MODELING ASPEN COVER TYPE DIAMETER DISTRIBUTIONS IN MINNESOTA

Curtis L. VanderSchaaf[1]

Abstract—An attempt was made to model diameter distributions of aspen (*Populus* spp) stands. The aspen (1,020,150 acres) cover type has the greatest acreage on DNR lands and it contains a variety of hardwood and softwood species. Aspen is a valuable pulpwood species, annually comprising around 50 percent of total timber harvest on DNR lands.

Modeling distributions of this cover type has been minimal; stands often contain a variety of tree sizes and species, making modeling difficult, particularly given a disproportionate amount of smaller trees. For this analysis, a three-parameter Weibull-based modeling approach was used, where the parameters are predicted using the 0, 25th, 50th, and 95th estimated percentiles of the distribution. The percentiles are predicted as functions of quadratic mean diameter and stand/plot age. Currently, species compositions were ignored.

Data used in model fitting were obtained from the Forest Inventory and Analysis (FIA) database and from all regions of the state.

Prediction errors are rather large for smaller diameter classes and onset of merchantability (e.g. 4 to 6 inches). When looking at the average error across all diameter classes at a particular site, each diameter class within a site had an average error of 20 trees per acre (smaller diameters generally had larger absolute tree per acre errors).

For common merchantable rotations (e.g. 40 to 50 years), total errors (total across all diameter classes) are less relative to younger and older ages. However, for diameter classes of most interest (6 to 12 inches), prediction errors are generally larger for these rotation ages relative to younger stands.

INTRODUCTION

As part of a project determining optimal economic rotation ages of aspen (*Populus* spp) cover types on Minnesota DNR lands for harvest scheduling analyses, it was attempted to model diameter distributions of these stands. By far the aspen (1,020,150 acres) cover type has the greatest acreage on DNR lands. Although some stands are dominated by aspen, a variety of species can be found including ash (*Fraxinus* spp), balsam poplar (*Populus balsamifera* L), birch (*Betula* spp), elm (*Ulmus* spp), maple (*Acer* spp), oak (*Quercus* spp), balsam fir (*Abies balsamea* (L.) Mill.), pine (*Pinus* spp), and spruce (*Picea* spp). Aspen is a valuable pulpwood species, recently averaging around $30 per cord, and in 2012 it comprised 63 percent of the pulpwood harvest and 50 percent of the total timber harvest.

Knowing the likely diameter distribution within a stand can provide more precise economic assessments since stumpage values are often related to diameter class. For example, the pulpwood and bolt class which usually occurs for d.b.h.s ranging from 8 to 11 inches, generally results in higher stumpage relative to merchandizing trees as pulpwood only (e.g. 8 inches and less), stumpage values can increase by nearly 60 percent. Currently, species compositions were ignored, but if

[1] Forest Modeler, Minnesota Department of Natural Resources, Division of Forestry, Resource Assessment Unit, Grand Rapids, MN 55744. To contact, call 218-322-2518 or email at curtis.vanderschaaf@state.mn.us.

future analyses are conducted they should concentrate on developing separate distributions by species.

Modeling distributions of this cover type has been minimal; stands often contain a variety of tree sizes and species, making modeling difficult, particularly given a disproportionate amount of smaller trees. Previous modeling has concentrated on estimating either total stand yields or has used an individual tree approach. For this analysis, a three-parameter Weibull-based modeling approach was used, where the parameters are predicted using the 0, 25th, 50th, and 95th estimated percentiles of the diameter distribution. The percentiles are predicted as functions of quadratic mean diameter and stand/plot age.

METHODS

Data used in model fitting were obtained from the Forest Inventory and Analysis (FIA) database. Survey data were obtained from all regions of the state, only plots measured from 2003 to 2007 (EVAL_GRP = 272007), and 2008 to 2012 (EVAL_GRP = 272012) were used (FORTYPCD = 901), and all four subplots within a plot had to have the same condition class. Since only a point in time distribution estimate is desired, temporal correlation when using data from the same plot was ignored.

Plots are clusters of four points arranged such that point 1 is central, with points 2 through 4 located 120 feet from point 1 at azimuths of 0, 120, and 240 degrees. Each cluster point is surrounded by a 24.0 foot fixed-radius subplot where trees 5.0 inches d.b.h. and larger are measured. Combined, the four subplots total approximately 1/6th acre. Each subplot contains a 6.8 foot fixed-radius microplot where saplings 1.0 to 4.9 inches are measured. The four microplots total approximately 1/75th acre. Blow-up factors associated with these different plot sizes could impact modeling ability. Condition classes are assigned to differentiate conditions occurring on a plot, a subplot can have more than one condition class. A condition class differentiates stand conditions given variables that FIA monitors, such as cover type, ownership, and stand density.

A percentile-based, parameter recovery, three-parameter Weibull distribution procedure was used to model diameter distributions (Brooks and others 1992). Due to correlation among residuals of the equations, parameters were estimated using seemingly unrelated regression (SUR). The following system of equations was estimated:

$$\text{Min}(D0) = a + b * Dq + c * Age \quad [1]$$
$$D25 = d + e * Dq \quad [2]$$
$$D50 = g + h * Dq + i * Age \quad [3]$$
$$D95 = j + k * Dq + l * Age \quad [4]$$

Where:

Min (D0) – minimum d.b.h. (inches) within a plot,
Di – predicted value for the d.b.h. (inches) at which the ith percentile occurs,
Dq – quadratic mean diameter (inches), and
Age – stand age.
The percentiles were then used to estimate parameters of the Weibull distribution:

$$a\ (\text{Location}) = (n^{1/3}D0 - D50)/(n^{1/3} - 1) \quad [5]$$
$$c\ (\text{Shape}) = 2.343088/(\ln(D95-a)-\ln(D25-a)) \quad [6]$$
$$b\ (\text{Scale}) = -(a\Gamma_1)/\Gamma_2 + \sqrt{(a/\Gamma_2)^2(\Gamma_1^2-\Gamma_2)+Dq^2/\Gamma_2} \quad [7]$$

Where:

$\Gamma_1 = \Gamma[1+(1/c)]$,
$\Gamma_2 = \Gamma[1+(2/c)]$, and
Γ = gamma function.

For the observations used to fit equations [1] to [4], the average age of plots was 39 ranging from 0 to 116 years, trees per acre ranged from 6 to 4,585 and averaged 1,039, basal area per acre averaged 76 ranging from 0.4 to 234 square feet, quadratic mean diameter ranged from 1.0 to 13.1 inches and averaged 4.4 inches, and site index (base age 50) averaged 63 feet and ranged from 2 to 112 feet.

RESULTS AND DISCUSSION

Parameter estimates for equations [1] to [4] are presented in Table 1. Perhaps future analyses can attempt to separate species.

Table 1.—Parameter estimates for equations [1] to [4]. Sample size was 1,876 plots.

Aspen	Intercept	Dq	Age
D0	0.328	0.472	-0.021
D25	-0.190	0.910	-
D50	0.238	0.911	0.029
D95	3.604	0.874	0.080

Table 2 presents verification results by diameter class. For all FIA plots used in model fitting, number of trees were predicted by diameter class using the Weibull distribution, the difference from the FIA observed was calculated, squared (to eliminate negative and positive errors), square rooted to produce the original units (trees per acre), and then averaged. Analyzing prediction errors by diameter class will help to identify if errors are greater at certain ranges of diameter classes – for example, perhaps the Weibull distribution is not flexible enough to model Aspen distributions across the entire range of stand conditions.

Prediction errors are rather large for smaller diameter classes and near the beginning of merchantability (e.g. 4 to 6 inches). Figure 1 shows in some cases there is an inability to model smaller diameter classes of merchantable rotation age Aspen stands – all plots have a site index of 65 feet. For distributions depicted in the figure plot ages ranged from 37 to 47 years.

Table 2.—Verification results for equations [1] to [4] by diameter class. Number of plots was 1,864. The number of plots is less relative to model fitting because some plots had too few trees to estimate Weibull parameters.

Diameter class (inches)	Error in Trees Per Acre
1	143.3
2	127.4
3	93.0
4	76.4
5	51.9
6	29.6
7	19.2
8	12.3
9	8.5
10	6.3
11	5.0
12	3.8
13	3.1
14	2.4
15	1.8
16	1.2
17	0.7
18	0.5
19	0.4
20	0.2
21	0.1
22	0.1
23	0.0
24	0.1
25	0.0
26	0.0
27	0.0
28	0.0
29	0.0
30	0.0

When looking at the average error across all diameter classes at an individual site, each diameter class within a site had an average error of 20 trees per acre (smaller diameters generally had larger absolute tree per acre errors). Average error across all diameter classes within a site was 585 (ranging from 13 to 3,379 trees per acre) – hence on average 585 trees were placed into the wrong diameter class at each site.

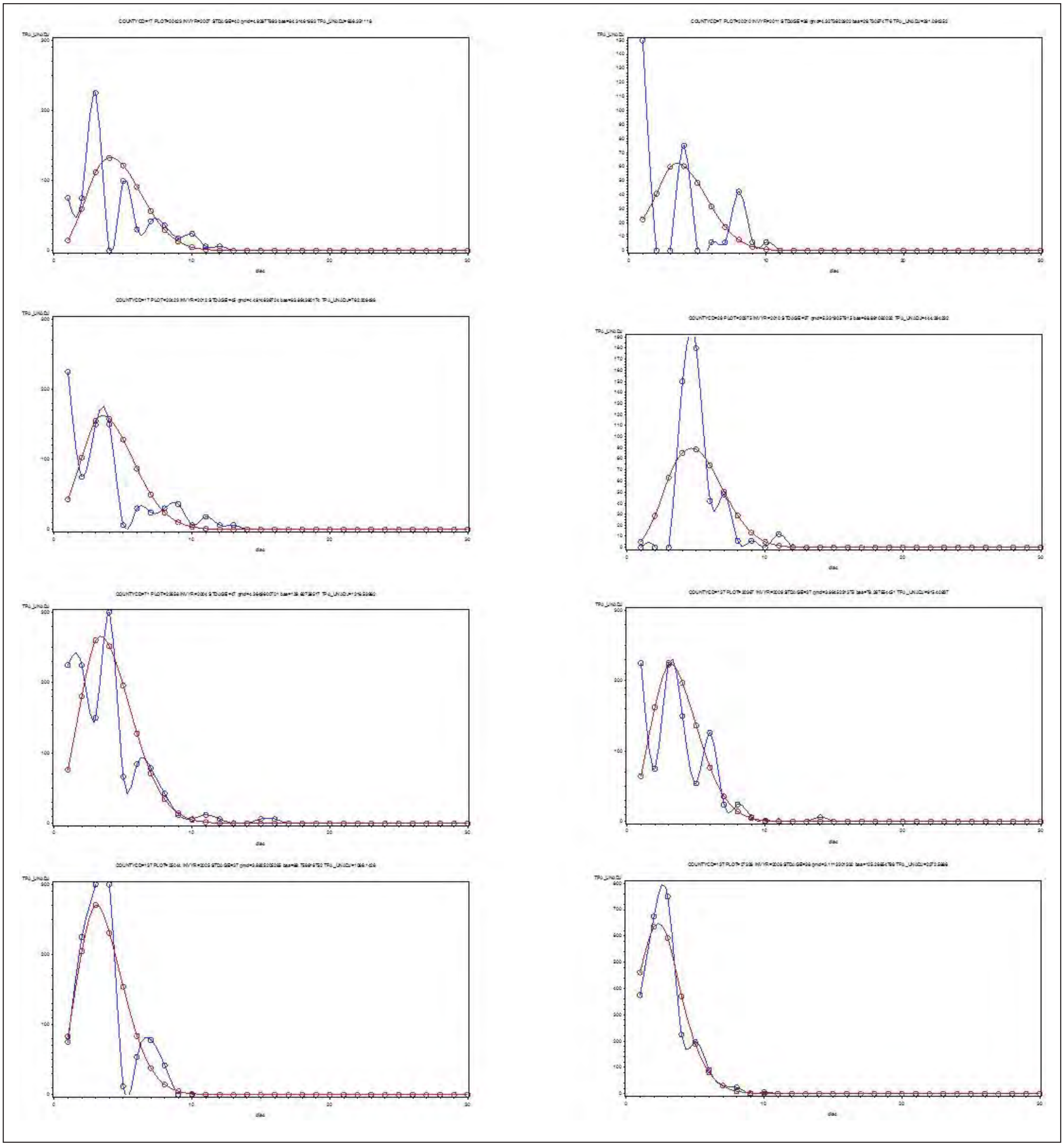

Figure 1—Observed Aspen forest type (FORTYPCD = 901) diameter distributions (from FIA – blue line) and those predicted using equations [1] to [4] for the Weibull distribution (red line), all plots have a site index (base age 50) of 65 feet, and range in age from 37 to 47 years.

Based on Table 2, it would be expected that younger stands, because of a larger number of smaller trees, would have larger prediction errors. To see if prediction errors varied by age, stands were separated into 10 year age-classes, and greater than 50 years old (Table 3). For common merchantable rotation ages (e.g. 40 to 50 years), total errors are less relative to younger and older ages. However, for diameter classes of most interest (6 to 12 inches), prediction errors are generally larger for these rotation ages relative to younger stands – absolute errors are based on number of trees per acre within a diameter class, since these ages have more trees in this range relative to younger ages, absolute errors are greater – hence absolute errors are likely a function of trees per acre (which is likely correlated with age, yes); but not directly related to age.

Table 3—Verification results for equations [1] to [4] by age-class and diameter class. Where n is number of plots within an age group.

Diameter class (inches)	Age Group					
	< 10.1	> 10 and < 20.1	> 20 and < 30.1	> 30 and < 40.1	> 40 and < 50.1	> 50.1
1	200.4	222.4	148.1	96.3	98.7	117.7
2	153.9	219.2	128.5	88.7	86.8	102.7
3	92.0	139.8	130.0	86.8	65.1	68.0
4	41.5	83.5	99.8	86.8	72.3	73.0
5	18.5	38.4	56.5	62.1	56.1	62.7
6	8.4	11.4	23.5	27.4	36.0	46.4
7	5.3	5.5	12.7	16.6	22.4	32.9
8	3.8	4.2	7.4	12.6	14.2	20.4
9	2.4	3.0	4.5	7.6	11.3	14.0
10	1.9	2.0	2.7	5.2	9.2	10.6
11	1.3	1.5	2.4	3.6	6.9	8.7
12	0.6	0.9	1.2	2.5	5.0	7.2
13	0.8	0.5	0.9	1.4	4.3	6.2
14	0.3	0.3	0.7	1.3	2.6	5.1
15	0.4	0.2	0.5	0.8	1.9	3.9
16	0.2	0.2	0.2	0.4	1.1	2.8
17	0.2	0.1	0.1	0.3	0.8	1.4
18	0.1	0.1	0.0	0.1	0.4	1.2
19	0.1	0.1	0.1	0.1	0.3	0.8
20	0.0	0.0	0.0	0.1	0.2	0.4
21	0.0	0.0	0.0	0.0	0.1	0.2
22	0.0	0.0	0.0	0.1	0.1	0.2
23	0.0	0.0	0.0	0.0	0.0	0.1
24	0.0	0.0	0.0	0.0	0.0	0.1
25	0.0	0.0	0.0	0.0	0.0	0.1
26	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0
N	215	304	258	226	225	636

LITERATURE CITED

Brooks, J.R.; Borders, B.E.; Bailey, R.L. 1992. Predicting diameter distributions for site-prepared loblolly and slash pine plantations. Southern Journal Applied Forestry 16: 130-133.