	0.1983	–2.2853	–3.4640	–0.6530	0.0204	0.0299
15	Hard maple	0.8211	–4.3175	0.2821	–2.1212	–3.0500	–0.7175	0.0406	0.0411
16	Soft maple	1.2410	–4.4017	0.1460	–2.9254	–2.4377	–0.5536	0.0286	0.0387
17	Beech	0.6889	–4.6325	0.3158	–1.9183	–4.2863	–0.4281	0.0896	0.0589
18	Sweetgum	1.2353	–4.3476	0.1618	–2.0122	–2.0042	–0.7278	*0.0145*	0.0000
19	Tupelo and blackgum	0.9228	–4.2780	0.1915	–2.3048	–3.4414	–0.4559	**0.0155**	0.0736
20	Ash	1.2976	–4.3743	0.1380	–2.8216	–2.7972	–0.5343	0.0386	0.0318
21	Cottonwood and aspen	1.6683	–4.4202	0.0402	–5.0822	–0.8311	–0.1950	0.0202	0.0314
22	Basswood	1.1765	–4.3006	0.1541	–2.9967	–1.7926	–0.5606	0.0473	0.0410
23	Yellow poplar	1.2729	–4.6294	0.1705	–3.3724	–1.0944	–0.5818	*0.0170*	0.0348
24	Black walnut	1.3850	–4.8044	*0.1026*	–3.2260	–1.6267	–0.3739	0.0602	*0.0204*
25	Other eastern soft hardwoods	1.1027	–4.6201	0.2342	–2.7022	–2.1610	–0.5749	0.0727	0.0420
26	Other eastern hard hardwoods	1.0988	–4.7961	0.2523	–2.6434	–2.4076	–0.6239	0.0694	0.0345
27	Eastern noncommercial hardwoods	1.0502	–4.2587	0.1661	–2.7280	–3.3206	–0.3108	0.0886	0.0728

Note: All parameter estimates were statistically significant at the 95% confidence level with P value < .0001, except statistically significant estimates having larger p-values (.05 < P < .0001) shown in italics and nonsignificant estimates (P > .05) given in bold.

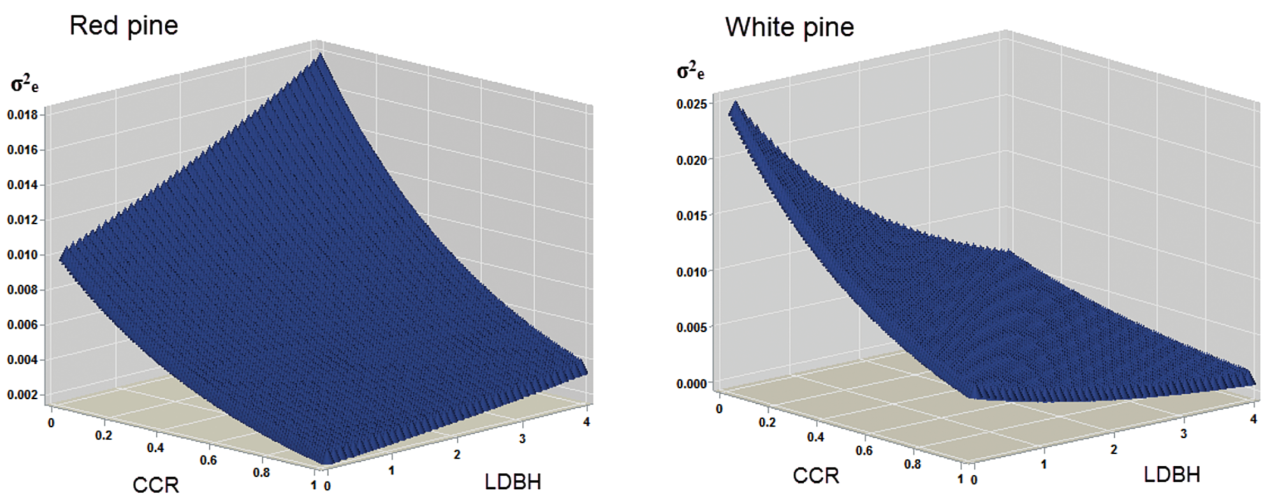

Figure 2. Change in residual error variance (σ^2_e) as a function of CCR and Ldbh (in.) using model 3 for red pine in contrast to all other species groups (as exemplified by eastern white pine).

very similar to those reported for other regions of the United States (Monleon et al. 2004, Toney and Reeves 2009, Randolph 2010). The inclusion of the random effects substantially reduced RMSE (RMSE* range 0.04–0.09; mean 0.07) and increased FI (FI* range 0.60–0.86; mean 0.73) (Table 3). MAE statistics were similar regardless of whether the random effects were included. This outcome suggests the use of random effects tends to reduce the largest prediction errors (both positive and negative); however, there is little influence on the central tendency of the absolute error distribution.

The models do not enforce the biological definition UNCR ≥ CCR. Other studies (Monleon et al. 2004, Randolph 2010) noted a small number of trees with $\widehat{\text{UNCR}}$ ≤ CCR. In this analysis, 0.07 percent of trees demonstrated this undesirable property. As expected, the preponderance (~70 percent) was found in cases where CCR ≥ 0.95, and this is where the largest discrepancies occurred (mean difference 0.03). Nearly 20 percent of the problematic predictions were associated with CCR ≤ 0.40, but the errors were smaller (mean difference 0.01). Overall, the anomaly occurred in 14 of the 27 groups with nearly 40 percent occurring in the spruce and balsam fir group (group 5). The other US studies avoided the issue for trees with high CCR by invoking $\widehat{\text{UNCR}}$ = CCR when CCR > 0.90; however, as Toney and Reeves (2009) point out, this approach has some bias. The recommendation from this study is to only require $\widehat{\text{UNCR}}$ = CCR when $\widehat{\text{UNCR}}$ < CCR to minimize bias for trees with CCR > 0.90 and also rectify any incongruous predictions that occur when CCR ≤ 0.90.

Table 3. RMSE, MAE, and FI for 27 species groups without and with (* designation) inclusion of random effects for prediction.

Group	Name	RMSE	MAE	FI	RMSE*	MAE*	FI*
1	Loblolly and shortleaf pines	0.08	0.06	0.78	0.05	0.07	0.85
2	Other yellow pines	0.12	0.09	0.66	0.07	0.10	0.79
3	Eastern white pine	0.11	0.08	0.75	0.06	0.08	0.86
4	Jack pine	0.11	0.08	0.73	0.06	0.08	0.86
5	Spruce and balsam fir	0.12	0.09	0.70	0.07	0.10	0.81
6	Eastern hemlock	0.12	0.10	0.59	0.07	0.10	0.75
7	Other eastern softwoods	0.13	0.10	0.60	0.08	0.10	0.75
8	Red pine	0.11	0.09	0.73	0.06	0.08	0.84
9	Select white oaks	0.13	0.10	0.44	0.07	0.10	0.68
10	Select red oaks	0.10	0.08	0.54	0.06	0.08	0.70
11	Other white oaks	0.13	0.10	0.45	0.07	0.10	0.64
12	Other red oaks	0.12	0.09	0.51	0.07	0.09	0.69
13	Hickory	0.13	0.10	0.50	0.08	0.10	0.68
14	Yellow birch	0.11	0.08	0.59	0.06	0.09	0.72
15	Hard maple	0.12	0.10	0.48	0.07	0.10	0.63
16	Soft maple	0.11	0.08	0.54	0.06	0.09	0.69
17	Beech	0.14	0.11	0.54	0.08	0.11	0.69
18	Sweetgum	0.12	0.09	0.57	0.08	0.12	0.60
19	Tupelo and blackgum	0.14	0.11	0.43	0.09	0.11	0.61
20	Ash	0.12	0.08	0.56	0.07	0.09	0.70
21	Cottonwood and aspen	0.08	0.06	0.74	0.04	0.06	0.86
22	Basswood	0.12	0.09	0.55	0.06	0.09	0.74
23	Yellow poplar	0.10	0.08	0.61	0.06	0.08	0.75
24	Black walnut	0.12	0.09	0.59	0.07	0.09	0.74
25	Other eastern soft hardwoods	0.14	0.10	0.53	0.08	0.11	0.72
26	Other eastern hard hardwoods	0.13	0.10	0.57	0.08	0.11	0.73
27	Eastern noncommercial hardwoods	0.15	0.12	0.44	0.08	0.11	0.70

Note: FI, fit index; MAE, mean absolute error; RMSE, root mean squared error.

The results from the Monte Carlo cross-validation showed no indication of overfitting and essentially no loss in predictive ability when fitted models were applied to the validation data. The mean of the RMSE and MAE (Table 4) were nearly identical to the values shown for the same model fitted to the entire data (Table 3). Also, the standard deviations of RMSE and MAE indicated very consistent outcomes for each of the 100 random splits of the data. Thus, there is little concern that substantial erosion in predictive accuracy would be encountered when applying the model to new observations obtained within the northern region.

Comparison with FVS and Southern Region Models

Differences in predicted UNCR between the FVS implementation of Holdaway (1986) models and those developed in this study were relatively large and consistent in direction. Essentially, the models from this study tended to predict a larger UNCR than FVS provides (Table 5). The range of mean differences across all species groups was approximately 0.02–0.40 for the central states variant, 0.06–0.31 for the lake states, and –0.10–0.29 for the northeast. Further investigation revealed a clear association between the prediction differences and tree size for trees having dbh ≤ 12 in., where the largest differences were found for the smallest trees in the data (Figure 3). For trees of size 12 < dbh ≤ 20 in., the correlation between prediction differences and tree size essentially dissipated, but there still existed a consistent mean difference of about 0.08 in. Trees having dbh > 20 in. exhibited substantial variability in model prediction differences, such that no particular pattern could be ascertained. Initially, it was suspected that perhaps the Holdaway (1986) model was being extrapolated beyond the range of the original fitting data; however, the paper suggests sufficient representation of small-diameter trees and stands of low basal area. Another consideration was that the geographic range of the data used by Holdaway only included the lake states of Wisconsin, Minnesota, and Michigan. There was a small, but discernible, effect of FVS variant, where the largest differences were in the lake states, with central states and northeast having smaller differences (Figure 3).

A number of species found in the northern region also occur in the southern United States. Thus, a comparison with the similarly constructed models developed for the southern region (Randolph 2010) is warranted for species common to both studies. An assessment of prediction differences between the southern and northern models by tree size indicated the northern models predicted slightly larger values for smaller trees (up to 7 in. dbh); whereas for trees having dbh > 7 in., the northern models consistently predicted approximately 0.01 less than the southern models (Figure 3). Twenty of the 27 species groups in this study contained species that were also encountered in the southern region (Table 5). The mean differences ranged from –0.030 to 0.057. Four groups with absolute value differences greater than 0.03 are worth noting: other yellow pines (group 2), eastern white pine (group 3), soft maple (group 16), and other eastern hard hardwoods (group 26). Randolph (2010) placed all *Pinus* species into a single group (PISP), whereas *Pinus* species common to both regions were spread across groups 1–3 of this study. The mean differences for these groups were 0.018, 0.045, and 0.057, respectively. The increase in the mean differences across these groups corresponds to the decreasing frequency of the species in PISP. That is, in the southern region study, PISP was composed of 75.7 percent loblolly and shortleaf pine, 4.6 percent pitch and Virginia pine, and 1.5 percent eastern white pine, with the remaining balance (18.2 percent) filled by six other species (unpublished data). Thus, the discrepancies found for these softwood species are not surprising given the compositional differences in species occurrence and aggregation between the two studies.

A similar scenario is likely in play for the soft maple group, although the evidence is less straightforward. For the southern region,

Table 4. Mean and standard deviation of RMSE and MAE from the Monte Carlo cross-validation analysis for 27 species groups.

		RMSE		MAE	
Group	Name	Mean	SD	Mean	SD
1	Loblolly and shortleaf pines	0.08	0.004	0.06	0.002
2	Other yellow pines	0.12	0.007	0.09	0.003
3	Eastern white pine	0.11	0.003	0.08	0.001
4	Jack pine	0.11	0.005	0.08	0.003
5	Spruce and balsam fir	0.12	0.001	0.09	0.001
6	Eastern hemlock	0.12	0.002	0.09	0.002
7	Other eastern softwoods	0.13	0.002	0.10	0.001
8	Red pine	0.11	0.003	0.08	0.002
9	Select white oaks	0.13	0.002	0.10	0.002
10	Select red oaks	0.10	0.003	0.08	0.001
11	Other white oaks	0.13	0.003	0.10	0.002
12	Other red oaks	0.12	0.003	0.09	0.002
13	Hickory	0.13	0.002	0.10	0.002
14	Yellow birch	0.11	0.003	0.08	0.002
15	Hard maple	0.12	0.001	0.09	0.001
16	Soft maple	0.11	0.001	0.08	0.001
17	Beech	0.14	0.002	0.11	0.002
18	Sweetgum	0.12	0.009	0.09	0.004
19	Tupelo and blackgum	0.14	0.005	0.11	0.003
20	Ash	0.12	0.003	0.08	0.001
21	Cottonwood and aspen	0.07	0.002	0.05	0.001
22	Basswood	0.11	0.003	0.09	0.002
23	Yellow poplar	0.10	0.004	0.07	0.002
24	Black walnut	0.12	0.006	0.09	0.003
25	Other eastern soft hardwoods	0.14	0.002	0.10	0.001
26	Other eastern hard hardwoods	0.13	0.003	0.10	0.002
27	Eastern noncommercial hardwoods	0.15	0.003	0.12	0.002

Note: MAE, mean absolute error; RMSE, root mean squared error.

Table 5. Mean difference ($\overline{D}$) and standard deviation of differences (σ_D) between model predictions from this study (Table 2) compared to Forest Vegetation Simulator variants (central states, lake states, and northeast) and Randolph (2010).

		Central states		Lake states		Northeast		Randolph	
Group	Name	$\overline{D}$	σ_D	$\overline{D}$	σ_D	$\overline{D}$	σ_D	$\overline{D}$	σ_D
1	Loblolly and shortleaf pines	0.025	0.096	–	–	0.126	0.162	0.018	0.015
2	Other yellow pines	0.106	0.144	0.134	0.153	–0.017	0.159	0.045	0.025
3	Eastern white pine	0.221	0.110	0.147	0.176	0.142	0.165	0.057	0.016
4	Jack pine	–	–	0.096	0.152	–0.097	0.064	–	–
5	Spruce and balsam fir	–	–	0.226	0.177	0.192	0.177	–	–
6	Eastern hemlock	–	–	0.289	0.123	0.232	0.146	–	–
7	Other eastern softwoods	0.059	0.155	0.190	0.146	0.188	0.161	–	–
8	Red pine	–	–	0.206	0.164	–0.012	0.156	–	–
9	Select white oaks	0.088	0.124	0.191	0.131	0.200	0.121	0.028	0.008
10	Select red oaks	0.042	0.111	0.146	0.124	0.078	0.111	–0.016	0.004
11	Other white oaks	0.046	0.118	–	–	0.129	0.124	0.003	0.010
12	Other red oaks	0.058	0.123	0.134	0.131	0.123	0.126	0.008	0.021
13	Hickory	0.138	0.133	0.223	0.134	0.196	0.141	0.028	0.035
14	Yellow birch	–	–	0.146	0.135	0.126	0.132	0.010	0.032
15	Hard maple	0.168	0.137	0.135	0.132	0.129	0.127	0.003	0.031
16	Soft maple	0.155	0.135	0.129	0.130	0.058	0.129	–0.030	0.017
17	Beech	0.405	0.161	0.231	0.175	0.287	0.176	–0.027	0.006
18	Sweetgum	0.165	0.126	–	–	0.158	0.149	–0.008	0.012
19	Tupelo and blackgum	0.220	0.141	0.272	0.057	0.242	0.136	0.017	0.011
20	Ash	0.094	0.117	0.183	0.140	0.114	0.147	0.025	0.006
21	Cottonwood and aspen	0.071	0.136	0.064	0.122	0.037	0.114	–	–
22	Basswood	0.199	0.120	0.143	0.135	0.116	0.125	–	–
23	Yellow poplar	0.137	0.128	–	–	0.109	0.137	0.012	0.021
24	Black walnut	0.156	0.128	0.212	0.145	0.114	0.133	–0.015	0.014
25	Other eastern soft hardwoods	0.141	0.142	0.176	0.153	0.121	0.146	0.011	0.047
26	Other eastern hard hardwoods	0.134	0.134	0.098	0.157	0.134	0.179	0.035	0.041
27	Eastern noncommercial hardwoods	0.130	0.156	0.308	0.138	0.222	0.144	0.019	0.026

Note: With the exception of red pine (group 8) in the northeast variant, all differences were statistically different from zero at the 95 percent confidence level.

Randolph (2010) combined all *Acer* species into a single group (ACSP), whereas this study created separate groups for hard maple (primarily sugar maple [*Acer saccharum* Marsh.]) and soft maple (essentially red maple [*Acer rubrum* L.]). The mean differences were small for hard maple (0.003), but relatively large for soft maple (–0.030). The ACSP group was predominantly red maple

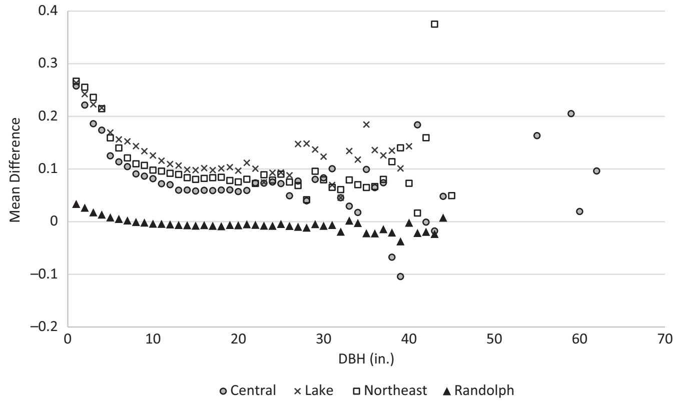

Figure 3. Mean difference in predictions between model 4 and (1) FVS (Holdaway 1986) for central states, lake states, and northeast variants, and (2) Randolph (2010) by 1-in. dbh classes.

(68.8 percent) so the reason for the relatively large difference between ACSP and the soft maple group is unclear. It may be due in part to less common *Acer* species, e.g., boxelder (*Acer negundo* L.), which were grouped with the maples in the southern study but with other groups, e.g., other eastern soft hardwoods, in this study. Of further note is that although both CCR and Ldbh were significant predictors of UNCR for the ACSP group in the southern region, Ldbh was not a significant predictor of UNCR for red maple (nor sugar maple) when the species was modeled individually (unpublished data). Therefore, the differences among the results for the northern and southern regions are likely due to the dissimilar aggregation of species.

Likewise, the prediction difference of 0.035 for the other eastern hard hardwoods group (group 26) was probably due to the inclusion of species that were modeled individually in the southern study but grouped together in this study, e.g., birch (*Betula* spp.), flowering dogwood (*Cornus florida* L.), common persimmon (*Diospyros virginiana* L.), and black locust (*Robinia pseudoacacia* L.). The extent of practical significance resulting from using one regional model over the other for this group, as well as for the pine and maple groups, depends on the specific application of the results, e.g., the sensitivity to crown parameters within systems that project stand development or fire behavior, but is likely to be minimal. More generally, it should also be noted that the statistically significant differences between the southern and northern models may be of little pragmatic concern given the relatively small magnitude of the prediction differences (Table 5).

Conclusion

The presentation of models for the northern region completes the geographic coverage of the conterminous United States for prediction of UNCR using CCR and the ubiquitously measured dbh.

Because of the importance of UNCR in various applications, these models should prove useful to analysts conducting assessments in the northern region and larger landscape-scale explorations when combined with other regional models. In cases where species occur in more than one region, e.g., northern and southern, there appears to be little difference in prediction because of model source; nevertheless, analysts should proceed with some caution if applying models outside the intended geographic range. This situation is likely to be encountered when using data from plots having nationally consistent spatial coverage. Other applications of the models include updating UNCR predictions in growth and yield models (e.g., FVS) and fire behavior simulators. Of course, the effects on overall performance need to be carefully examined, as unanticipated outcomes can occur within systems that rely on numerous inter-related models.

There is considerable merit in having both CCR and UNCR values available, as usually one is preferable depending on the analytical purpose. Typically, only one ratio is assessed because of resource limitations or the unanticipated future need for both ratios. Given the wide range of conditions observed on the forest inventory plots, the models presented here have application across the Northern United States and are recommended for use when data collection does not include UNCR or modeling efforts are not feasible for local inventories conducted in the region. Further, similar modeling efforts should be pursued for in other countries to provide users with crown information suited to their needs.

Literature Cited

Alvarez, A., M. Gracia, and J. Retana. 2012. Fuel types and crown fire potential in *Pinus halepensis* forests. *Eur. J. For. Res.* 131(2):463–474.

Bechtold, W.A., and C.T. Scott. 2005. *The forest inventory and analysis plot design.* USDA Forest Service Gen. Tech. Rep. SRS-102, Southern Research Station, Asheville, NC. 37–52 p.

Budhathoki, C.B., T.B. Lynch, and J.M. Guldin. 2008. Nonlinear mixed modeling of basal area growth for shortleaf pine. *For. Ecol. Manage.* 255(8–9):3440–3446.

Burkhart, H.E., R.L. Amateis, J.A. Westfall, and R.F. Daniels. 2008. *PTAEDA4.0: Simulation of individual tree growth, stand development, and economic evaluation in loblolly pine plantations.* Department of Forestry, Virginia Tech, Blacksburg, VA. 23 p.

Burrill, E.A., A.M. Wilson, J.A. Turner, S.A. Pugh, J. Menlove, G. Christiansen, B.L. Conkling, and W. David. 2017. *The forest inventory and analysis database: Database description and user guide version 7.2 for Phase 2.* USDA Forest Service. 946 p. Available online at http://www.fia.fs.fed.us/library/database-documentation/; last accessed April 20, 2019.

Chen, C., A.R. Weiskittel, M. Bataineh, and D.A. MacLean. 2017. Evaluating the influence of varying levels of spruce budworm defoliation on annualized individual tree growth and mortality in Maine, USA and New Brunswick, Canada. *For. Ecol. Manage.* 396:184–194.

Dixon, G.E. (comp.). 2002 (revised 2018). *Essential FVS: A user's guide to the forest vegetation simulator. Internal Rep*. USDA Forest Service, Forest Management Service Center, Fort Collins, CO. 226 p.

Dixon, G.E., and C.E. Keyser (comps.). 2008a (revised 2018). *Northeast (NE) variant overview—forest vegetation simulator. Internal Rep.* USDA Forest Service, Forest Management Service Center, Fort Collins, CO. 55 p.

Dixon, G.E., and C.E. Keyser (comps.). 2008b (revised 2018). *Lake States (LS) variant overview—forest vegetation simulator. Internal Rep.* USDA Forest Service, Forest Management Service Center, Fort Collins, CO. 53 p.

Dixon, G.E., and C.E. Keyser (comps.). 2008c (revised 2018). *Central States (CS) variant overview—forest vegetation simulator. Internal Rep.* USDA Forest Service, Forest Management Service Center, Fort Collins, CO. 58 p.

Ducey, M.J. 2009. Predicting crown size and shape from simple stand variables. *J. Sust. For.* 28:5–21.

Ex, S., F.W. Smith, and T.L. Keyser. 2015. Characterizing crown fuel distribution for conifers in the interior western United States. *Can. J. For. Res.* 45(7):950–957.

Fu, L., H. Zhang, J. Lu, H. Zang, M. Lou, and G. Wang. 2015. Multilevel nonlinear mixed-effect crown ratio models for individual trees of Mongolian Oak (*Quercus mongolica*) in Northeast China. *PLoS One* 10:e0133294.

Gregoire, T.G., and O. Schabenberger. 1996. Nonlinear mixed-effects modeling of cumulative bole volume with spatially correlated within-tree data. *J. Agric. Biol. Environ. Stat.* 1(1):107–119.

Hann, D.W., and M.L. Hanus. 2002. *Enhanced diameter-growth-rate equations for undamaged and damaged trees in southwest Oregon. Research contribution*, vol. 39. Oregon State University, Forest Research Laboratory, Corvallis, OR. 54 p.

Hevia, A., A. Crabiffosse, J.G. Álvarez-González, A.D. Ruiz-González, and J. Majada. 2018. Assessing the effect of pruning and thinning on crown fire hazard in young Atlantic maritime pine forests. *J. Environ. Manage*. 205:9–17.

Hilt, D.E., and R.M. Teck. 1989. NE-TWIGS: An individual tree growth and yield projection system for the Northeastern United States. *Compiler* 7(2):10–16.

Holdaway, M.R. 1986. Modeling tree crown ratio. *For. Chron.* 62:451–455.

Kozak, A., and R. Kozak. 2003. Does cross validation provide additional information in the evaluation of regression models? *Can. J. For. Res.* 33(6):976–987.

Leites, L.P., A.P. Robinson, and N.L. Crookston. 2009. Accuracy and equivalence testing of crown ratio models and assessment of their impact on diameter growth and basal area increment predictions of two variants of the Forest Vegetation Simulator. *Can. J. For. Res.* 39(3):655–665.

McPherson, E.G., and J.R. Simpson. 1999. *Carbon dioxide reduction through urban forestry: Guidelines for professional and volunteer tree planters.* USDA Forest Service Gen. Tech. Rep. PSW-171, Pacific Southwest Research Station, Albany, CA. 237 p.

Monleon, V.J., D. Azuma, and D. Gedney. 2004. Equations for predicting uncompacted crown ratio based on compacted crown ratio and tree attributes. *West. J. Appl. For.* 19(4):260–267.

Morin, R.S., K.C. Randolph, and J. Steinman. 2015. Mortality rates associated with crown health for eastern forest tree species. *Environ. Monit. Assess.* 187:87.

Nowak, D.J., S. Hirabayashi, A. Bodine, and E. Greenfield. 2014. Tree and forest effects on air quality and human health in the United States. *Environ. Pollut.* 193:119–129.

Popoola, F.S., and P.O. Adesoye. 2012. Crown ratio models for *Tectona grandis* (Linn. f) stands in Osho Forest Reserve, Oyo State, Nigeria. *J. For. Sci.* 28(2):63–67.

Randolph, K.C. 2010. Equations relating compacted and uncompacted live crown ratio for common tree species in the South. *South. J. Appl. For.* 34(3):118–123.

Reams, G.A., W.D. Smith, M.H. Hansen, W.A. Bechtold, F.A. Roesch, and G.G. Moisen. 2005. *The forest inventory and analysis sampling frame.* USDA Forest Service Gen. Tech. Rep. SRS-102, Southern Research Station, Asheville, NC. 21–36 p.

Rohner, B., P. Waldner, H. Lischke, M. Ferretti, and E. Thürig. 2018. Predicting individual-tree growth of central European tree species as a function of site, stand, management, nutrient, and climate effects. *Eur. J. For. Res.* 137(1):29–44.

Saud, P., T.B. Lynch, K.C. Anup and J.M. Guldin. 2016. Using quadratic mean diameter and relative spacing index to enhance height–diameter and crown ratio models fitted to longitudinal data. *Forestry* 89(2):215–229.

Schomaker, M.E., S.J. Zarnoch, W.A. Bechtold, D.J. Latelle, W.G. Burkman, and S.M. Cox. 2007. *Crown-condition classification: A guide to data collection and analysis.* USDA Forest Service Gen. Tech. Rep. SRS-102, Southern Research Station, Asheville, NC. 78 p.

Shao, J. 1993. Linear model selection by cross-validation. *J. Am. Stat. Assoc.* 88(422):486–494.

Sharma, M., and J. Parton. 2007. Height–diameter equations for boreal tree species in Ontario using a mixed-effects modeling approach. *For. Ecol. Manage.* 249(3):187–198.

Soares P., and M. Tomé. 2001. A tree crown ratio prediction equation for eucalypt plantations. *Ann. For. Sci.* 58(2):193–202.

Tedeschi, L.O. 2006. Assessment of the adequacy of mathematical models. *Agric. Syst.* 89(2–3):225–247.

Temesgen, H., V. LeMay, and S.J. Mitchell. 2005. Tree crown ratio models for multi-species and multi-layered stands of southeastern British Columbia. *For. Chron.* 81(1):133–141.

Toney, C., and M.C. Reeves. 2009. Equations to convert compacted crown ratio to uncompacted crown ratio for trees in the Interior West. *West. J. Appl. For.* 24(2):76–82.

US Forest Service. 2018a. *Forest Inventory and Analysis national core field guide. Vol. 1: Field data collection procedures for phase 2 plots*, version 7.2, Northern Research Station edition. Available online at https://www.nrs.fs.fed.us/fia/data-collection/field-guides/ver7.2/FGpercent20NRSpercent207.2-Completepercent20Document.pdf; last accessed April 20, 2019.

US Forest Service. 2018b. *Forest Inventory and Analysis national core field guide. Vol. 1 Supplement: Field data collection procedures for phase 2+ plots*, version 7.2, Northern Research Station edition. Available online at https://www.nrs.fs.fed.us/fia/data-collection/field-guides/ver7.2/NRSpercent20FGpercent207.2-Aprilpercent202018-Completepercent20Document_NRSP2plus.pdf; last accessed April 20, 2019.

Valentine, H.T., and T.G. Gregoire. 2001. A switching model of bole taper. *Can. J. For. Res.* 31(8):1400–1409.

Vargas-Larreta, B., F. Castedo-Dorado, J.G. Álvarez-González, M. Barrio-Anta, and F. Cruz-Cobos. 2009. A generalized height–diameter model with random coefficients for uneven-aged stands in El Salto, Durango (Mexico). *Forestry* 82(4):445–462.

Vospernik, S., R.A. Monserud, and H. Sterba. 2010. Do individual-tree growth models correctly represent height:diameter ratios of Norway spruce and Scots pine? *For. Ecol. Manage*. 260:1735–1753.

West, P.W., D.A. Ratkowsky, and A.W. Davis. 1984. Problems of hypothesis testing of regressions with multiple measurements from individual sampling units. *For. Ecol. Manage.* 7(3):207–224.

Westfall, J.A., and C.T. Scott. 2010. Taper models for commercial tree species in the Northeastern United States. *For. Sci.* 56(6):515–528.

Yang, Y., S. Huang, G. Trincado, and S.X. Meng. 2009. Nonlinear mixed-effects modeling of variable-exponent taper equations for lodgepole pine in Alberta, Canada. *Eur. J. For. Res.* 128:415–429.