Mathematical Modeling of Forest Fire Initiation in Three Dimensional Setting

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Abstract—*In this study, the assignment and theoretical investigations of the problems of forest fire initiation were carried out, including development of a mathematical model for description of heat and mass transfer processes in overterrestrial layer of atmosphere at crown forest fire initiation, taking into account their mutual influence. Mathematical model of forest fire was based on an analysis of experimental data and using concept and methods from reactive media mechanics. In the context of the general mathematical model of forest fires, this study gives a new mathematical setting and method of numerical solution of a problem of a forest fire modeling. The boundary-value problem is solved numerically using the method of splitting according to physical processes. In this paper the assignment and theoretical investigations of the problems of forest fire initiation are carried out.*

Introduction

One of the objectives of these studies is the improvement of knowledge on the fundamental physical mechanisms that control forest fire initiation. A great deal of work has been done on the theoretical problem of forest fire initiation. Crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism. The first explanation of this process was given by Van Wagner (1977). The theory proposed there depends on three simple crown properties: crown base height, bulk density, and moisture content of forest fuel. Also, crown fire initiation and hazard have been studied and modeled in detail (see for example Alexander 1998; Van Wagner 1999; Xanthopoulos 1990; Rothermel 1991a,b; Cruz and others 2002; Albini and others 1995; Scott and Reinhardt 2001).The more complete discussion of the problem of modeling forest fires is provided by a cycle of works produced by a group of coworkers at Tomsk University (Grishin 1997; Grishin and Perminov 1998; Perminov 1995 and 1998). In particular, a mathematical model of forest fires was obtained by Grishin (1997) based on an analysis of known and original experimental data (Grishin 1997; Konev 1977), and using concepts and methods from reactive media mechanics. The physical two-phase models used in Morvan and Dupuy (2001 and 2004) may be considered as a continuation and extension of the formulation proposed by Grishin and Perminov. However, the investigation of crown fires has been limited mainly to cases of forest fires initiation without taking into account the mutual interaction of the forest fires and three dimensional atmosphere flows.

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Problem Formulation

The basic assumptions adopted during the deduction of equations and boundary and initial conditions:

1. The forest represents a multiphase, multistoried, spatially heterogeneous medium.
2. In the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium.
3. The forest canopy is supposed to be nondeformed media (trunks, large branches, small twigs, and needles) and affects only the magnitude of the force of resistance in the equation of conservation of momentum in the gas phase—that is, the medium is assumed to be quasisolid (almost nondeformable during wind gusts).
4. Let there be a so-called "ventilated" forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products).
5. The flow has a developed turbulent nature and molecular transfer is neglected.
6. Gaseous phase density doesn't depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound.

Let the coordinate reference point x_1, x_2 , x_3= 0 be situated at the centre of the surface forest fire source at the height of the roughness level, axis $0x_1$ directed parallel to the Earth's surface to the right in the direction of the unperturbed wind speed, axis $0x_2$ directed perpendicular to $0x_1$ and axis $0x_3$ directed upward (fig. 1).

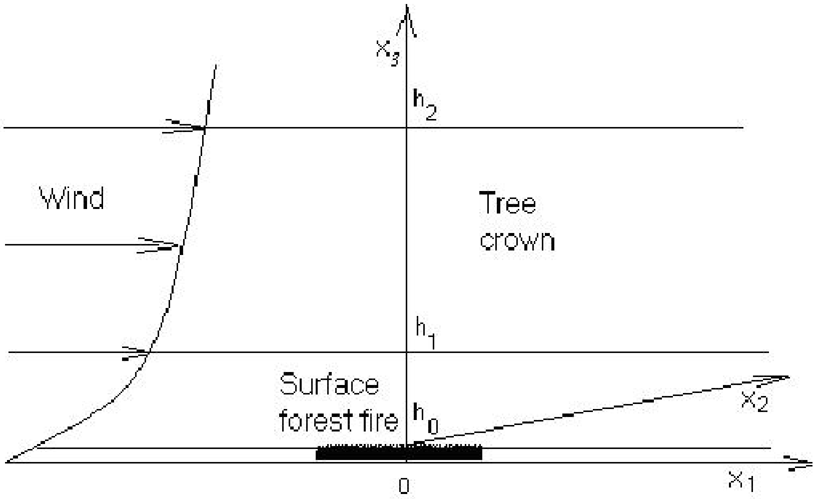

Figure 1—Coordinate reference point information.

Using the results of Grishin and Perminov from their studies conducted 1995 through 1998, and known experimental data (Konev 1977), we have the following sufficiently general equations, which define the state of the medium in the forest fire zone, written using tensor notation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = m\,, \quad j = 1,2,3, \quad i = 1,2,3; \tag{1}$$

$$\rho \frac{dv_i}{dt} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}(-\rho \overline{v_i' v_j'}) - \rho s c_d v_i \,|\, v \,|\, - \rho g_i - m v_i\,; \tag{2}$$

$$\rho c_p \frac{dT}{dt} = \frac{\partial}{\partial x_j}(-\rho c_p v_j' \overline{v_i' T'}) + q_5 R_5 - \alpha_v (T - T_s) + k_g (cU_R - 4\sigma T^4); \tag{3}$$

$$\rho \frac{dc_\alpha}{dt} = \frac{\partial}{\partial x_j}(-\rho \overline{v_j' c_\alpha'}) + R_{5\alpha} - m c_\alpha \ , \alpha = 1,5; \tag{4}$$

$$\frac{\partial}{\partial x_j}\left(\frac{c}{3k}\frac{\partial U_R}{\partial x_j}\right) - kcU_R + 4k_S \sigma T_S^4 + 4k_g \sigma T^4 = 0, k = k_g + k_S; \tag{5}$$

$$\sum_{i=1}^{4} \rho_i c_{pi} \varphi_i \frac{\partial T_S}{\partial t} = q_3 R_3 - q_2 R_2 + k_s (cU_R - 4\sigma T_S^4) + \alpha_V (T - T_S); \tag{6}$$

$$\rho_1 \frac{\partial \varphi_1}{\partial t} = -R_1\,, \ \rho_2 \frac{\partial \varphi_2}{\partial t} = -R_2\,, \rho_3 \frac{\partial \varphi_3}{\partial t} = \alpha_C R_1 - \frac{M_C}{M_1} R_3\,, \ \rho_4 \frac{\partial \varphi_4}{\partial t} = 0; \tag{7}$$

$$\sum_{\alpha=1}^{5} c_\alpha = 1, \ p_e = \rho R T \sum_{\alpha=1}^{5} \frac{c_\alpha}{M_\alpha}, \ v = (v_1, v_2, v_3), g = (0,0,g), m = (1 - \alpha_c) R_1 + R_2 + \frac{M_c}{M_1} R_3 + R_{54} + R_{55}.$$

Here and above $\frac{\mathrm{d}}{\mathrm{d}t}$ is the symbol of the total (substantial) derivative; α_v is the coefficient of phase exchange; ρ - density of gas – dispersed phase, t is time; v_i - the velocity components; T, T_S, - temperatures of gas and solid phases, U_R – density of radiation energy, k – coefficient of radiation attenuation, P – pressure; c_p – constant pressure specific heat of the gas phase, c_{pi}, ρ_i, φ_i – specific heat, density and volume of fraction of condensed phase: 1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel), R_i – the mass rates of chemical reactions, q_i – thermal effects of chemical reactions; k_g , k_S – radiation absorption coefficients for gas and condensed phases; T_e – the ambient temperature; c_α – mass concentrations of α - component of gas – dispersed medium, index α=1,2,...,5, where 1 corresponds to the density of oxygen, 2 – to carbon monoxide CO, 3 – to carbon dioxide and inert components of air, 4 – to particles of black, 5 – to particles of smoke; R – universal gas constant; M_α , M_C, and M molecular mass of α – components of the gas phase, carbon and air mixture; g is the gravity acceleration; c_d is an empirical coefficient of the resistance of the vegetation, s is the specific surface of the forest fuel in the given forest stratum, v_g – mass fraction of gas combustible products of pyrolysis, α_4 and α_5 – empirical constants. To define source terms that characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase, the following formulae were used for the rate of formulation of the gas-dispersed mixture $\dot{m}$, outflow of oxygen R_{51}, changing carbon monoxide R_{52}, generation of black R_{54} and smoke particles R_{55}.

$$R_{51} = -R_3 - \frac{M_1}{2M_2} R_5, R_{52} = v_g (1-\alpha_c) R_1 - R_5, R_{53} = 0, R_{54} = \alpha_4 R_1, R_{55} = \frac{\alpha_5 v_3}{v_3 + v_{3*}} R_3 .$$

Reaction rates of these various contributions (pyrolysis, evaporation, combustion of coke, and volatile combustible products of pyrolysis) are approximated by Arrhenius laws whose parameters (preexponential constant k_i and activation energy E_i) are evaluated using data for mathematical models (Grishin and Perminov 1995-1998):

$$R_1 = k_1 \rho_1 \varphi_1 \exp\left(-\frac{E_1}{RT_s} \right), R_2 = k_2 \rho_2 \varphi_2 T_s^{-0.5} \exp\left(-\frac{E_2}{RT_s} \right),$$

$$R_3 = k_3 \rho \varphi_3 s_\sigma c_1 \exp\left(-\frac{E_3}{RT_s} \right), R_5 = k_5 M_2 \left(\frac{c_1 M}{M_1} \right)^{0.25} \frac{c_2 M}{M_2} T^{-2.25} \exp\left(-\frac{E_5}{RT} \right).$$

Coefficients of multiphase (gas and solid phase) heat and mass exchange are defined $\alpha_V = \alpha S - \gamma C_P m, S = 4\varphi_S / d_S$. Here $\alpha = Nu\lambda/d_S$ – coefficient of heat exchange for sample of forest combustible material (for example, needle), Nu – Nusselt number for cylinder, λ – coefficient of heat conductivity for pine needle; γ – parameter, which characterizes the relation between molecular masses of ambient and inflow gases.

The system of equations 1 through 7 must be solved taking into account the initial and boundary conditions:

$$t = 0 : v_1 = 0, v_2 = 0, v_3 = 0, T = T_e, c_\alpha = c_{\alpha e}, T_s = T_e, \varphi_1 = \varphi_{ie}; \tag{8}$$

$$x_1 = -x_{1e} : v_1 = V_e, v_2 = 0, \frac{\partial v_3}{\partial x_1} = 0, T = T_e, c_\alpha = c_{\alpha e}, -\frac{c}{3k}\frac{\partial U_R}{\partial x_1} + cU_R / 2 = 0 ; \tag{9}$$

$$x_1 = x_{1e} : \frac{\partial v_1}{\partial x_1} = 0, \frac{\partial v_2}{\partial x_1} = 0, \frac{\partial v_3}{\partial x_1} = 0, \frac{\partial c_\alpha}{\partial x_1} = 0, \frac{\partial T}{\partial x_1} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_1} + \frac{c}{2} U_R = 0 ; \tag{10}$$

$$x_2 = x_{20} : \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, -\frac{c}{3k}\frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0 ; \tag{11}$$

$$x_2 = x_{2e} : \frac{\partial v_1}{\partial x_2} = 0, \frac{\partial v_2}{\partial x_2} = 0, \frac{\partial v_3}{\partial x_2} = 0, \frac{\partial c_\alpha}{\partial x_2} = 0, \frac{\partial T}{\partial x_2} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_2} + \frac{c}{2} U_R = 0 ; \tag{12}$$

$$\begin{gathered} x_3 = 0 : v_1 = 0, v_2 = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, -\frac{c}{3k}\frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0 , v_3 = v_{30}, T = T_g , \left| x_1 \right| \le \Delta, \left| x_2 \right| \le \Delta, \\ v_3 = 0, \ T = T_e , \left| x_1 \right| > \Delta, \left| x_2 \right| > \Delta; \end{gathered} \tag{13}$$

$$x_3 = x_{3e} : \frac{\partial v_1}{\partial x_3} = 0, \frac{\partial v_2}{\partial x_3} = 0, \frac{\partial v_3}{\partial x_3} = 0, \frac{\partial c_\alpha}{\partial x_3} = 0, \frac{\partial T}{\partial x_3} = 0, \frac{c}{3k}\frac{\partial U_R}{\partial x_3} + \frac{c}{2} U_R = 0 . \tag{14}$$

The conditions of symmetry are used because of the patterns of flow and distributions of all scalar functions are symmetrical relatively to the axes Ox_2.

The source of ignition is defined as a function of time at $\left| x_1 \right| \le \Delta_{x_1}, \left| x_2 \right| \le \Delta_{x_2}$ and turned off after the forest fire initiation. It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is "grey"), and the so-called diffusion approximation for radiation flux density was used for a mathematical description of radiation transport during forest fires. The components of the tensor of turbulent stresses, as well as the turbulent fluxes of heat and mass, are written in terms of the gradients of the average flow properties (Grishin 1997). It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy, and the space above it, while the appropriate components of the data base are used to calculate the specific properties of the various forest strata and the near-ground layer of atmosphere. This approach substantially simplifies the technology of solving problems of predicting the state of the medium in the fire zone numerically. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different type of forest; for example, pine forest (Grishin 1997; Perminov 1995).

Calculation Method and Results

The boundary-value problems 1 through 7 we solve numerically using the method of splitting according to physical processes (Perminov 1995). In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting was then integrated. A discrete analog was obtained by means of the control volume method using the SIMPLE like algorithm (Patankar 1981). The accuracy of the program was checked by the method of inserted analytical solutions. The time step was selected automatically. Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically.

Figure 2 illustrates the time dependence of dimensionless temperatures of gas and condensed phases (a), concentrations of components (b), and relative volume fractions of solid phases (c) at crown base of the forest

$$a)(1-\overline{T}=T/T_e, 2-\overline{T}_s=T_s/T_e, T_e=300K),$$

$$b)(1-\overline{C}_1=C_1/C_{1e}, 2-\overline{C}_2=C_2/C_{1e}, C_{1e}=0.23),$$

$$c)(1-\overline{\varphi}_1=\varphi_1/\varphi_{1e}, 2-\overline{\varphi}_2=\varphi_2/\varphi_{2e}, 2-\overline{\varphi}_3=\varphi_3/\varphi_{3e}).$$

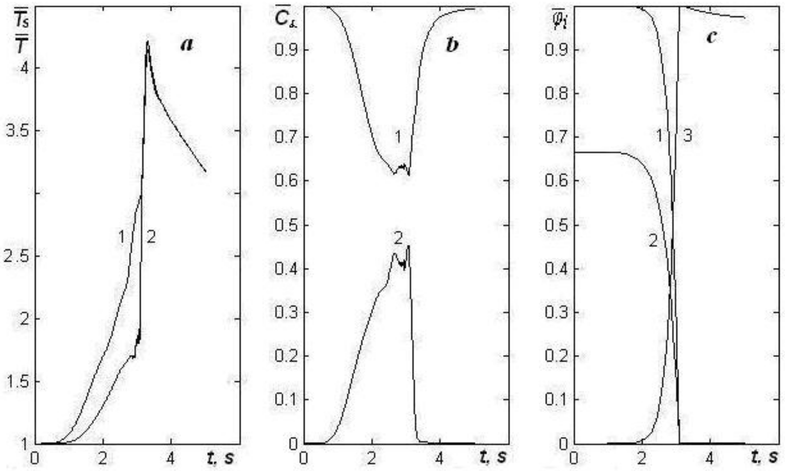

Figure 2—Relationships of (a) dimensionless temperatures, (b) concentrations, and (c) volume fractions in the lower boundary of the forest canopy.

At the moment of ignition the gas combustible products of pyrolysis burn away, and the concentration of oxygen is rapidly reduced. The temperatures of both phases reach a maximum value at the point of ignition. The ignition processes are of a gas-phase nature—that is, initial heating of solid and gaseous phases occurs and moisture is evaporated. Then the decomposition process into condensed and volatile pyrolysis products starts, the latter being ignited in the forest canopy. Note also that the transfer of energy from the fire source takes place due to radiation; the value of radiation heat flux density is small compared to that of the convective heat flux. As a result of heating of forest fuel elements, moisture evaporates, and pyrolysis occurs accompanied by the release of gaseous products, which then ignite. The effect of the wind on the zone of forest fire initiation is shown in figures 3 through 5, which present the space distribution of field of temperature for gas phase for different instants of time when a wind velocity V_e= 7 m/s. We can note that the isosurfaces are deformed by the action of wind. The isosurfaces of the temperature of gas phase 1,2,3 и 4 correspond to the temperatures $\overline{T}$ = 1.2., 2, 3 and 4. In the vicinity of the source of heat and mass release, heated air masses and products of pyrolysis and combustion float up. The wind field in the forest canopy interacts with the gas-jet obstacle that forms from the surface forest fire source and from the ignited forest canopy base. Recirculating flow forms beyond the zone of heat and mass release, while on the windward side the movement of the air flowing past the ignition region accelerates. Under the influence of the wind the tilt angle of the flame is increased. As a result this part of the forest canopy, which is shifted in the direction of the wind from the center of the surface forest fire source, is subjected to a more intensive warming up. The isosurfaces of the gas phase are deformed in the direction of the wind. Figures 4 and 5 present the distribution of the velocity and isosurfaces of the temperature at the different instants of time when a wind velocity V_e= 7 m/s.

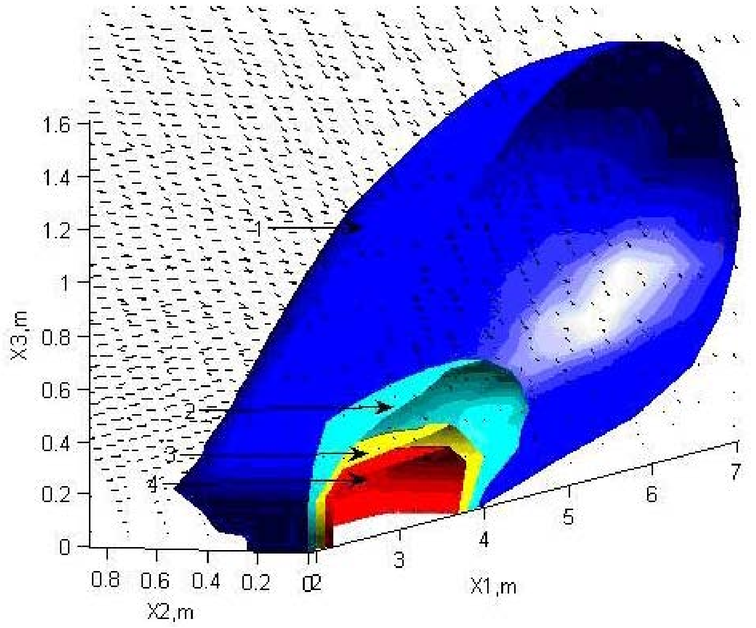

Figure 3—The vectorial field of velocity and temperature at *t=3.3 s.*

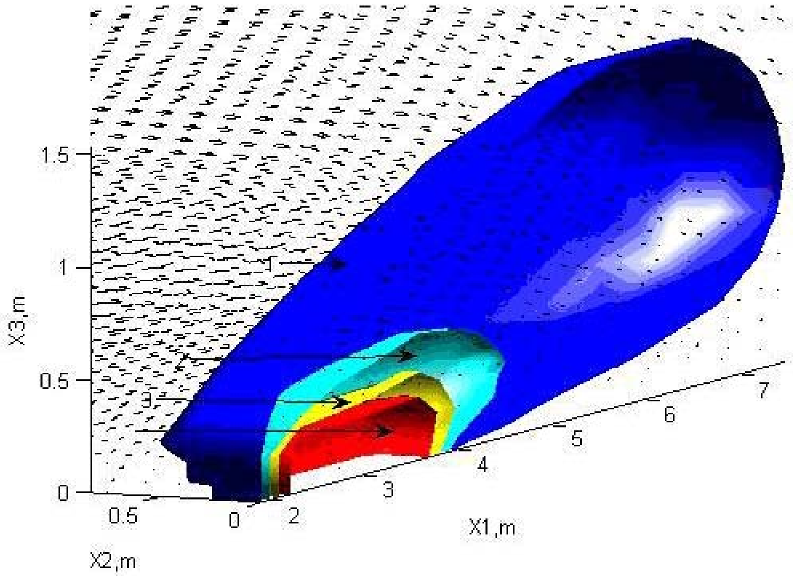

Figure 4—The vectorial field of velocity and temperature at *t=3.8 s.*

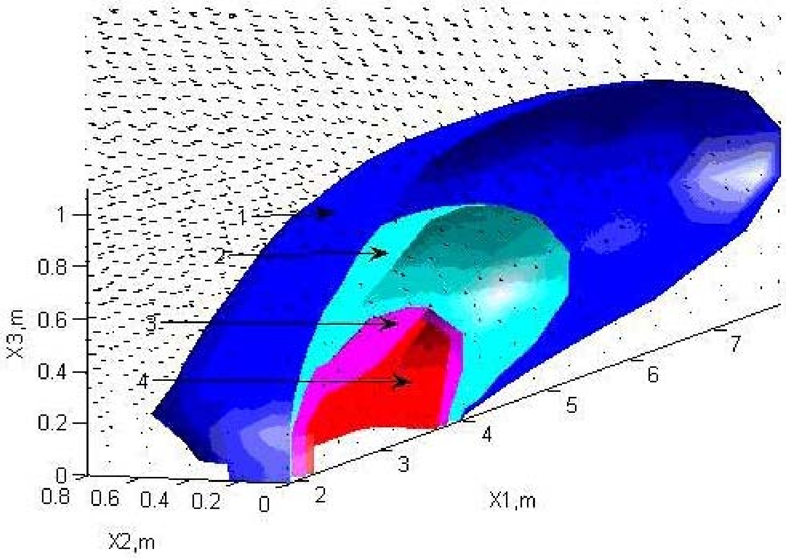

Figure 5—The vectorial field of velocity and temperature at *t=4.8 s.*

Conclusions

Mathematical model and the result of the calculation give an opportunity to evaluate critical condition of the forest fire initiation and spread, which allows applying the given model for preventing fires. The model overestimates the rate of the crown forest fires spread. The results obtained agree with the laws of physics and experimental data (Grishin 1997; Konev 1977). This work represents the attempt for application of three dimensional models for description of crown forest fires initiation and spread.

Acknowledgment

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