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Chapter 50 - Sample Scaling

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Digest: Following is an explanation of the changes throughout the directive by section.

50: This new chapter provides direction and technical descriptions for the various sampling systems approved for use in sample scaling of National Forest System timber.

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This chapter provides direction and technical descriptions for the various sampling systems approved for use in sample scaling of National Forest System timber. The approved sampling systems are set out in section 54. Basic statistical methods that can be used in sample scaling are also described in this chapter. The purpose of sample scaling is to estimate actual timber sale volume and/or value using a representative sample of logs. Although actual sale volume and value could be obtained with 100 percent scaling, estimates based on representative samples of logs can be constructed to any desired level of accuracy using appropriate statistical methods. Sample scaling, if properly done, can provide reliable estimates of timber sale volume and value at a lower cost than 100 percent scaling.

The examples given are intended to illustrate the arithmetic procedures involved in each sample scaling system. The examples are brief and are not based on actual data.

50.2 - Objective

The objective of sample scaling is to provide reliable estimates of timber sale volume and value at a lower cost than 100 percent scaling.

50.3 - Policy

Forest Service officers shall use only those sampling systems set out in this chapter and approved for use in sample scaling of National Forest System timber.

50.4 - Responsibility

1. The Regional Forester is responsible for approving sampling systems authorized in this Handbook for use in sample scaling of National Forest System timber, as set out in section 54. This authority may be redelegated to Forest Supervisors.

2. The Director of Forest and Rangeland Management, Washington Office may approve additional sampling systems.

51 - Statistical Concepts

Knowledge of sampling concepts and elementary statistical methods, such as population, variation, sampling error, sampling statistics, and the central limit theorem, is needed to choose and implement sample scaling systems. Knowledge of these concepts and methods is also necessary for proper sampling design (sec. 55).

1. The three basic elements that must be considered in sample scale design are:

    a. The population to be sampled.

    b. The degree of variation within the population.

    c. The sampling error objective.

2. Sections 51.1 through 51.9 provide an overview of the statistical concepts and basic formulas needed for log scaling sample design. For more detailed information about using statistical methods in sample scaling refer to the following publications or to college texts on forest mensuration and statistics:

a. Freese, Frank. 1962. Elementary Forest Sampling. Agric. Handb. 232. Washington, DC: U. S. Department of Agriculture.

b. Freese, Frank. 1967. Statistical Methods for Foresters. Agric. Handb. 317. Washington, DC: U. S. Department of Agriculture (Reprinted March 1974).

c. Grosenbaugh, L. R. 1965. Three-Pee Sampling Theory and Program "THRP" For Computer Generation of Selection Criteria. Research Paper PSW-21. Berkeley, CA: U. S. Department of Agriculture, Forest Service. Pacific Southwest Forest and Range Experiment Station.

d. National Institute of Standards and Technology (NIST) Handbook 44, Washington, DC: U. S. Department of Commerce.

51.1 - Statistical Notation

The following symbols and abbreviations are used in place of full, descriptive explanations of the generic statistical symbology used in the formulas in this chapter, as appropriate. These symbols and abbreviations are used to indicate what is being measured or calculated for each sampling method:

x	Individual measurement
N	Number of units in the population
n	Number of units in a sample
Σ	Summation sign, read as "sum of"
$\overline{x}$	Mean (or average value)
V	Variance (also referred to as "s^2")
SD	Standard deviation (also referred to as "*s*")
SE	Standard error (also referred to as "$s_{\overline{x}}$")
E	Sampling error in percent
CV	Coefficient of variation in percent
t	Constant based on the sample size and probability level (sec. 51.9, ex. 01)
df	Degrees of freedom or *n*-1 (sec. 51.9, ex. 01)
MV	Measured volume of a log
LMV	Measured volume of a load
KPI	The estimated or predicted volume in a log
KPIL	Sum of estimated gross log volume of all trees in a sample load
LKPI	The estimated or predicted volume in a load; typically derived by summing the *KPI*'s of all logs on a load
R	In 3P sampling (sec. 54.3), the *M/P* ratio or the ratio of the measured to predicted volume in a log

KZ	The sampling rate in 3P sampling (sec. 54.3)
VAL	Dollar value of a load of logs
W	Weight of a load of logs
VWR	Value to weight ratio for a load of logs
N_L	Number of loads in the population
N_{3P}	Number of logs on all sample loads available for 3P sampling (sec. 54.3)
n_L	Number of loads picked for sampling
n_{3P}	Number of logs selected for sampling

51.2 - Population

1. A population is a set of units from which a sample is drawn. Each unit in the sample becomes a basis for which an observation is made. Population units are rarely identical, and representative samples are critical because of the inherent variability among population units. If each unit in the population has the same volume, for example, only one sample unit would need to be selected and measured to obtain an estimate for the total population, providing the number of units in the population is known. Populations are never that uniform, however, and the values of the units comprising the population vary. Generally, when there is less variation in the measured variable from unit to unit, fewer sampling units are needed to get a reliable estimate. The reliability of estimates is usually measured by the sampling error.

For example, out of a population of all the truckloads of logs for a sale, a selected number of loads (sample units) are scaled to estimate a mean load volume. Mean load volume for an entire population (where all loads are scaled) is called a parameter. The mean volume is the statistic used to estimate the parameter.

2. In sample scaling, the population parameter of primary interest is usually mean load volume or mean load dollar value. The type of material being scaled and the sampling methodology being used determines when mean load volume or mean load dollar value should be used. The term “value” is used to represent volume, value, and dollar value. When either volume value, or dollar value should be given preference it should be noted before the sample scaling begins.

3. Sampling statistics are calculated in terms of mean load value, and sample size is based upon the variation in load values. Stratify large heterogeneous populations into homogeneous subpopulations based on correlated characteristics and load value when it will reduce the sampling cost. These characteristics include species, species price groups, products, truck or bunk size, and long versus short log loads.

The elements to be computed from the sample data are:

a. The estimate (usually in terms of volume or value).

b. Sampling error (sec. 52.11).

51.21 - Sample Statistics

Statistics are descriptive values computed from sample data. Common statistics computed from samples obtained in sample scaling are the arithmetic mean, standard deviation, coefficient of variation, standard error (of the mean), and sampling error.

51.3 - Arithmetic Mean

The arithmetic mean is the average value of the sample unit values obtained by dividing the sum by the number of sample units using the following formula:

$$\bar{x} = \frac{\sum^{n} x}{n}$$

The arithmetic mean is used to estimate the population mean.

51.31 - Standard Error of Mean

The standard error of the mean is calculated as follows:

$$SE = s_{\bar{x}} = \frac{SD}{\sqrt{n}} = \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n(n-1)}}$$

The standard error of the mean depends on the sample size. As the sample size increases, the standard error decreases. Since the reliability of the sample mean as an estimate of the population mean depends in part on the standard error, it is possible to achieve a higher level of reliability by increasing the sample size.

When simple random sampling is used and each sample unit appears only once in the sample (called sampling without replacement), a finite population correction can be applied to the standard error using the following formula:

$$SE = \frac{SD}{\sqrt{n}}\sqrt{1-\frac{n}{N}}$$

$$= \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n(n-1)} \times \left(1-\frac{n}{N}\right)}$$

The purpose of the finite population correction is to prevent an inflated statement of standard error when the number of sampling units is a large proportion (0.05 or greater) of the total population.

The finite population correction is not needed if the sampling fraction (*n/N*) is small, or less than 0.05.

51.4 - Frequency Distribution

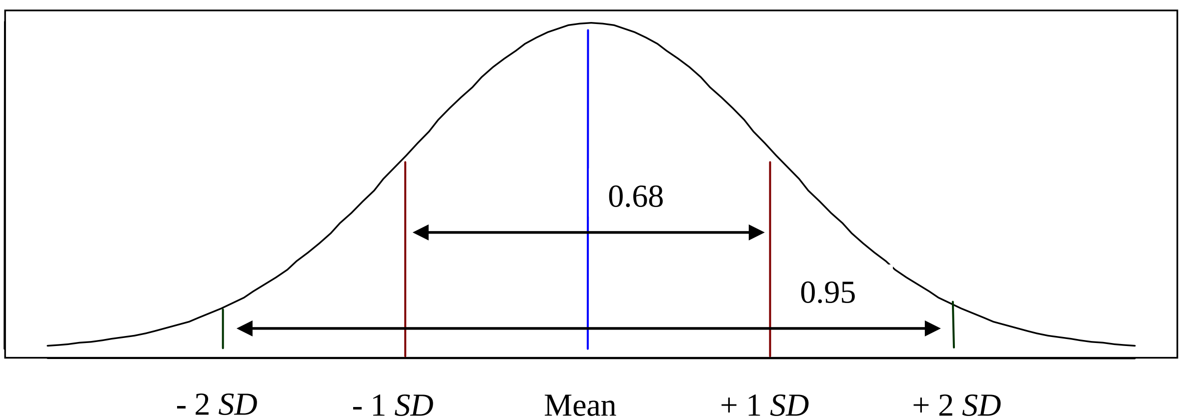
A frequency distribution displays a summary of data showing the frequency of occurrence of various values of a variable in a given population set. The most commonly used variables are the mean and the variance, a measure of the variability of the distribution. The frequency distribution for a given population is seldom known, as this would require complete enumeration of all population units. Therefore, for most applications, estimate the population frequency distribution parameters, such as the mean and variance.

51.41 - Normal Distribution

A normal distribution of data is characterized by a bell-shaped curve in a graph. With repeated sampling, the estimated sample means will form a bell-shaped curve with the peak of the bell occurring at the true population mean. With a sufficiently large sample, an estimate of the mean will be within 1 standard deviation of the population mean 68 percent of the time, and will be within 2 standard deviations of the population mean 95 percent of the time.

The standard deviation measures the dispersion of individual observations about their mean. The standard deviation of sample means is called the standard error of the mean.

Normal Distribution



51.5 - Central Limit Theorem

The central limit theorem states that for large samples, the distribution of the sample mean has approximately a normal distribution centered at the population mean. As a consequence of this theorem, it is possible to essentially ignore the underlying population frequency distribution. For most applications, a sample size of 30 or more sampling units is large enough for the central limit theorem to be applied. Estimates are based on samples. Repeated estimates of the same parameter will have a frequency distribution. For example, the population mean is estimated using the mean of the samples, or the sample average. The sample average depends on the samples that are chosen.

51.6 - Variance

Variance (*V*) is a parameter that measures the dispersion (scatter) of individual unit values about their mean. The sample variance is a statistic that estimates the parameter for the population variance and is determined using the following formula:

$$V = s^2 = \frac{\sum^{n}(x-\overline{x})^2}{n-1} = \frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n-1}$$

The sampling frequency needed to achieve a target sampling error depends on the variation within the defined population.

51.7 - Standard Deviation

The standard deviation is a measure of dispersion that is related to the variance (sec. 51.6). The sample standard deviation is used to estimate the population standard deviation and is calculated using the following formula:

$$SD = \sqrt{V} = s = \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n-1}}$$

51.8 - Coefficient of Variation

The coefficient of variation is a relative measure of dispersion where the standard deviation of the mean is expressed as a percentage of the mean. Calculate the coefficient of variation using the following formula:

$$CV = \frac{SD}{\bar{x}} \times 100$$

Because the coefficient of variation is a measure of relative variability, it can be used to compare the degree of variation between different populations. For example, if the following information is known for two timber sales:

<u>Timber Sale A</u>

$\bar{x}$ = 900 cubic feet per load
SD = 250 cubic feet per load

Then:

$$CV = \frac{250}{900} \times 100 = 27.8\%$$

<u>Timber Sale B</u>

$\bar{x}$ = 675 cubic feet per load
SD = 220 cubic feet per load

Then:

$$CV = \frac{220}{675} \times 100 = 32.6\%$$

Then, timber sale B exhibits greater variation than timber sale A and requires a larger number of samples to achieve the sampling error.

51.9 - Confidence Interval Estimates

The sample mean is called a point estimate of the population mean. Point estimates alone are often inadequate because there is no way to assess their reliability without additional information such as the standard error (sec. 51.31). Interval estimates provide an alternative. The most commonly used interval estimate is the confidence interval.

Confidence interval estimates, like other statistics, depend on the sample. If two independent samples of the same size are chosen and 95 percent confidence intervals are computed for each sample, it is likely that the two intervals will be different. When repeated samples of the same size are chosen and 95 percent confidence intervals are computed for each sample, then 95 percent of these intervals will contain the true mean.

A confidence interval for the population mean is calculated using the following formula where $\bar{x}$ and *SE* are known and *t* is a constant that depends on the sample size and probability level:

Confidence interval = $\bar{x} \pm (t \times SE)$

See exhibit 01 for the distribution of *t* for 90 percent, 95 percent, and 99 percent confidence interval. The first column labeled "*df*" is the degrees of freedom, or *n*-1; the second column labeled 0.10 is the 90 percent probability level; the third column labeled 0.05 is the 95 percent probability level; and the fourth column labeled 0.01 is the 99 percent probability level.

The following is an example of a confidence interval calculation:

Given:

$n = 211$

$\bar{x} = 631 \text{ cubic feet}$

$SD = 241 \text{ cubic feet}$

$t = 1.96 \text{ (tabulated at a 95\% confidence interval)}$

Then:

$$SE = \frac{241}{\sqrt{211}} = 16.59 \text{ cubic feet}$$

$$\text{Confidence Interval} = \bar{x} \pm (t \times SE) = 631 \pm (1.96 \times 16.59) = 631 \pm 32.52$$

This can be interpreted as follows:

Unless a 1 in 20 (or 5 percent) chance of error has occurred, the confidence interval of 598 cubic feet to 664 cubic feet will contain the true population mean value.

51.9 - Exhibit 01

t Distribution Table

	Probability		
df	0.10	0.05	0.01
1	6.314	12.706	63.657
2	2.920	4.303	9.925
3	2.353	3.182	5.841
4	2.132	2.776	4.604
5	2.015	2.571	4.032
6	1.943	2.447	3.707
7	1.895	2.365	3.499
8	1.860	2.306	3.355
9	1.833	2.262	3.250
10	1.812	2.228	3.169
11	1.796	2.201	3.106
12	1.782	2.179	3.055
13	1.771	2.160	3.012
14	1.761	2.145	2.977
15	1.753	2.131	2.947
16	1.746	2.120	2.921
17	1.740	2.110	2.898
18	1.734	2.101	2.878
19	1.729	2.093	2.861
20	1.725	2.086	2.845
21	1.721	2.080	2.831
22	1.717	2.074	2.819
23	1.714	2.069	2.807
24	1.711	2.064	2.797
25	1.708	2.060	2.787
26	1.706	2.056	2.779
27	1.703	2.052	2.771
28	1.701	2.048	2.763
29	1.699	2.045	2.756
30	1.697	2.042	2.750
40	1.684	2.021	2.704
60	1.671	2.000	2.660
120	1.658	1.980	2.617
∞	1.645	1.960	2.576

52 - Sampling Statistics

Sample estimates are used to determine the merchantable volume of timber cut and removed, by species, from the sale area for payment purposes.

52.1 - Determining Sample Size

When calculating sample size, if the estimated coefficient of variation (sec. 51.8) for a population is known, the number of sampling units required for a specified sampling error (*E*) can be calculated. Two sampling conditions occur:

1. Sampling finite populations. The total number of sampling units in the population is approximately known. In this case, use the following formula:

$$n = \frac{1}{\frac{E^2}{t^2CV^2} + \frac{1}{N}} = \frac{(tCV)^2}{E^2 + \frac{(tCV)^2}{N}}$$

2. Sampling infinite populations. The number of sampled units is a small proportion (less than 0.05) of the total population. In this case, use the following formula:

$$n = \frac{t^2CV^2}{E^2} = \frac{(tCV)^2}{E^2}$$

52.11 - Sampling Error

The sampling error is a relative expression of the confidence interval. It is the standard error of the mean times the *t* value expressed as a percentage of the mean. Calculated sampling error using the following formula:

$$E = \frac{SE}{\bar{x}} \times t \times 100$$

53 - Stratification

The statistical calculations discussed in sections 51.3 through 52.11 assume simple random sampling. In certain cases, it may be appropriate to use stratified random sampling. In stratification, divide the heterogeneous population into homogenous subpopulations (strata) where individuals with similar characteristics, such as species or dollar value classes, are grouped into a stratum.

Create strata so that the variation within each stratum is less than it would be for the (unstratified) population as a whole. This enables taking fewer sampling units for a specified sampling error. With stratification, a given number of sample units generally provides a more reliable estimate than if the same number of sample units were taken for the unstratified population.

For example, where sampling units are truckloads, a population of truckloads might be stratified on the basis of value (dollar value of species groups or products) or truck size. This may reduce the sample size needed for a stated objective, compared to the number needed for the unstratified population.

When stratification is used, the stratum must be clearly defined prior to selecting samples. Post stratification must be avoided.

Two methods of sample allocation in stratified sampling are optimum allocation and proportional allocation. These are described in sections 53.1 and 53.2.

53.1 - Optimum Allocation

When using the optimum allocation method of stratified sampling, the numbers of sample units are allocated to each stratum so as to produce the smallest standard error given a total number of sample units. Optimum allocation requires an estimate of the number of units in each stratum, as well as the variation in each stratum. Based on this information, calculate a separate sampling intensity for each stratum and the total number of sample units prorated (allocated) by strata. Refer to section 54.12 for examples on using optimum allocation.

53.2 - Proportional Allocation

When using the optimum allocation method of stratified sampling, sample units are allocated to each stratum according to the proportion of the population in each stratum (sec. 54.12).

54 - Sample Scaling Systems

Use sample scaling when the volume of business warrants and scaling costs can be reduced (compared to 100 percent scaling) and when accuracy acceptable to the Regional Forester can be maintained (FSM 2443.1). See section 55.3 for sampling error standards. The general procedure to use in sample scaling is to count and, when necessary, weigh all loads. Scale a random sample of all loads or logs within loads. The load averages from the sample are applied to the total number of loads to estimate total sale value and volume.

The sample scaling systems approved for use in sample scaling of National Forest System timber are:

1. Sample load log scaling (sec. 54.1).

2. Sample load log scaling with sample load weight (sec. 54.2).

3. 3P sample scaling (sec. 54.3).

4. Sample load with 3P subsample log scaling (sec. 54.4).

5. Sample load with load weight and 3P subsample log scaling (sec. 54.5).

When appropriate, use stratification with these systems to improve sampling efficiency. The period over which estimates are developed is ordinarily the life of the sale for flat rate sales and a calendar quarter for stumpage rate adjustment sales. There can be no retroactive adjustments (to previous quarterly value estimates) on stumpage rate adjustment sales unless technique errors, such as computation errors, are discovered.

In flat rate sales, use a running mean calculation procedure. Load averages to date are calculated using the sample data for the current month plus the sample data for all previous months. In stumpage rate adjustment sales, a running mean calculation procedure based over the life of the sale may be used when sample populations are composed of a single species (or with uniform value).

54.1 - Sample Load Log Scaling

In this scaling system, a sample of (n_L) loads is randomly selected for scaling from the population (N_L) to be sampled (all loads from the sale or within a stratum). The variable of interest, mean load value, is subject to sampling error. All loads are counted and sample loads are scaled. The estimated mean load value is determined from the sample and multiplied by the total load count to determine total estimated value.

For this sampling method, load dollar value is typically used to calculate the coefficient of variation and sample size. There may be a significant difference between the dollar value of species in the sale and variation of the mix of species on each load. Weighting the load volume by value gives a higher coefficient of variation and requires a larger sample size than using volume alone.

54.11 - One-Stratum Sales

Methods for calculating the number of sample loads needed for one-stratum sales and for stratified sales are described by the following examples.

1. Use the finite population formula (sec. 52.1) to calculate the total number of sample loads needed over the life of the sale.

$$n = \frac{1}{\frac{E^2}{t^2 CV^2} + \frac{1}{N}} = \frac{(tCV)^2}{E^2 + \frac{(tCV)^2}{N}}$$

Where: n_L = Number of sample loads
t = Student's t (with a sufficiently large sample, t is about 2 for a 95% probability)
CV = Coefficient of variation in percent
E = Desired sampling error in percent for the sale as a whole
N_L = Number of loads expected to be hauled from the sale area

The coefficient of variation, which is based on load value, can be estimated in several ways:

a. Estimate *CV* based on experience from similar past sales.

b. Estimate *CV* based on a random sample of loads. An example of a calculation is shown in the following table, assuming species values are $85/CCF for DF (Douglas-fir) and $48/CCF for WF (white fir) (where CCF is hundred cubic feet):

Load	DF Vol. in CCF (*MV* DF)	Value (*VAL* DF)	WF Vol. in CCF (*MV* WF)	Value (*VAL* WF)	Load Value (*VAL*)	Load Value Squared
1	10.46	889	1.72	83	972	944,784
2	10.16	864	1.00	48	912	831,744
3	10.25	871	1.75	84	955	912,025
4	8.67	737	1.96	94	831	690,561
5	2.30	196	8.90	427	623	388,129
Total					4,293	3,767,243

$$SD = \sqrt{\frac{\sum^{n} VAL^2 - \frac{\left(\sum^{n} VAL\right)^2}{n_L}}{n_L - 1}}$$

$$= \sqrt{\frac{3{,}767{,}243 - (4{,}293^2 \div 5)}{5-1}}$$

$$= 142.5$$

$$CV = \frac{SD}{\overline{VAL}} \times 100$$

$$CV = \frac{142.5}{(4{,}293 \div 5)} \times 100 = 16.6\%$$

= 17%, rounded

2. Calculate the number of sample loads needed and the sampling rate:

Where: Total sale volume (from the sale contract) = 7,869 CCF
Estimated average load volume = 8.50 CCF
CV = 17% (from preceding example)
N_L = Total loads in sale = 7,869/8.50 = 926
t = 2 (95% probability level)
E = Desired sampling error of 4%

$$n_L = \frac{(tCV)^2}{E^2 + \frac{(tCV)^2}{N_L}}$$

$$= \frac{(2 \times 17)^2}{4^2 + \frac{(2 \times 17)^2}{926}}$$

$$= \frac{1156}{16 + 1.2500} = 67 \text{ loads}$$

Sampling rate or frequency $= \frac{926}{67} = 14$, or an average of 1 load scaled for every 14 hauled.

3. Recalculate the sampling intensity based upon a revised coefficient of variation determined from at least 20 loads scaled. Recalculate the sampling intensity again after the 50th sample load is scaled and at least yearly thereafter.

4. Do not sample less than 4 percent of the sale (1:25 frequency).

54.11a - Stratum Sampling Error

The sampling error for each stratum or component is first computed, and a sampling error is computed for the sale as a whole.

Use the tabulated *t* value for the 95 percent probability level with the appropriate degrees of freedom (n-1) when calculating sampling error. The following are two examples to illustrate this process.

Example 1: Sampling error calculation for one stratum.

Sample Load Number	Load Value (VAL)	Load Value Squared (VAL^2)
1	$972	944,784
2	912	831,744
3	955	912,025
4	831	690,561
5	623	388,129
Total	$4,293	3,767,243

Compute the mean load value, standard deviation, and standard error of the mean:

$$\overline{VAL} = \frac{\$4{,}293}{5} = \$858.60$$

$$SD = \sqrt{\frac{\sum^{n} VAL^2 - \frac{\left(\sum^{n} VAL\right)^2}{n_L}}{(n-1)}}$$

$$= \sqrt{\frac{3{,}767{,}243 - (4{,}293^2 \div 5)}{4}}$$

$$= 142.54$$

$$SE = \frac{SD}{\sqrt{n_L}}$$

$$= \frac{142.54}{\sqrt{5}} = 63.75$$

Compute the stratum sampling error:

$$E = \frac{SE}{\overline{VAL}} \times 100 \times t$$

$$= \frac{63.75}{858.60} \times 100 \times 2.571 = 19.1\%$$

Example 2: For a sale with multiple strata, aggregate the errors using the following procedure:

1. Given stratum sampling errors and value, compute the sampling error for the sale as a whole.

Stratum No.	Value in M$ (VAL_i)	Sampling Error % (E_i)	Value × Error % ($VAL_i \times E_i$)	$(VAL_i \times E_i)^2$
1	$15.21	12	182.52	33,313.55
2	8.15	17	138.55	19,196.10
3	2.20	35	77.00	5,929.00
Total	$25.56			58,438.65

$$E_T = \frac{\sum^{n_t}\left(VAL_i \times E_i\right)^2}{\sum^{n_t} VAL_i}$$

$$= \sqrt{\frac{(VAL_1 \times E_1)^2 + (VAL_2 \times E_2)^2 + (VAL_3 \times E_3)^2}{\left(V_1 + V_2 + V_3\right)^2}}$$

$$= \sqrt{\frac{33{,}313.55 + 19{,}196.10 + 5{,}929.00}{25.56^2}} = 9.5\%$$

2. Use the same procedure to combine quarterly sampling data.

54.12 - Stratified Sales

Use the following method to calculate sample size with optimum allocation of sample units among strata (sec. 53.1).

1. To calculate the sample size:

   a. Specify the sampling error objective for the sale as a whole.

   b. Divide (stratify) the sale population into sampling components.

   c. Calculate coefficient of variation by stratum and a weighted *CV* over all strata.

   d. Calculate number of sample loads by stratum.

Example: Given the following information, determine the number of sample loads by stratum.

Stratum	Timber Characteristic	Est. $/Load	Est. No. Loads	Est. M$	% Value *(a)*	Est. *CV* *(b)*	*CV* Fraction *(a)* × *(b)*
1	Ponderosa Pine	1,500	200	300	.54	15	8.1
2	Fir	800	150	120	.22	20	4.4
3	Lodgepole Pine	450	300	135	.24	10	2.4
Total			650	555	1.00		14.9

Sale sampling error objective: $E = 4\%$
Weighted coefficient of variation: $CV = 14.9\%$

The total number of sample loads needed for all strata, or the sale as a whole, (n_T):

$$n_T = \frac{(tCV)^2}{E^2 + \frac{(tCV)^2}{N}}$$

$$= \frac{(2 \times 14.9)^2}{4^2 + \frac{(2 \times 14.9)^2}{650}}$$

$$= \frac{888.04}{17.37}$$

$$= 51.1 = 52 \text{ loads}$$

The total number of samples in the sale can be allocated to strata, (n_s) using optimal allocation:

$$n_s = \frac{(CV \text{ fraction})(n_T)}{\text{weighted } CV}$$

Where: n_s = Stratum sampling units

$$n_1 = \frac{(8.1)(52)}{14.9} = 28$$

$$n_2 = \frac{(4.4)(52)}{14.9} = 15$$

$$n_3 = \frac{(2.4)(52)}{14.9} = 9$$

2. To calculate the sampling rate (frequency) use the following formula:

Frequency = Estimated total loads in stratum $\div$ sample loads in stratum

$$\text{Stratum 1:} \quad \text{Frequency} = \frac{200}{28} = 7.1 \; or \; 1 \; in \; 7$$

$$\text{Stratum 2:} \quad \text{Frequency} = \frac{150}{15} = 10.0 \; or \; 1 \; in \; 10$$

$$\text{Stratum 3:} \quad \text{Frequency} = \frac{300}{9} = 33.3 \; or \; 1 \; in \; 33$$

3. As an alternative, allocate the total number of samples in the sale to strata (n_s) using the proportional allocation formula:

$$n_s = \frac{\text{Strata size} \times n_T}{\text{Total sale size}}$$

$$n_1 = \frac{200 \times 52}{650} = 16$$

$$n_1 = \frac{150 \times 52}{650} = 12$$

$$n_1 = \frac{300 \times 52}{650} = 24$$

54.13 - Calculating Sample Expansion

For all sales, keep a monthly record of each load hauled. For scaled loads, record at least the truck ticket number, date scaled, scaled volume by species, and, if being weighed, the net weight. For count loads, record the truck ticket number, date hauled, and, if being weighed, the net weight.

For flat rate sales, calculate means over the life of the sale. Combine the sample scale data for the current month with those of the preceding months, and use the new averages to determine the estimated volume for that period (month). For stumpage rate adjustment sales, quarterly volumes and values shall be final.

54.2 - Sample Load Log Scaling with Sample Load Weights

All loads are weighed and a sample of (n) loads is randomly selected from the population for scaling. Two variables are combined, load value and load weight, to establish the load value/weight ratio. The load value is sample-based and subject to sampling error. The mean

load value/weight ratio determined from the sample is multiplied by the total weight of all loads for the period to determine total estimated value.

When there is variation in load weight (due to different bunk sizes, for example), using weight instead of load count to expand samples may reduce the coefficient of variation and hence the total number of loads to be scaled. The effect of different size loads on statistical variation is reduced when weight is used. Overall timber accountability is improved when all loads are weighed.

The Contracting Officer shall approve scales on which National Forest System timber will be weighed (contract provision B6.814). Only full platform scales, which can weigh the entire load of logs in a single operation, are acceptable. Scales must meet all the requirements for weighing commercial vehicles set forth in the current edition, including amendments of the United States Department of Commerce, National Institute of Standards and Technology, NIST Handbook 44 (sec. 51).

54.21 - Calculating Sample Size, Sample Load with Weight

The procedure for calculating sample size for this scaling system is similar to the procedure described in section 54.11 for sample load log scaling.

Use the following formula to calculate the coefficient of variation:

$$CV = \frac{SD}{\overline{VWR}} \times 100$$

Where:

$$SD = \sqrt{\frac{\sum^{n_L} VWR^2 - \frac{\left(\sum^{n_L} VWR\right)^2}{n_L}}{n_L - 1}}$$

Where: VWR = \$/ton = Value/weight ratio, or value of the load divided by load weight
n = Number of sample loads
$\overline{VWR}$ = Mean value/weight ratio

Example: Calculate *CV* given the following data:

Load Number	Value (*VAL*)	Weight in tons (*W*)	\$/Ton (*VWR*)	\$/Ton^2 (VWR^2)
1	844.94	27.0	31.29	979.06
2	801.51	25.3	31.68	1003.62
3	756.23	26.1	28.97	839.26
4	779.55	26.7	29.20	852.64
5	781.88	25.9	30.19	911.44
Total			151.33	4,586.02

$$SD = \sqrt{\frac{4{,}586.02 - (151.33^2) \div 5}{5-1}}$$

$$= 1.211$$

$$CV = \frac{1.211}{151.33 \div 5} \times 100 = 4.002\%$$

= 4%, rounded

54.22 - Calculating Sample Expansion, Sample Load with Weight

Keep a monthly record of each load hauled. For scaled loads, record at least date, truck ticket number, net load weight, and volume scaled by species. For count-and-weigh-only loads, record date, truck ticket number, and net load weight.

For flat rate sales, calculate means over the life of the sale. Combine the sample scale data for the current month with those of the preceding months. The new averages are used to determine the estimated volume for that period (month). For stumpage rate adjustment sales, quarterly volumes and values shall be final.

54.23 - Sampling Error, Sample Load with Weight

Sampling error can be computed using the following formulas:

$$E = \frac{SE}{\overline{\overline{VWR}}} \times 100 \times t$$

Where:

$$SE = \frac{SD}{\sqrt{n_L}}\sqrt{1-\frac{n_L}{N_L}}$$

$$= \sqrt{\frac{\sum^{n_L} VWR^2 - \frac{\left(\sum^{n_L} VWR\right)^2}{n_L}}{n_L(n_L-1)} \times \left(1-\frac{n_L}{N_L}\right)}$$

And: VWR = \$/ton = Value/weight ratio, or value of the load divided by load weight
N_L = Total number of loads in stratum or sale
n_L = Number of sample loads
$\overline{VWR}$ = Mean value/weight ratio

Example: Calculate E given the following data. In this example the actual t value is used.

ΣVWR^2 = 19,500
ΣVWR = 605.34
$\overline{VWR}$ = 30.27
N_L = 100
n_L = 20
t = 2.093 (95% confidence interval)

$$SE = \sqrt{\frac{19500 - \frac{605.34^2}{20}}{20 \times 19} \times \left(1-\frac{20}{100}\right)} = 1.57$$

$$E = \frac{1.57}{30.27} \times 100 \times 2.093 = 10.9\%$$

54.3 - 3P Sample Scaling

1. The 3P sampling system is a form of variable probability sampling that involves observing each log from a timber sale presented for scaling. Gross log volume (*KPI*) is estimated and the estimate is compared with a random number. If the estimated volume (*KPI*) is equal to or greater than the random number, the log is scaled as a sample unit.

2. The probability of a log being selected as a sample unit is proportional to its predicted volume (*KPI*), hence 3P: probability proportional to prediction.

The larger the predicted volume (*KPI*), the greater chance a log has of being selected as a sample. A log with a *KPI* of 10 has twice the chance of being selected as does a log with a *KPI* of 5. Therefore, the larger logs are favored for sample log selection.

3. The variable of interest in 3P scaling is the *M/P* ratio (measured volume/predicted volume). Determine the *M/P* ratio for each sample log by dividing the scaled volume of the log by the predicted volume. This procedure is the same when calculating either gross or net volume.

Determine total estimated sale volume by multiplying predicted log volumes (*KPI*'s) by the mean *M/P* ratio. The *M/P* ratio is the sample base that is subject to sampling error.

54.31 - Calculating Sample Size

The coefficient of variation of the *M/P* ratio is normally low. Therefore, few sample logs are needed to achieve satisfactory sampling errors.

At a minimum, the sample design shall require 30 or more 3P sample logs.

Note: In the following calculations to determine the scaler's *CV*, a ratio or gross volume to gross predicted volume is used. This may result in a lower scaler's *CV* than would be encountered on a sale with that has a highly variable defect percentage from log to log. If the sale in question has highly variable defect (for example, no defect in log 1, 50 percent defect in log 2, and 30 percent defect in log 3) then the scaler's *CV* should be increased.

1. Determine the coefficient of variation from the scaler's actual *CV* experienced on previous sales, or assume a *CV* to start and calculate the scaler's actual *CV* as logs are scaled (an experienced scaler can usually achieve a *CV* of less than 30 percent).

An example of determining *CV* for a scaler follows:

Gross Scaled Cubic Ft. (*MV*)	Gross Predicted Cubic Ft. (*KPI*)	*M/P* Ratio (*R*)	*M/P* Ratio Squared (R^2)
20	22	0.9091	0.8265
50	50	1.0000	1.0000
25	20	1.2500	1.5625
18	20	0.9000	0.8100
14	12	1.1667	1.3612
23	30	0.7667	0.5878
20	30	0.6667	0.4445
20	28	0.7143	0.5102
22	24	0.9167	0.8403
10	15	0.6667	0.4445
Total		8.9569	8.3875

$$SD = \sqrt{\frac{\sum^{n_{3P}} R^2 - \frac{\left(\sum^{n_{3P}} R\right)^2}{n_{3P}}}{n_{3P} - 1}}$$

$$SD = \sqrt{\frac{8.3875 - \frac{(8.9569)^2}{10}}{10-1}} = 0.2014$$

$$\overline{R} = \frac{\sum R}{n_{3P}} = \frac{8.9569}{10} = 0.8957$$

$$CV = \frac{SD}{\overline{R}} \times 100$$

$$CV = \frac{0.2014}{0.8957} \times 100 = 22.49\% = 23\% \text{ rounded}$$

Where: *MV* = Scaled log volume
KPI = Predicted or estimated log volume
R = Measured to predicted ratio (*MV/KPI*)
R^2 = Ratio squared

2. Compute the number of sample logs to be scaled to meet sampling error requirements as follows; using the finite population formula (sec. 51.4) to calculate the total number of sample logs needed for the life of the sale:

Where: n_{3P} = Number of sample logs
t = Student's *t* for the 95% probability level = 2
CV = Coefficient of variation (percent) of ratios (*M/P*) = 22
E = Desired sampling error in percent for the sale as a whole = 4.0
N_{3P} = Number of logs expected to be hauled from the sale area = 2,500

$$n_{3P} = \frac{(tCV)^2}{E^2 + \frac{(tCV)^2}{N_{3P}}}$$

$$= \frac{(2 \times 22)^2}{4^2 + \frac{(2 \times 22)^2}{2500}} = 115 \text{ logs}$$

54.31a - Calculating *KZ*

KZ is the sampling rate and is equal to the estimated stratum volume divided by the desired number of 3P sample logs. Calculate *KZ* as follows:

$$KZ = \frac{\text{stratum volume}}{\text{desired number of 3P sample logs}}$$

There are several ways to calculate an estimate of stratum volume. One example follows:

Where: n_L = Number of sample loads = 50
L = Average number of logs per load = 50
$\overline{MV}$ = Average volume/log = 20 cubic feet

Then: Stratum volume = $n_L \times L \times \overline{MV} = 50 \times 50 \times 20$ = 50,000 cubic feet

The *KZ* for the desired 115 3P sample logs is:

$$KZ = \frac{50{,}000}{115} = 434.78 = 435 \text{ cubic feet}$$

In this example, the sampling frequency is about one log for each sum of 435 *KPI*. In other words, for each 435 cubic feet of estimated log volume, one sample log is scaled. Monitor the *KZ* after scaling five (5) sample loads to assure that the proper sample is being achieved. If it is clear after scaling five loads that there are definitely not enough sample logs, or too many sample logs, adjust the *KZ* to correct the under or over sampling only before the sixth (6) load or at the end of the quarter.

54.32 - Calculating Sample Expansion

1. Calculate estimated total sale or stratum volume from a 3P sample by multiplying the stratum or sale sum of *KPI*'s by the mean *M/P* ratio for the sale or stratum.

$$VOL_T = \left(\sum^{n_{3P}} KPI \right) x\ \overline{R}$$

Where: VOL_T = Total estimated sale or stratum volume
KPI = Estimated log volume within the sale or stratum
$\overline{R}$ = Mean M/P ratio for the sale or stratum

The mean *M/P* ratio, by stratum, is an adjustment factor to be applied to the predicted stratum volume or sum of *KPI*'s.

The calculations for volume expansion of gross and net scale by species are shown as follows for five loads of logs. Under normal circumstances, approximately 33 3P sample logs would be scaled from these five loads to meet the 16.2 percent calculated sample size. However, for ease

of display, only 10 samples are used in the following example for Douglas-fir (DF) and lodge pole pine (LP).

Example:

Species	Scaled Gross (*MVG*)	Predicted Gross (*KPI*)	Gross Ratio (*RG*)	Net Scaled (*MVN*)	Net Ratio (*RN*)
DF	24	22	1.0909	20	0.9091
DF	26	20	1.3000	25	1.2500
DF	18	20	0.9000	18	0.9000
DF	28	30	0.9333	23	0.7667
DF	13	15	0.8667	10	0.6667
DF	24	24	1.0000	22	0.9167
Total			6.0909		5.4092

Species	Scaled Gross (*MVG*)	Predicted Gross (*KPI*)	Gross Ratio (*RG*)	Net Scaled (*MVN*)	Net Ratio (*RN*)
LP	20	20	1.0000	20	1.0000
LP	35	30	1.1667	35	1.1667
LP	12	14	0.8571	10	0.7143
LP	8	6	1.3333	4	0.6667
Total			4.3571		3.5477

Number of sample logs = 6 DF and 4 LP as follows:

2. Calculate average ratios $(\overline{RG} \text{ and } \overline{RN})$ for each species by dividing the sum of ratios (ΣRG and ΣRN) by the number of sample logs.

	Species	Sum Ratio (Σ*R*)	No. Sample Logs	Mean Ratio $\overline{R}$
Gross	DF	6.0909	6	1.0152
Net		5.4092	6	0.9015
Gross	LP	4.3571	4	1.0893
Net		3.5477	4	0.8869

3. Calculate the expanded volumes for each species by multiplying the sum of *KPI*'s (Σ*KPI*) by the mean ratio ($\overline{R}$). Assume the sum of the estimates (*KPI*'s) for DF totals 3,500 and for LP, the total is 1,560. Convert cubic foot volumes to CCF by dividing by 100.

	Species	Σ*KPI*	$\overline{R}$	Cubic Volume (Cu. Ft.)	Cubic Volume (CCF)
Gross	DF	3500	1.0152	3,553.2	35.532
Net		3500	0.9015	3,155.3	31.553
Gross	LP	1560	1.0893	1,699.3	31.553
Net		1560	0.8869	1,383.6	13.836

4. For flat rate sales, calculate means over the life of the sale. Combine the sample scale data for the current month with those of the preceding months. The new averages are used to determine the estimated volume for that period (month). For stumpage rate adjustment sales, quarterly volumes and values shall be final.

5. When *KZ* is changed, treat the data collected under the old *KZ* as belonging to a separate and completed stratum. Sample expansion and sampling error calculations are completed for these data. The sample scaling process begins anew under the new *KZ* as if for a new sale. The reason for this is that when the sampling rate is changed, the weight or contribution to total estimated sale volume represented by each sample log changes.

54.33 - 3P Sampling Error

The sampling error of a 3P sample is based on the variance of the *M/P* ratio. It is calculated from the following formula using gross to net ratios.

$$E = \frac{SE}{\overline{R}} \times 100 \times t$$

Where: E = Sampling error in percent, and

$$SE = \sqrt{\frac{\sum^{n} R^2 - \frac{\left(\sum^{n} R\right)^2}{n_{3P}}}{(n_{3P}-1)n_{3P}}}$$

Where: SE = Standard error
R = *M/P* ratio for a 3P sample log
N_{3P} = Number of 3P sample logs
t = Tabulated t value of 95% probability level

The sampling error calculations are shown in the following example for a one-stratum timber sale.

Sample Log No.	Gross to Net *M/P* Ratio (*R*)	Gross to Net M/P^2 Ratio
1	1.0909	1.1901
2	1.0000	1.0000
3	1.3000	1.6900
4	0.9000	0.8100
5	1.1667	1.3612
6	0.9333	0.8710
7	0.8667	0.7512
8	0.8571	0.7346
9	1.0000	1.0000
10	1.3333	1.7777
Total	10.4480	11.1858

$$SE = \sqrt{\frac{11.1858 - \frac{(10.4480)^2}{10}}{(10-1)10}}$$

$$SE = \sqrt{\frac{0.2697}{90}} = 0.0547$$

$$E = \frac{SE}{\overline{R}} \times 100 \times t$$

$$E = \frac{0.0547}{1.0448} \times 100 \times 2.262$$

E = 11.8% (95% probability level)

54.34 - 3P Scaling Procedure

The general scaling procedure is to predict the gross volume of each log and to scale those logs selected as samples. Experience indicates that predictions are best made from the small end of the log, proceeding up one side and down the other side of the load (in roll-out scaling). When a log is a 3P sample, write the log number on the small end of the log with a bright-colored keel or paint.

54.35 - Field Procedures Using Automated Sample Selection and Recording Methods

Use a data recorder with a random number generator that automatically selects samples to be scaled. Record scale data on the data recorder and download to a personal computer for volume computations.

54.4 - Sample Load with 3P Subsample

This is a two-stage sampling method. The first stage is a load sample. The second stage is a 3P subsample of the logs on each of the sample loads. The first stage sample loads are selected randomly, within groups, with equal probability.

The purpose of the first stage is to estimate the sale or stratum sum *KPI* (the estimated or predicted volume in a log). The purpose of the second stage is to estimate the 3P, net and gross *M/P* ratio. Determine estimated stratum volume by multiplying the estimated stratum sum *KPI* from the first stage by the estimated mean *M/P* ratio of the second stage.

54.41 - Calculating Sample Size, 3P Subsample

This sample scaling method includes two forms of sampling; equal probability for the selection of sample loads in the first stage, and 3P or probability proportional to prediction at the second stage.

There are two sources of statistical error in this sample scaling procedure:

1. The estimated sum of stratum *KPI* and

2. The estimate of the *M/P* ratio (measured volume/predicted volume). In the first stage, the population parameter for which the sampling error is estimated is the mean sum estimated load volume (mean load sum *KPI*). In the second or 3P stage, the sampling error is estimated for

the mean net *M/P* ratio, which (for each log) is net scale log volume divided by gross predicted volume (*KPI*).

The stratum sampling error combines both sources of error.

The formula for calculating the sampling error for the sample load/3P sample scaling procedure is:

$$E_T = \sqrt{E_L^{\,2} + E_{3P}^{\,2}}$$

Where: E_T = Combined sampling error in percent
E_L = Sampling error of load sample in percent
E_{3P} = Sampling error of the 3P subsample in percent

Determine sample size for the load sample (number of loads) and for the 3P subsample (number of logs) to satisfy the target combined sampling error specified for the stratum.

To calculate sample size for each stage, the desired sampling error, in percent, must be proportioned between the two stages. For example, given a desired sampling error of 5 percent for the sale and a 2 percent stage two 3P sampling error, calculate the stage-one error as follows:

$$E_L = \sqrt{E_T^{\,2} - E_{3P}^{\,2}}$$

$$= \sqrt{5^2 - 2^2}$$

$$= \sqrt{21} = 4.6\%$$

Determine the coefficient of variation for the first stage from the variance of the sum *KPI* of the individual loads. A good source for data is previous sales. Use the coefficient of variation of individual load volumes as an approximation of the load sum *KPI*. For the second or 3P stage, use the coefficient of variation of the *M/P* ratio. The following is an example of a calculation of the number of sample units for each stage:

<u>First Stage (Loads)</u>

Where: CV_L = 30%
t = 2
E_L = 4.6%
N_L = 1500 loads

$$n = \frac{(tCV_L)^2}{E_L^{\,2} + \frac{(tCV_L)^2}{N_L}}$$

<u>Second Stage (3P Logs)</u>

Where: CV_{3P} = 20%
t = 2
E_{3P} = 2%
N_{3P} = 153 x 50 = 7650 logs

$$n = \frac{(tCV_{3P})^2}{E_{3P}^{\,2} + \frac{(tCV_{3P})^2}{N_{3P}}}$$

$$n = \frac{(2\text{x}30)^2}{4.6^2 + \frac{(2\text{x}30)^2}{1500}}$$

= 152.8 or 153 loads

$$n = \frac{(2\text{x}20)^2}{2^2 + \frac{(2\text{x}20)^2}{7650}}$$

= 380 logs

54.42 - Calculating Sample Expansion, 3P Subsample

For all sales, keep a monthly record of each load hauled. For scaled loads, record at least truck ticket number, date scaled, sum *KPI* for the load, and the *KPI* and scaled net and gross volume for each 3P sample log in the load. For count loads, record the truck ticket number and date hauled. Calculate estimated total sale (or stratum) volume from a sample load/3P sample by multiplying the number of loads hauled by the average estimated load *KPI* and by the mean *M/P* ratio using the following formulas:

$$V_T = N_L \times \overline{KPIL} \times \overline{R}$$

Where: V_T = Total estimated sale or stratum volume

$$\overline{KPIL} = \frac{\sum KPIL}{n_L}$$ = Mean load sum of *KPI*

Where: *KPIL* = Sum of estimated gross log volume of all trees in a sample load
N_L = Total number of loads hauled
$\mathrm{n_L}$ = Number of first stage sample loads
$\overline{R}$ = Mean *M/P* ratio of all second stage 3P sample trees

The mean load sum *KPI* and the mean *M/P* ratio are sample based and subject to sampling error.

The following is an example of a calculation of estimated gross and net volume for a one-month period:

Where: Total loads counted = N_L = 100
Number of sample loads = n_L = 10
Number of 3P sample logs = n_{3P} = 25

Load No.	Load *KPI* (*KPIL*)	3P Sample Log No.	Est. Log Vol. (*KPI*)	Net Log Volume	Gross Log Volume	Net *M/P* Ratio	Gross *M/P* Ratio	Net Ratio Squared	Gross Ratio Squared
1	1022	1	12	14	14	1.167	1.167	1.362	1.362
		2	16	14	16	0.875	1.000	0.766	1.000
2	992	3	10	10	15	1.000	1.500	1.000	2.250
		4	24	22	26	0.917	1.083	0.841	1.173
		5	18	20	20	1.111	1.111	1.234	1.234
3	1056	6	20	20	20	1.000	1.000	1.000	1.000
		7	28	26	30	0.929	1.071	0.863	1.147
4	956	8	6	8	8	1.333	1.333	1.777	1.777
		9	16	16	16	1.000	1.000	1.000	1.000
		10	14	10	12	0.714	0.857	0.510	0.734
		11	18	12	14	0.667	0.778	0.445	0.605
5	962	12	22	26	26	1.182	1.182	1.397	1.397
		13	12	14	14	1.167	1.167	1.362	1.362
6	1030	14	32	28	28	0.875	0.875	0.766	0.766
		15	30	20	30	0.667	1.000	0.445	1.000
7	1004	16	10	10	12	1.000	1.200	1.000	1.440
		17	32	30	30	0.938	0.938	0.880	0.880
8	986	18	10	10	12	1.000	1.200	1.000	1.440
		19	14	12	12	0.857	0.857	0.734	0.734
9	974	20	34	40	40	1.059	1.176	1.121	1.383
		21	9	6	6	0.667	0.667	0.445	0.445
		22	18	10	12	0.556	0.667	0.309	0.445
10	1032	23	32	24	26	0.750	0.812	0.562	0.659
		24	20	18	18	0.900	0.900	0.810	0.810
		25	24	24	24	1.000	1.000	1.000	1.000
Total	10014					23.331	25.541	22.629	27.043

Mean load $\overline{KPIL} = \frac{\sum KPIL}{n_L} = \frac{10014}{10} = 1001.4$

Total *KPI* for month = $\overline{KPIL} \times N_L = 1001.4 \times 100 = 100{,}140$

Mean net *M/P* ratio $= \frac{23.331}{25} = 0.933$

Mean gross *M/P* ratio $= \frac{25.541}{25} = 1.022$

Total estimated net volume for month $= \sum KPIL \times \overline{R}(net)$

= 100,140 x 0.933

= 93,430.62 cubic feet

= 934.31 CCF (net)

Total estimated gross volume for month $= \sum KPIL \times \overline{R}(gross)$

= 100,140 x 1.022

= 102,343 cubic feet = 1,023.43 CCF (Gross)

Percent defect = $\frac{1{,}023.43 - 934.31}{1{,}023.43} \times 100 = 8.7\%$

1. For flat rate sales, calculate means over the life of the sale. Combine the sample scale data for the current month with those of the preceding months. The new averages are used to determine the estimated volume for that period (month).

2. For stumpage rate adjustment sales, make quarterly volumes and values final.

3. For either flat rate or stumpage rate adjustment sales, follow the general procedure defined below for computing estimated net sale volume for the second and succeeding months:

   a. Calculate an updated mean load sum *KPI* using all sample load data to date.

   b. Calculate an updated sale sum (to date) *KPI* by multiplying mean load sum *KPI* (para. 3a) by the number of loads hauled (scaled plus count) to date.

   c. Calculate an updated mean net *M/P* ratio by dividing the sum of the net ratios of all 3P sample logs scaled to date by the number of 3P sample logs to date.

   d. Similarly, calculate an updated mean gross *M/P* ratio.

   e. Calculate a new total (to date) estimated net sale volume by multiplying the updated mean net *M/P* ratio (para. 3c) by the sale sum (to date) *KPI* (para. 3b).

   f. Calculate net sale volume for the current month by subtracting old total net sale volume from net sale volume to date (para. 3e).

54.43 - Sampling Error, 3P Subsample

There are two sources of sampling error for the sample load/3P sample scaling process:

1. The estimated sum of stratum or sale *KPI* and

2. The estimate of the 3P *M/P* ratio.

The formula for calculating the sampling error for this scaling procedure is:

$$E_T = \sqrt{{E_L}^2 + {E_{3P}}^2}$$

Where: E_T = Combined sampling error in percent
E_L = Sampling error in percent of mean load sum *KPI*

$$= \frac{1}{\overline{KPIL}} \sqrt{\frac{\sum KPIL^2 - \frac{\left(\sum KPIL\right)^2}{n_L}}{(n_L - 1)(n_L)}} \times 100 \times t$$

Where: *KPIL* = Sum *KPI* for any sample load
n_L = Number of sample loads
t = Tabulated *t* value for 95% probability level for *m*-1 *df*
E_{3P} = Sampling error in percent of mean *M/P* ratio

$$= \frac{1}{\overline{R}} \sqrt{\frac{\sum R^2 - \frac{\left(\sum R\right)^2}{n_{3P}}}{(n_{3P} - 1)(n_{3P})}} \times 100 \times t$$

Where: R = *M/P* ratio for any 3P sample tree
N_{3P} = Number of 3P sample trees in all sample loads
t = Tabulated *t* value for 95% probability level for *n*-1 *df*

The data from the sample expansion example in section 54.42 is used in the following sampling error calculation example.

<u>Calculating Sampling Error of Mean Load Sum *KP*</u>

$\Sigma KPIL = 10{,}014$
$\Sigma KPIL^2 = 10{,}037{,}876$
$n_L = 10$ loads

$$\overline{KPIL} = \frac{10014}{10} = 1001.4$$

$$SE_L = \sqrt{\frac{\sum KPIL^2 - \frac{\left(\sum KPIL\right)^2}{n_L}}{(n_L - 1)(n_L)}}$$

$$= \sqrt{\frac{10{,}037{,}876 - \frac{(10{,}014)^2}{10}}{(10-1)(10)}}$$

$$= \sqrt{\frac{9{,}856.4}{90}} = 10.46$$

$$E_L = \frac{10.46}{1{,}001.4} \times 100 \times 2.262 = 2.36\%$$

<u>Calculating Sampling Error of Mean Net *M/P* Ratio</u>

$\Sigma R = 23.331$
$\Sigma R^2 = 22.629$
$n_{3P} = 25$ logs

$$\overline{R} = \frac{23.331}{25} = 0.933$$

$$SE_{3P} = \sqrt{\frac{\sum R^2 - \frac{\left(\sum R\right)^2}{n_{3P}}}{(n_{3P} - 1)(n_{3P})}}$$

$$= \sqrt{\frac{22.629 - \frac{(23.331)^2}{25}}{(25-1)(25)}}$$

$$= \sqrt{\frac{0.856}{600}} = 0.038$$

$$E_{3P} = \frac{0.038}{0.933} \times 100 \times 2.064 = 8.41\%$$

$$E_T = \sqrt{E_L^{\,2} + E_{3P}^{\,2}}$$

$$= \sqrt{2.36^2 + 8.41^2} = 8.7\%$$

54.5 - Sample Load with Load Weight and 3P Subsample Log Scaling

This is a two-staged sampling method. All loads (N_L) in the sale or stratum are weighed and a sample of (m) loads is randomly selected, with equal probability, for within load 3P sample log scaling. Statistical variation of total estimated sale volume is a function of the variation in mean load sum *KPI*, mean load weight, and the co-variation of load sum *KPI* and weight plus the variation of the mean *M/P* ratio of the 3P subsample.

In practice, sample loads are selected with equal probability in the manner described for sample load/weight scaling (sec. 54.2). The within load 3P sample log scaling is done using the

procedures described in 3P sample scaling (sec. 54.3) and sample load with 3P subsampling (sec. 54.4).

54.51 - Calculating Sample Size, Load/Weight 3P

The formula for calculating the sampling error of the sample load/weight, 3P subsampling system is:

$$E_T = \sqrt{{E_L}^2 + {E_{3P}}^2}$$

Where: E_T = Combined sampling error in percent
E_L = Sampling error in percent of load sample
E_{3P} = Sampling error in percent of the 3P subsample

Determine sample size for the load sample (number of loads) and for the 3P subsample (number of logs) to satisfy the target combined sampling error. To calculate sample size for each stage, the target combined sampling error (E_T), in percent, must be proportioned between the two stages. For example, given a target sampling error of 5 percent for the sale and a 2 percent stage two 3P sampling error, the stage one or sample load error is calculated as follows:

$$E_L = \sqrt{{E_T}^2 - {E_{3P}}^2}$$

$$= \sqrt{5^2 - 2^2}$$

$$= \sqrt{21} = 4.6\%$$

The following is an example calculation of the number of sample units for each stage. Use the following formula to calculate the coefficient of variation for the load sample:

$$CV_L = \sqrt{\frac{\sum KPIL^2 + \left(VWR^2 \times \sum W^2\right) - \left(2 \times VWR \times \sum KPIL \times W\right)}{VWR^2 \times (n_L - 1) \times \overline{W}^2}}$$

Where: $KPIL$ = Estimated load volume for a sample load (The sum of log KPI's)
W = Weight of a sample load
VWR = Volume to weight ratio $\Sigma KPIL/\Sigma W$, where $\Sigma KPIL$ is the sum of the estimated load volumes and ΣW is the sum of load weights
n_L = Number of sample loads
$\overline{W}$ = Average weight of the sample loads, or $\Sigma W/n_L$

Following is an example of this calculation using the following information:

Load	Volume in CCF (*KPIL*)	Weight in M lbs. (*W*)	$KPIL^2$	W^2	Vol. × Wt. (*KPIL*×*W*)
1	8.6	58.1	73.96	3,375.61	499.66
2	9.8	62.6	96.04	3,918.76	613.48
3	8.4	45.3	70.56	2,052.09	380.52
4	8.4	62.6	70.56	3,918.76	525.84
5	7.2	54.9	51.84	3,014.01	395.28
6	7.8	47.6	60.84	2,265.76	371.28
7	7.0	39.9	49.00	1,592.01	279.30
8	9.0	39.9	81.00	1,592.01	359.10
9	7.4	63.0	54.76	3,969.00	466.20
10	7.8	58.3	60.84	3,398.89	454.74
Totals	81.4	532.2	669.40	29,096.90	4,345.40

Then using the values from the table, calculate the *CV*:

$$VWR = \frac{81.4}{532.2} = 0.1529 \text{ CCF per M pounds}$$

$$\overline{W} = \frac{532.2}{10} = 53.22 \text{ M pounds per load}$$

Calculate the number of samples for the first stage (loads) and the second stage (logs) as follows:

$$CV_L = \sqrt{\frac{\sum KPIL^2 + \left(VWR^2 \times \sum W^2\right) - \left(2 \times VWR \times \sum KPIL \times W\right)}{VWR^2 \times (n_L - 1) \times \overline{W}^2}}$$

$$= \sqrt{\frac{669.40 + (0.1529^2 \times 29{,}096.9) - (2 \times 0.1529 \times 4{,}345.4)}{0.1529^2 \times (10-1) \times 53.22^2}}$$

$$= \sqrt{0.035} = 0.187 \textit{ or } 19\%$$

First stage (loads) sample size:

Where: CV_L = 19%
$t = 2$
E_L = 4.6%
Expected loads = N_L = 1,500

Then:

$$n_L = \frac{(tCV_L)^2}{E_L{}^2 + \frac{(tCV_L)^2}{N_L}}$$

$$= \frac{(2 \times 19)^2}{4.6^2 + \frac{(2 \times 19)^2}{1{,}500}}$$

$$= \frac{1{,}444}{21.16 + \frac{1{,}444}{1{,}500}} = 65.3 \text{ or } 66 \text{ loads}$$

Second stage (logs) sample size:

Where: Avg. # logs/load = 50
CV_{3P} = 20%
$t = 2$
E_{3P} = 2%
$N_{3P} = 66 \times 50 = 3{,}300$ logs

Then:

$$n_{3P} = \frac{(tCV_{3P})^2}{E_{3P}{}^2 + \frac{(tCV_{3P})^2}{N_{3P}}}$$

$$= \frac{(2 \times 20)^2}{2^2 + \frac{(2 \times 20)^2}{3{,}300}}$$

$$= \frac{1{,}600}{4 + \frac{1{,}600}{3{,}300}} = 356.8 \text{ or } 357 \text{ logs}$$

54.52 - Calculating Sample Expansion, Load/Weight 3P

For all sales, keep a monthly record of each load hauled. For scaled loads, record at a minimum the truck ticket number, date scaled, sum *KPI*, and net weight for the load. For each 3P sample log in the load, record the *KPI* and scaled net and gross volume. For weight only loads, record truck ticket number, net weight, and date hauled.

Calculate estimated total sale or stratum volume for the period using a sample load/weight with 3P subsample sampling method using the total sum of *KPI* (gross volume), the ratio of net weight hauled to net sample weight, and the mean *M/P* ratio. The formula is:

$$\text{Total Estimated Volume} = \sum^{n_L} KPIL \times \frac{\sum^{\mathrm{N_L}} W}{\sum^{n_L} W} \times \frac{\sum^{n_{3P}} R}{n_{3P}}$$

Where: N_L = Number of loads hauled in period
$KPIL$ = Estimated gross volume of sample load (Sum log *KPI*)
n_L = Number of sample loads in period
R = *M/P* ratio for each 3P sample log, or the measured net volume over estimated gross volume
n_{3P} = Number of 3P sample logs on all sample loads
W = Net weight of any load

The following is an example calculation of estimated gross and net volume for a one-month period:

Where: Total loads weighed, $N_L = 100$
Total net weight on 100 loads = 5,290.0 M pounds
Number of sample loads, $n_L = 10$
Number of 3P sample logs on loads, $n_{3P} = 25$
Total net weight of 10 sample loads = 532.2 M pounds
$\Sigma KPIL$ (where *KPIL* is the sum of log *KPI*) on 10 sample loads = 81.4 CCF
Sum of the net *M/P* ratios of 25, 3P sample logs, $\Sigma R = 23.331$

Then:

$$\text{Total estimated net volume} = 81.4 \times \frac{5{,}290.0}{532.2} \times \frac{23.331}{25}$$

$$= 81.4 \times 9.94 \times 0.933 = 754.91 \text{ CCF}$$

1. For flat rate sales, calculate means over the life of the sale. Combine the sample scale data for the current month with those of the preceding months. The new averages are used to determine the estimated volume for that period (month).

2. For stumpage rate adjustment sales, quarterly volumes, and values shall be final.

3. For either flat rate or stumpage rate adjustment sales, follow the general procedure defined as follows for computing estimated net sale volume for the second and succeeding months:

a. Calculate the total net weight hauled to date.

b. Calculate the total net weight of the sample loads hauled to date.

c. Calculate the $\Sigma KPIL$ of the sample loads hauled to date.

d. Calculate an updated $\Sigma KPIL$/Net weight ratio: The $\Sigma KPIL$ to date from paragraph 3c divided by net weight of the sample loads hauled to date from paragraph 3b.

e. Calculate an updated mean *M/P* ratio by dividing the sum of the net ratios of all the 3P sample logs scaled to date by the number of 3P sample logs scaled to date.

f. Calculate a new total (to date) estimated net sale volume.

g. Calculate net sale volume for the current month by subtracting old total net sale volume from net sale volume to date (para. 3f.)

54.53 - Sampling Error, Load/Weight 3P

The combined sampling error is calculated as shown in the following steps for this sample scaling method:

Step 1: Calculate the load sample error (E_L):

$$E_L = \frac{\sqrt{\dfrac{\sum KPIL^2 + \left(VWR^2 \times \sum W^2\right) - \left(2 \times VWR \times \sum KPIL \times W\right) \times N_L(N_L - n_L)}{n_L(n_L - 1)}}}{ETV} \times 100 \times t$$

Where:
- $KPIL$ = Sum of log *KPI* of any sample load
- W = Weight of any sample load
- VWR = $\Sigma KPIL/\Sigma W$
- N_L = Total number of loads
- n_L = Number of sample loads
- ETV = Estimated *KPI* for sale, or *VWR* × total hauled weight

Step 2: Calculate the 3P sample error (E_{3P}):

$$E_{3P} = \frac{\sqrt{\dfrac{\sum^{n_{3P}} R^2 \times \dfrac{\left(\sum^{n_{3P}} R\right)^2}{n_{3P}}}{n_{3P}(n_{3P} - 1)}}}{\overline{R}} \times 100 \times t$$

Where:
- R = *M/P* ratio for each 3P sample tree
- $\overline{R}$ = Average *M/P* ratio
- n_{3P} = Number of 3P sample trees
- t = Tabulated *t* value

Step 3: Calculate the combined sampling error (E_T):

$$E_T = \sqrt{E_L^{\ 2} + E_{3P}^{\ 2}}$$

Calculate the load sampling error (E_L).

Calculate the sampling error of the mean net *M/P* ratio (E_{3P}). Given the data from section 54.51:

$$\Sigma KPIL^2 = 669.4$$
$$VWR = 0.1529 \text{ CCF/M pounds}$$
$$\Sigma W^2 = 29{,}096.9$$
$$\Sigma KPIL \times W = 4{,}345.40$$
$$n_L = 10$$
$$N_L = 100$$
$$ETV = 0.1529 \times 5{,}290.0 = 808.84 \text{ CCF}$$

Then:

$$E_L = \sqrt{\frac{669.4 + (0.1529^2)(29{,}096.9) - 2(0.1529)(4{,}345.4) \times 100(100-10)}{10(10-1)}} \times \frac{2}{808.84}$$

$$= \sqrt{\frac{20.82\ x\ 9{,}000}{90}} \times \frac{2}{808.84} = 11.3\%$$

Calculate the sampling error of the mean net *M/P* ratio (E_{3P}). Given the data from section 54.42:

$$\Sigma R = 23.331$$
$$\Sigma R^2 = 22.629$$
$$n_{3P} = 25$$
$$\overline{R} = 0.933$$

$$E_{3P} = \sqrt{\frac{22.629 - \frac{23.331^2}{25}}{(25-1)\,(25)}} \times \frac{1}{0.933} = \frac{0.038}{0.933} \times 2 = 8.1\%$$

Calculate the combined sampling error as (E_T):

$$E_T = \sqrt{{E_L}^2 + {E_{3P}}^2}$$

$$= \sqrt{11.3^2 + 8.1^2} = \sqrt{127.69 + 65.61}$$

$$= \sqrt{193.3} = 13.9\%$$

55 - Sample Design

Specify the appropriate formulas, values, and costs necessary to determine the allowable sampling error for individual timber sales. The Contracting Officer or delegate shall prepare a sample design for each timber sale to be sample scaled.

55.1 - Sampling System Selection

Select the sampling system that considers scaling costs, timber characteristics of the sale, scaling location conditions, scaling skills available for executing the various systems, and availability of approved scales.

55.2 - Sampling Intensity

Plan to achieve the sampling error objective for the sale. Do not prescribe a greater sampling intensity than is needed to meet the sampling error objective. Consider efficiency measures, such as stratification and cost versus values at risk in determining sampling intensity for the sale.

55.3 - Sampling Error Standards

Design scaling samples to meet a sampling error standard of 4 percent or less. The standard sampling error limits are at the 95 percent confidence interval (two standard errors or $t = 2$ in the appropriate statistical formulas for sufficiently large sample sizes). Sampling error standards are a percentage of the total estimated sale value, except for 3P sampling systems where the standards are in percent of total estimated sale volume.

There may be situations where a 4 percent sampling error is too costly to obtain. In these cases, document the reasons, including the purchaser's concurrence, for using a higher sampling error and obtain Regional Office approval to use a higher sampling error.

56 - General Sample Scaling Considerations

56.1 - Sample Scaling Road Right-of-Way Timber

Consider establishing a sampling stratum for right-of-way timber if the right-of-way timber has a significantly different coefficient of variation from the timber in other strata and the right-of-way timber is estimated to be a significant part of the sale value.

56.2 - Memorandum of Understanding

Execute a memorandum of understanding by individual sale that documents procedures to enable sample scaling. Following are important points to consider:

1. <u>Load Sizes</u>. For trucks with different configurations or capacity haul from the same sale, it is necessary to stratify (segregate) them according to width and to compute volumes, respectively. This is done by issuing truck ticket books according to bunk size. The same truck

ticket book must not be used for more than one stratum. Do not be concerned about difference in bunk widths when weight scales are used.

2. Method of Conveyance. This is important only when weight sales are not used. Long log trucks predominate in the western Regions. In addition, there are short log trucks, trucks with trailers, and railroad cars.

3. Presentation. Spread loads to be scaled on the ground in a single layer arrangement in the assigned scaling area so they may be scaled in an economical and safe manner.

4. Sorting. If woods sorting will be done, the purchaser should specify whether by species or product; such as stud logs, peelers, and sawlogs. It is important in sorting not to change the pattern once it has been established. If there is a change, it must be known well in advance. Partial sorting, done as the woods crew finds convenient, can be dangerous. On multi-product or multi-value sales, when appropriate, issue a separate truck ticket book for mixed loads.

5. Delivery to Different Mills. If material from a timber sale is delivered to different mills, stratify the sale according to the volume to be delivered to each mill. If this situation changes, such as all material is delivered to a single mill, a new sample frequency must be established.

6. Random Selection of Loads to Sample Scale. It must not be known whether a load is to be scaled or decked until it arrives at the scaling point and a random number is drawn. There are a number of acceptable systems for randomly selecting loads to be scaled. Among these are automated sample selection system, envelope, card, locked bin, and ticket dispenser.

7. Non-Forest Service Sample Scaling. Include provisions for sample scaling requirements in the memorandum of agreement (FSH 2409.15) with the scaling organization providing this service. Require that the scaling organization furnish the Forest Service certificates or other documents which identify volumes and weight (if necessary) referenced to load removal receipts. Also require statistical information generated from the sample data to be included in the appropriate documents.

56.3 - Monitoring and Changing Sampling Frequency

Periodically review the accumulated sample scale data on the active timber sales to determine if previously estimated coefficients of variation are still valid. If the review indicates a significant difference between experienced *CV* and assumed *CV* for the sale or for a stratum, consider changing the sampling frequency.

On stumpage rate adjustment sales, change the sampling rate when necessary, at the beginning of a calendar quarter, unless unusually serious sampling problems suggest quicker action.

When the sampling frequency is changed, a new sampling period begins; therefore, for the timber sale or the stratum, computations of volume, value, and sampling statistics are concluded

for the previous period. Accumulation of data and computations begin anew for the new sampling period, based on the new sampling frequency.

56.4 - Accuracy, Precision, and Bias

1. Estimates should be accurate or close to the true parameter value of a population or sample.

2. Precision refers to the clustering of parameter estimates; precise estimates are tightly clustered.

3. Bias is a systematic error. If the estimate is biased, it might be precise, but it is not accurate. Bias, lack of precision, or both, can result in inaccurate estimates. Examples of bias are:

   a. A defective tape that mis-measures each log by a fixed amount (measurement bias).

   b. A scaler who consistently over- or underestimates extent of defect (estimation bias).

   c. Prior knowledge of the loads to be scaled (selection bias).

57 - Records and Recording

1. Records. Maintain complete sample scaling documentation in the timber sale folder for each sale being sample scaled. As a minimum, documentation regarding the following items is to be included:

   a. Initial sample design.

   b. Monitoring of factors affecting sampling frequency.

   c. Rationale and computations for changing sampling frequency.

   d. Determination of sampling statistics, including coefficient of variation and sampling error by sampling period, stratum, and sale as a whole.

   e. Load accountability records.

   f. Volume and value sample expansion calculations.

Refer to FSM 2443.32 for information on retention of records.

2. Recording. Record all sample scale information promptly and accurately, using procedures applicable for the Region or Forest to which the timber is accountable.