Forest Service Handbook
National Headquarters - Washington Office
Washington, DC

Forest Service Handbook 2409.12 – Timber Cruising Handbook
Chapter 30 - Cruising Systems

Amendment: 2409.12-1993-1

Effective date: February 23, 1993

Duration: This amendment is effective until superseded or removed.

Approved by:

Date approved:

Responsible Staff:

Last Change:

Superseded Document(s):

Digest:

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During the design phase, choose the appropriate and most efficient cruise system from those described in this chapter. The numerical examples given are intended to illustrate the arithmetic procedures involved in each cruising system. The examples are necessarily brief, are not based on actual data, and in most cases would not meet error standards due to the small number of samples taken.

30.5 - Definitions

Accuracy. Accuracy refers to how well the sample estimates the true value of a quantity. Either bias, lack of precision, or both, can impair accuracy of an estimate. An estimate may be very precise, but because of bias, may still be inaccurate.

Bias. Bias is a systematic error.

Coefficient of Variation. This is a measure of relative dispersion. It is the standard deviation expressed as a percentage of the mean.

Expansion Factor. In any sampling scheme, only selected trees are completely measured. The trees that are measured also represent those that were not measured and must be expanded to get a total stand value. A blow-up, or expansion factor can be calculated based on the sampling scheme that allows each sampled tree to be expanded to represent the entire stand. The factors (denoted by F) can be calculated to expand each sample tree to units such as trees per acre or volume per acre; the type of factor is denoted by subscripting the F.

Frequency Distribution. A frequency distribution divides the population into a relatively small number of classes, listing the number of observations belonging to each.

Mean. This is the average value obtained from dividing the sum of sample unit values by the number of sample units.

Normal Distribution. The normal distribution is a frequency distribution where the distribution of a random measuring error or random error for repeated measurements (for large n) of physical quantities exhibits a particular symmetric bell shape, with a typical maximum, the curve falling off on both sides, and large deviations from the measured value being rare. Assume a normal distribution governs the distribution of population values dealt with in timber cruises.

Parameter. Characteristics such as relative frequency, mean, or standard deviation which refer to the population, are called parameters. Mean tree volume for the entire population is called a parameter, and the sample-based mean tree volume is a statistic which estimates the parameter.

Population. The defined total set of all possible observations, about which information is desired is termed a population.

Precision. Precision describes the clustering of sample values about their mean.

Sampling Error. The sampling error expresses the precision of the inventory. It is the percent error of an estimated mean at a desired probability level. It is calculated as the standard error of the mean times a student t value, divided by the mean and multiplied by 100 to express a percent.

Standard Deviation. This commonly used measure of dispersion is calculated by taking the square root of the variance.

Standard Error. The standard error of the mean provides an estimate of the sampling error. It describes the variation of multiple sample means about the population mean. It can be estimated from a single sample as the square root of the sample variance divided by the sample size.

Tree Factor. A factor that expands each sampled tree to the number of trees that it represents.

Variance. Variance is a measure of how the sample unit values, such as tree volumes are dispersed (clustered, scattered) around the mean of the unit values.

Volume Factor. A factor that expands each sampled tree to the volume that it represents.

31 - Statistical Methods in Cruising

To make valid estimates, use statistical methods together with the basic formulas to design a cruise. For detailed information about statistics, refer to Freese (1962 and 1967) or college text books on forest mensuration and statistics.

31.1 - Statistical Notation and Formulas

There are several methods of writing many of the formulas. While an effort has been made to provide a consistent approach in this handbook, other references may use different notations and formulations for the same equations. Some mensuration and statistical texts use the notation as given in parenthesis. The following describes standard statistical notation used in this chapter:

x	*Individual measurement*
n	*Number of sampling units in a sample*
N	*Number of sampling units in the population*

Σ	*Summation sign, read as sum of*
$\bar{x}$	*Arithmetic mean of the sample*
V	*Variance (s^2)*
SD	*Standard deviation (s)*
SE	*Standard error of the mean ($s_{\bar{x}}$)*
E	*Sampling error in percent*
CV	*Coefficient of variation in percent*

31.2 - Statistical Concepts

Use the statistical concepts of population, population variance, and sampling error objective as a starting point in cruise design. The units to be sampled within each population exhibit a characteristic variation. Populations are never uniform, and the values of the units (trees, points, plots) comprising the population vary between certain limits, depending on how the population is defined. Therefore, the less variation there is in the measured variable from sampling unit to sampling unit, the fewer sampling units are needed to get a reliable estimate. Determine the reliability of estimates using the sampling error.

Use three basic elements in designing a cruise:

1. The population to be sampled.

2. The degree of variation within the population.

3. The sampling error objective.

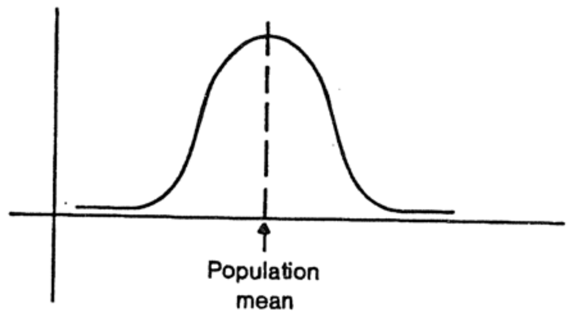
Once the cruise is completed, compute the estimate, the sampling error of the estimate (within 95 percent confidence limits), and the coefficient of variation.

31.21 - Frequency Distribution

The mathematical form of a frequency distribution is expressed as a distribution function. Each population has a distinctive distribution function and the form of the distribution function determines the suitable statistical treatment of sample data. The form of the distribution is seldom known and large samples tend to approximate a normal distribution; consider a large sample to be one consisting of 30 or more sampling units.

31.22 - The Normal Distribution

Assume that the normal distribution governs population values dealt with in timber cruises. The normal distribution is characterized by a bell-shaped curve (fig. 01).



31.22 - Figure 01
Normal Distribution

The total area under the curve above the x-axis is equal to 1. The detailed line erected at the mean divides the area under the curve into two halves.

See section 31.32 for a more detailed description of the characteristics of the normal curve.

31.23 - Sampling

From a population of the units to be measured, some are selected, such as trees in a stand or timber sale. Each unit in the sample becomes a sampling unit upon which an observation is made to determine the value for that unit. For example, out of a population of sawtimber trees, a selected number will be measured for the characteristics needed to determine a mean tree volume. Usually, these are diameter breast height (DBH) and height.

31.24 - Examples of Bias

The following are bias examples:

1. Measurement bias occurs when an instrument is out of adjustment, and mismeasures each sampling unit by a fixed amount.

2. Estimation bias occurs when a volume estimator consistently overestimates or consistently underestimates actual tree volume.

3. Selection bias occurs when plots over-represent a timber condition, such as taking a string of plots along a drainage instead of across a drainage.

31.3 - Sampling Statistics

Common statistics computed from samples obtained in cruising are the arithmetic mean, standard deviation, coefficient of variation, the standard error (of the mean), and the sampling error.

31.31 - Calculating a Mean

The mean is the average value obtained from dividing the sum of sample unit values by the number of sample units:

$$\bar{x} = \frac{\sum^{n} x}{n}$$

where:

$\bar{x}$ = the sample mean
x = the sample value for the i^{th} individual
n = the number of samples

The mean by itself is not an adequate measure of the 'goodness' of the sample estimate. It must be assessed in association with a measure of dispersion.

31.32 - Calculating Variance

Variance is calculated as:

$$V = \frac{\sum^{n} (x-\bar{x})^2}{n-1} = \frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n-1}$$

where:

V = variance
x = the sample value
$\bar{x}$ = the average sample value
n = the number of samples

If all trees in a population and sample were exactly the same size they would have the same volume. In such a case, the sample mean volume is the same as the volume of each tree and there is no dispersion of sample units around the mean. That is, the variance is zero and only one sample is necessary to estimate the mean. In reality, there is variation and as individual tree volumes differ from the mean by ever larger dispersions, more samples are needed to obtain a reliable estimate of the mean.

31.33 - Calculating Standard Deviation

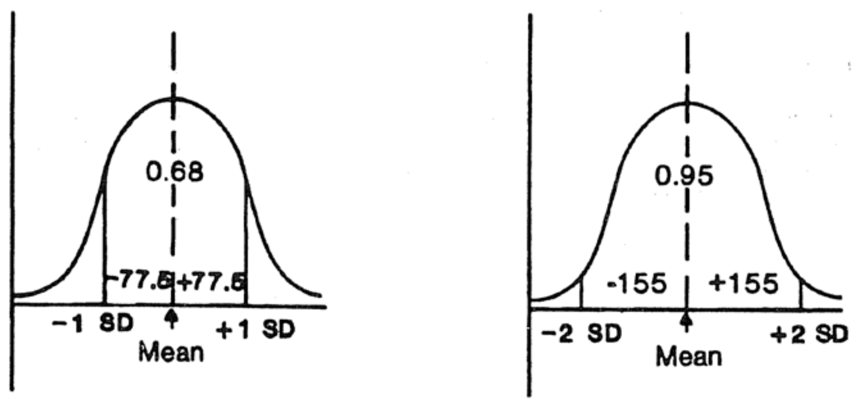
This commonly used measure of dispersion is calculated by taking the square root of the variance:

$$SD = \sqrt{V} = \sqrt{\frac{\sum^{n} x^2 - \frac{(\sum^{n} x)^2}{n}}{n-1}}$$

where:

SD = standard deviation
V = variance
x = sample value for i^{th} individual
n = number of samples

The normal curve displays a relationship between the sample mean and standard deviation. For example, it is found that in one cruise, the standard deviation is ±77.5 BF. The probability is 0.68 (68 times out of 100) that an observed value will fall within +77.5 BF of the sample mean, and 0.95 (95 times out of 100) that an observed value will fall within +155 BF of the sample mean.



31.33 - Figure 01
Standard Deviation

The standard deviation measures the dispersion of individual observations about their mean. A similar relationship holds for the dispersion of sample means about their mean. The standard deviation of sample means is called the standard error of the mean.

31.34 - Calculating Coefficient of Variation

The coefficient of variation is calculated as:

$$CV = \frac{SD}{\bar{x}} \times 100$$

where:

CV = coefficient of variation
SD = standard deviation
$\bar{x}$ = mean value of all individuals sampled

Because the coefficient of variation is a measure of relative variability, it can be used to compare the degree of variation between different populations. For example, the following is known for two timber tracts:

Tract A	Tract B
$\bar{x}$ = 8.5 cubic feet per tree	$\bar{x}$ = 6.4 cubic feet per tree
SD = ±4.7 cubic feet per tree	SD = ±4.0 cubic feet per tree

then:

$$CV = \frac{100(4.7)}{8.5} = 55.3\% \qquad CV = \frac{100(4.0)}{6.4} = 62.5\%$$

The calculations show tract B exhibits greater variation than tract A and requires a larger sample for a given sampling error.

31.35 - Standard Error and Confidence Limits

The standard error can be thought of as the standard deviation of sample means about their mean. The distribution of large numbers of sample means (30 or more) follows the normal curve and has the same relationship for sample means as for individual values about their mean.

For large samples of 30 or more observations, expect the true mean to fall within +1 standard error of the sample mean with a probability of 0.68, or within ±2 standard errors of the sample mean with a probability of 0.95 (sec. 31.33, fig. 01). These are the confidence limits of a sample-based estimate of a population mean, since the true population mean is never known.

The standard error is calculated as:

$$SE = \frac{SD}{\sqrt{n}} = \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n(n-1)}}$$

where:

SE = standard error of the estimate
SD = standard deviation
x = sample value for i^{th} individual
n = number of samples

When simple random sampling is used and each sample unit appears only once in the sample, which is called sampling without replacement, a finite population correction can be applied to the standard error.

$$SE = \frac{SE}{\sqrt{n}} \sqrt{1 - \frac{n}{N}}$$

$$= \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n(n-1)} \times \left(1 - \frac{n}{N}\right)}$$

where:

SE = standard error of the estimate
SD = standard deviation
x = sample value for i^{th} individual
n = number of sampling units selected
N = number of units in the population.

The finite population correction is not needed if n/N is less than 0.05. That is, if the number of sample units is less than 5 percent of the total number of units in the population, the population correction factor is not needed. The purpose of the finite population correction is to prevent an inflated statement of standard error when the number of sampling units is a large proportion (0.05 or greater) of the total population.

In calculating the confidence limit, use the "Student t" distribution. For practical purposes, consider the value of t constant for a given degree of confidence.

These confidence limits are expressed as:

68 percent confidence limits (t = 1). Unless a 1-in-3 chance has occurred, the true mean will be within $\pm$1 standard error of the sample mean.

95 percent confidence limits (t = 2). Unless a 1-in-20 chance has occurred, the true mean will be within ± 2 standard errors of the sample mean.

99.7 percent confidence limits (t = 3). Unless a 1-in-300 chance has occurred, the true mean will be within ± 3 standard errors of the sample mean.

Determine the confidence limit by:

1. Finding the mean and standard deviation,
2. Calculating the standard error,
3. Multiplying the standard error by the appropriate Student t value,
4. Subtracting this product from the mean to set the lower limit,
5. Adding the product to the mean to set the upper limit.

These limits set the range of values expected to include the true population mean. This is called the confidence interval. The probability that the population will fall within these limits is set by the t value chosen. For example, select a sample of trees from a stand and measure the cubic volume of each sample tree. Then calculate the confidence limits as follows:

Example:

$$\begin{aligned} \textit{Given:}\quad SD &= 8.318\ ft^3 \\ \overline{x} &= 31.53\ ft^3 \\ n &= 211 \\ \textit{then:}\quad SE &= \frac{8.318}{\sqrt{211}} = 0.573\ ft^3 \end{aligned}$$

The Student t value for the 95 percent confidence limits is approximately 2; therefore:

$$\begin{aligned} \text{Confidence limit} &= \overline{x} \pm (t \times SE) \\ &= 31.53 \pm (2 \times 0.573) \\ &= 31.53 \pm 1.146\ ft^3 \end{aligned}$$

This can be interpreted as follows:

$$\begin{gathered} (\overline{x} - t \times SE)\ \textit{and}\ (\overline{x} + t \times SE) \\ (31.53 - 1.146)\ \textit{and}\ (31.53 + 1.146) \\ 30.384\ \textit{and}\ 32.676 \end{gathered}$$

Unless a 1-in-20 chance has occurred, the true population mean falls between these confidence limits.

31.36 - Calculating Sampling Error

Calculate sampling error by dividing the standard error by the mean, then multiplying by 100, and finally multiplying by the student t value appropriate to the confidence level:

$$E = \frac{SE}{\bar{X}} \times 100 \times t$$

In the above example with a mean of 31.53 and a SE of 0.573, the sampling error at the 95 percent confidence level is:

$$E = \frac{0.573}{31.53} \times 100 \times 2$$

$$= 3.63\%$$

We can expect, with 95 percent probability, the true population mean to lie within 3.63 percent of our sample mean.

31.4 - Determining Sample Size

Knowing the coefficient of variation (sec. 31.34) for a population, the number of sampling units (observations) needed for a specified sampling error objective can be calculated. Recognize two sampling conditions:

1. Sampling finite populations. (For example, the total number of sampling units in the population is approximately known).

$$n = \frac{1}{\frac{E^2}{t^2 CV^2} + \frac{1}{N}} = \frac{(tCV)^2}{E^2 + \frac{(tCV)^2}{N}}$$

2. Sampling infinite populations (total number of sampling units in population not known) or if number of sampling units is a small proportion (less than 0.05) of the total population.

$$n = \frac{t^2 CV^2}{E^2} = \frac{(t\ CV)^2}{E^2}$$

31.5 - Stratification

The statistical calculations discussed up to this point assumed simple random sampling conditions. In certain cases, it is advantageous to use stratified random sampling. The chief purpose in stratification is to divide the population into subpopulations (strata or sample groups) based on certain similar characteristics.

The aim of stratification is to create strata or sample groups such that the variation within each is less than it would be for the (unstratified) population as a whole. This enables taking fewer sampling units for a specified sampling error objective, or, conversely, a given number of sample units provides a more precise (stratified) estimate than if taken for the unstratified population.

Where sampling units are single trees, a population of sawtimber trees for example, might be stratified on the basis of 2-inch or 4-inch groups. Either option may reduce the sample size needed for a stated objective over the number needed for the unstratified population.

Similarly, sample plots might be stratified by timber types or by other groupings that are likely to increase homogeneity within groups.

Optimum allocation requires some precruise estimates of the number of sample units in each stratum, as well as the variation in each stratum. Based on this information, calculate a separate sampling intensity for each stratum and the total number of sample units prorated (allocated) by strata.

An example of optimum allocation is given in section 35.3.

32 - 100 Percent Cruise

Every member of the population is visited and measured. The variable of interest, generally expressed as volume or value, is an exact measure of the strata. Apply this method to finite populations of trees where each tree in the population is measured, but restrict its use to special cases where sampling is not practical. For example, the trees to be cut from an access road right-of-way which lies outside the boundaries of the cutting units may be an appropriate use. Consider 100 percent measurement for sales of species or products with exceptionally high values.

33 - Sample-Tree with Complete Tally

33.1 - Sample-Tree With Complete Tally Method

In this method, single trees are sampled with equal probability of selection. Define the population(s) and components to be sampled. Some form of stratification may be used either based on species, species groups, diameter breast height (DBH) classes, or a combination of these. In some cases, define a non-sampled (100-percent measured) component. Such a component might consist of trees above a certain DBH class, or trees of certain species estimated to have too few trees to yield the required minimum number of samples at the prescribed sampling frequency. To use this cruise method, make a complete tally of the population being sampled.

33.11 - Operational Features

Select trees without bias. Select trees randomly to meet the requirement for unbiased sample unit selection. Ensure randomness by using dice, poker chips, marbles, or random numbers in the selection process.

Systematic sampling (sampling every nth tree) may be used in certain controlled situations, where the cruiser does not keep track of their own samples. In such applications, someone other than the cruiser records the cruise data. This is to control personal bias on the part of the cruiser.

Sample-tree cruising is best done with a crew of three or four marker-cruisers and a tallier. This system loses its advantage in scattered timber. Crew members function as both marker and cruiser. The main advantage is that the tallier keeps the tree count and calls the sample trees to be measured as they come up, thereby removing a possible source of bias in sample selection.

The cruiser receiving the sample-tree call measures the sample-tree and either records the data or calls it in to the tallier. Identify cruise trees for later check cruising with the cruiser's identification and the tree number.

Conduct an accurate tree count and avoid missing trees or double-counting to ensure accurate results from this method. The best way to do this is to mark the tree, then report it to the tallier.

33.12 - Statistical Features

This cruising method may be used with simple random sampling or with stratified random sampling.

Use simple random sampling for single-product situations, such as pulpwood, where the need for sampling precision is less critical, or where the population is not large enough to make stratification worthwhile.

Prescribe stratified sampling where it is possible to subdivide the population into fairly homogeneous groups (sec. 31.5).

Generally, for sample-tree with complete tally, the variable of interest subject to sampling error, is volume per tree.

33.2 - Calculating Sample Size

Calculate optimum sample size using precruise data and the series of steps illustrated. In the example, the population is stratified (divided into sample groups) by species or combinations of species. Calculate the sample statistics by species group (ex. 02-03) from the precruise data (ex. 01).

33.2 - Exhibit 01

Precruise Data

Tree ID	Species	Sample Group	DBH	Height	Net ft^3 Volume/Tree Sample Group 1	Net ft^3 Volume/Tree Sample Group 2
1	A	1	10.0	48	10.7	
2	A	1	11.0	74	24.2	
3	A	1	13.0	67	29.0	
4	B	2	13.0	73		26.9
5	B	2	8.0	45		4.9
6	B	2	13.0	61		23.0
7	B	2	15.0	88		43.8
8	A	1	12.0	59	21.0	
9	B	2	10.0	56		11.5
10	B	2	9.0	61		10.1
11	B	2	16.0	66		29.4
12	B	2	12.0	43		6.7
13	A	1	13.0	67	41.4	
14	A	1	17.0	61	21.0	
15	A	1	12.0	74	30.9	
16	A	1	9.0	92	66.3	
17	B	2	11.0	63		19.3
18	B	2	13.0	54		8.8
19	C	2	11.0	54		14.0
20	A	1	14.0	77	31.8	
21	C	2	11.0	52		13.0
22	A	1	14.0	70	33.4	
23	C	2	11.0	63		16.8
24	A	1	14.0	69	32.4	
25	A	1	16.0	72	45.6	
26	A	1	12.0	74	27.8	
27	B	2	10.0	55		11.0
28	B	2	9.0	63		10.7
29	B	2	11.0	56		14.2
30	B	2	13.0	63		24.2
31	B	2	12.0	70		21.6
32	A	1	15.0	69	38.4	
33	C	2	12.0	57		17.7
34	A	1	11.0	63	19.7	
35	A	1	10.0	65	17.3	
36	B	2	13.0	73		26.9
37	B	2	8.0	45		4.9
38	B	2	13.0	61		23.0
39	B	2	15.0	88		43.8

33.2 - Exhibit 02

Standard Deviation and Coefficient of Variation of Sample Group 1

Tree ID	Net ft^3 Volume (x)	Net ft^3 Volume Squared (x^2)
1	10.7	114.49
2	24.2	585.64
3	29.0	841.00
8	21.0	441.00
13	41.4	1713.96
14	21.0	441.00
15	30.9	954.81
16	66.3	4395.69
20	31.8	1011.24
22	33.4	1115.56
24	32.4	1049.76
25	45.6	2079.36
26	27.8	772.84
32	38.4	1474.56
34	19.7	388.09
35	17.3	299.29
n = 16	490.9	17678.29

$$Mean = \overline{x} = \frac{\overset{n}{\Sigma} x}{n} = \frac{490.9}{16} = 30.68$$

$$Standard\ Deviation = SD = \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{17678.29 - \frac{490.9^2}{16}}{15}} = 13.21$$

$$Coefficient\ of\ Variation = CV = \frac{SD}{\overline{x}} \times 100$$

$$= \left(\frac{13.20}{30.68}\right)(100)$$

$$= 43.02 = 43\%$$

33.2 - Exhibit 03

Standard Deviation and Coefficient of Variation of Sample Group 2

Tree ID	Net ft^3 Volume (x)	Net ft^3 Volume Squared (x^2)
4	26.9	723.61
5	4.9	24.01
6	23.0	529.00
7	43.8	1918.44
9	11.5	132.25
10	10.1	102.01
11	29.1	846.81
12	6.7	44.89
17	19.3	372.49
18	8.8	77.44
19	14.0	196.00
21	13.0	169.00
23	16.8	282.24
27	11.0	121.00
28	10.7	114.49
29	14.2	201.64
30	24.2	585.64
31	21.6	466.56
33	17.7	313.29
36	26.9	723.61
37	4.9	24.01
38	23.0	529.00
39	43.8	1918.44
n = 23	425.9	10415.87

$$Mean = \overline{x} = \frac{\sum^{n} x}{n} = \frac{425.9}{23} = 18.52$$

$$Standard\ Deviation = SD = \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{10415.87 - \frac{425.9^2}{23}}{22}} = 10.72$$

$$Coefficient\ of\ Variation = CV = \frac{SD}{\overline{x}} \times 100$$

$$= \frac{10.72}{18.52}\ (100)$$

$$= 57.88 = 58\%$$

Develop volume per tree estimates by processing the precruise data as a cruise or from previous sale volume summaries of comparable sales.

Given the sale sampling error objective: E = 10% with 95% confidence (t = 2), then subdivide (stratify) the sale population into sample groups. For example, suppose that a sale of the following trees is being prepared:

Sample Group	Species Component	Estimated No. of Trees in Sale
1	A	152
2	B,C	276
Total		428

Calculate the coefficient of variation by sample group and a weighted CV over all sample groups.

Sample Group	Estimated Volume	(a) Percent Volume	(b) Est.CV%	CV Fraction (a) x (b)
1	4663	48	43	20.6
2	5111	52	58	30.2
	9774			50.8 Weighted CV

For the desired sale sampling error objective (E_T) of 10% and a weighted coefficient of variation of 50.8, determine the number of sample trees needed for sale as a whole, n_T:

$$n_T = \frac{t^2 (Weighted\ CV)^2}{(E_T)^2}, \text{ where } t = 2 \text{ standard errors}$$

$$n_T = \frac{2^2 (50.8)^2}{10^2} = 103.2 = 104 \text{ sample trees}$$

Allocate number of trees for each j^{th} sample group, (there are s sample groups, and in this example, s = 2):

$$n_j = \frac{(CV\ Fraction)\, n_T}{Weighted\ CV} = \text{Sample trees for } j^{th} \text{ group}$$

Allocate the trees to the sample groups; calculate the number of sample trees needed for sample group 1.

$$n_1 = \frac{(20.6)\ (104)}{50.8} = 42.2 = 43 \text{ sample trees}$$

Calculate the number of sample trees needed for sample group 2.

$$n_2 = \frac{(30.2)\ (104)}{50.8} = 61.8 = 62 \text{ sample trees}$$

33.3 - Calculating Sampling Statistics

33.31 - Sample Expansion

Calculate the expansion factors for each species (total number of trees in the group divided by the number sampled):

		Sample Group 1	Sample Group 2
Total number of trees marked (N)	=	152	276
Total number of trees in sample (n)	=	16	23
Expansion factor (F_t) = N/n	=	9.5	12.0

Expand the volumes for each sample group as in exhibits 01 - 02.

33.31 - Exhibit 01
Sample Group 1 Volume

Tree ID	Net ft^3 Volume in tree (x)	Tree Factor (F_t)	Expanded net ft^3 Vol.	Squared net ft^3 (x^2)
1	10.7	9.5	101.65	114.49
2	24.2	9.5	229.90	585.64
3	29.0	9.5	275.50	841.00
8	21.0	9.5	199.50	441.00
13	41.4	9.5	393.30	1713.96
14	21.0	9.5	199.50	441.00
15	30.9	9.5	293.55	954.81
16	66.3	9.5	629.85	4395.69
20	31.8	9.5	302.10	1011.24
22	33.4	9.5	317.30	1115.56
24	32.4	9.5	307.80	1049.76
25	45.6	9.5	433.20	2079.36
26	27.8	9.5	264.10	772.84
32	38.4	9.5	364.80	1474.56
34	19.7	9.5	187.15	388.09
35	17.3	9.5	164.35	299.29
n = 16	490.9	152.0	4663.55	17678.29

Total volume for sample group 1 = 4664

Alternately:

Estimated Volume = Estimated Average Tree Volume x Total Number of Trees Marked

Average tree volume = 490.9/16 = 30.681

Total volume for sample group 1 = 30.681 x 152
= 4664 ft^3

33.31 - Exhibit 02

Sample Group 2 Volume

Tree ID	Net ft^3 Volume (x)	Tree Factor (F_t)	Expanded net ft^3 Volume	Squared net ft^3 (x^2)
4	26.9	12.0	322.80	723.61
5	4.9	12.0	58.80	24.01
6	23.0	12.0	276.00	529.00
7	43.8	12.0	525.60	1918.44
9	11.5	12.0	138.00	132.25
10	10.1	12.0	121.20	102.01
11	29.1	12.0	349.20	846.81
12	6.7	12.0	80.40	44.89
17	19.3	12.0	231.60	375.49
18	8.8	12.0	105.60	77.44
19	14.0	12.0	168.00	196.00
21	13.0	12.0	156.00	169.00
23	16.8	12.0	201.60	282.24
27	11.0	12.0	132.00	121.00
28	10.7	12.0	128.40	114.49
29	14.2	12.0	170.40	201.64
30	24.2	12.0	290.20	585.64
31	21.6	12.0	259.20	466:56
33	17.7	12.0	212.40	313.29
36	26.9	12.0	322.80	723.61
37	4.9	12.0	58.80	24.10
38	23.0	12.0	276.00	529.00
39	43.8	12.0	525.60	1918.44
n = 23	425.9	276.0	5110.80	10415.87

Total volume for sample group 2 = 5111

Alternately:

Estimated Volume = Estimated Average Tree Volume x
Total Number of Trees Marked

Average tree volume = 425.9/23 = 18.517

Total volume for sample group 2 = 18.517 x 276
= 5111 ft^3

Add the volumes for the two sample groups together for the sale volume.

Volume for sample group 1 = 4664 ft^3
Volume for sample group 2 = 5111 ft^3
Total volume for the sale = 9775 ft^3

33.32 - Sampling Error

Compute the sampling error for each sampling group first, and then compute a combined sampling error for the sale as a whole. Given the sample tree data from section 33.31, and given the following information for sample group 1:

$$n = 16$$

$$Sum\ Tree\ Volume = \sum^{n} x = 490.9$$

$$Sum\ Tree\ Volume^2 = \sum^{n} x^2 = 17678.29$$

$$\overline{x} = \frac{490.9}{16} = 30.68$$

Total Number of Trees Marked = N = 152

Calculate the standard error for sample group 1:

$$SE = \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n(n-1)}\left(\frac{1-n}{N}\right)}$$

$$= \sqrt{\frac{17678.29 - \frac{490.9^2}{16}}{16\,(15)}\left(1-\frac{16}{152}\right)}$$

$$= \sqrt{\frac{2616.86}{240}\,(.89)}$$

$$= 3.12\ ft^3$$

Calculate sampling error for the sample group 1:

$$E_1 = \frac{SE}{\overline{x}} \times 100 \times t$$

$$= \frac{3.12}{30.68} \times 100 \times 2$$

$$= 20.34$$

$$= 20.3\%$$

Given the following information for sample group 2:

$$n = 23$$

$$Sum\ Tree\ Volume = \sum^{n} x = 425.9$$

$$Sum\ Tree\ Volume^2 = \sum^{n} x^2 = 10415.87$$

$$\overline{x} = \frac{425.9}{23} = 18.52$$

Total Number of Trees Marked = N = 276

Calculate the standard error for sample group 2:

$$SE = \sqrt{\frac{\sum^{n} x^2 - \frac{\left(\sum^{n} x\right)^2}{n}}{n(n-1)}\left(1-\frac{n}{N}\right)}$$

$$= \sqrt{\frac{10415.87 - \frac{425.9^2}{23}}{23(22)}\left(1-\frac{23}{276}\right)}$$

$$= \sqrt{\frac{2529.31}{506}(.917)}$$

$$= 2.14\ ft^3$$

Calculate sampling error for sample group 2:

$$E_2 = \frac{SE}{\overline{x}} \times 100 \times t$$

$$= 100\ \frac{2.14}{18.52} \times 100 \times 2$$

$$= 23.11$$

$$= 23.1\%$$

Combine the strata errors to get the total sale error (ex. 01).

33.32 - Exhibit 01
Sampling Error for the Strata

Sample Group	Volume ft^3 (V)	Sampling Error (%) (E)	Volume x % Error/100 (VxE)	$(VxE)^2$
1	4663	20.4	951.252	904880.37
2	5111	23.1	1180.641	1393913.17
	9774			2298793.54

Compute combined sampling error for sale:

$$E_T = \sqrt{\frac{(V_1E_1)^2 + (V_2E_2)^2}{V_T}}$$

$$= \sqrt{\frac{904880.37 + 1393913.17}{4663 + 5111}}$$

$$= 15.33\%$$

33.4 - Additional Population Characteristics

In addition to estimating the product volumes, it may be necessary to make estimates of other population characteristics. Determine the average diameter, the quadratic mean diameter, and the average height using the sample-tree data set (ex. 01).

33.4 - Exhibit 01

Cruise Data Necessary to Estimate Additional Population Characteristics

Sample Group	Tree ID	DBH	DBH^2	HT	Tree Fact (F_t)	Expanded DBH ($DBH x F_t$)	Expanded DBH^2 ($DBH^2 x F_t$)	Expanded HT ($HT x F_t$)
1	1	10	100	48	9.5	95.0	950.0	456.0
1	2	11	121	74	9.5	104.5	1149.5	703.0
1	3	13	169	67	9.5	123.5	1605.5	636.5
1	8	12	144	59	9.5	114.0	1368.0	560.5
1	13	16	256	67	9.5	152.0	2432.0	636.5
1	14	12	144	61	9.5	114.0	1368.0	579.5
1	15	13	169	74	9.5	123.5	1605.5	703.0
1	16	17	289	92	9.5	161.5	2745.5	874.0
1	20	13	169	77	9.5	123.5	1605.5	731.5
1	22	14	196	70	9.5	133.0	1862.0	665.0
1	24	14	196	69	9.5	133.0	1862.0	655.5
1	25	16	256	72	9.5	152.0	2432.0	684.0
1	26	12	144	74	9.5	114.0	1368.0	703.0
1	32	15	225	69	9.5	142.5	2137.5	655.5
1	34	11	121	63	9.5	104.5	1149.5	598.5
1	35	10	100	65	9.5	95.0	950.0	617.5
2	4	13	169	73	12.0	156.0	2028.0	876.0
2	5	8	64	45	12.0	96.0	768.0	540.0
2	6	13	169	61	12.0	156.0	2028.0	732.0
2	7	15	225	88	12.0	180.0	2700.0	1056.0
2	9	10	100	56	12.0	120.0	1200.0	672.0
2	10	9	81	61	12.0	108.0	972.0	732.0
2	11	14	196	66	12.0	168.0	2352.0	792.0
2	12	9	81	43	12.0	108.0	972.0	516.0
2	17	12	144	63	12.0	144.0	1728.0	756.0
2	18	9	81	54	12.0	108.0	972'0	648.0
2	19	11	121	54	12.0	132.0	1452.0	648.0
2	21	11	121	52	12.0	132.0	1452.0	624.0
2	23	11	121	63	12.0	132.0	1452.0	756.0
2	27	10	100	55	12.0	120.0	1200.0	660.0
2	28	9	81	63	12.0	108.0	972.0	756.0
2	29	11	121	56	12.0	132.0	1452.0	672.0
2	30	13	169	63	12.0	156.0	2028.0	756.0
2	31	12	144	70	12.0	144.0	1728.0	840.0
2	33	12	144	57	12.0	144.0	1728.0	684.0
2	36	13	169	73	12.0	156.0	2028.0	876.0
2	37	8	64	45	12.0	96.0	768.0	540.0
2	38	13	169	61	12.0	156.0	2028.0	732.0
2	39	15	225	88	12.0	180.0	2700.0	1056.0
Sum					428.0	5117.5	63298.5	27379.5
Count	39							

Calculate the additional population characteristics:

$$Mean\ DBH = \frac{\sum^{n}(DBH \times F_t)}{\sum^{n} F_t} = \frac{5117.5}{428} = 12.0\ inches$$

$$Quadratic\ Mean\ Diameter = \sqrt{\frac{\sum^{n}(DBH^2 \times F_t)}{\sum^{n} F_t}}$$

$$= \sqrt{\frac{63298.5}{428}}$$

$$= 12.2\ inches$$

$$Mean\ Height = \frac{\sum^{n}(HT \times F_t)}{\sum^{n} F_t} = \frac{27379.5}{429} = 64\ feet$$

33.5 - Application

Apply this method in the following cases:

1. Stands requiring partial cutting, in which the trees to be left are marked, or stands requiring overstory removal where trees to be cut are marked.

2. Small clearcut areas, usually less than 25 acres with a high coefficient of variation (plots or points), for which the number of sample points or plots needed for prescribed accuracy standards would be excessive (more than 1 point per acre).

3. Sales having numerous species, for which stratification can reduce variation, resulting in a light sample intensity relative to other cruising systems.

34 - Fixed Plots

34.1 - Fixed Plot Method

This is an area sampling method using plots of a specified area so trees in the population are selected with equal probability. The plots may be either circular or rectangular. The population is the total number of plots of specified area contained in the proposed area to be cruised. For example, the population of fifth-acre plots on a 48-acre tract is 240. Cruises using fixed area plots are customarily described in terms of the percentage of tract area covered. On this same tract, a 10-percent cruise would sample 4.8 acres, the equivalent of 24 fifth-acre plots.

34.11 - Operational Features

Establish a specified number of sample plots of equal area in an unbiased manner over the tract of timber to be cruised. Measure each tree located within each sample plot boundary as a sample tree.

The cruise results in an estimate of average volume per acre. Determine the area of the tract by survey (ch. 50). Determine total estimated tract volume by multiplying estimated average volume per acre by the number of acres in the tract.

34.12 - Statistical Features

In this cruising method, each tree has an equal probability of being selected.

Stratification can be an advantage in plot sampling to reduce sampling variation and to reduce the number of plots needed. Ordinarily, stratify by similarity (homogeneity) of timber conditions. Subdivide the tract to be cruised into areas having similar conditions of density, product components, or other characteristics. Sample each stratum separately, and compute the sample statistics as for a stratified sample.

The use of sample groups (stratification of individual trees on a point) is not recommended due to of the difficulty in determining a sampling error.

The variable of interest, subject to sampling error, is average volume or value per acre.

34.2 - Field Procedures

Use the following field procedures appropriate to the situation. Evaluate the plot shape using the following criteria:

1. Circular plots. The center of each plot is definitively marked so it can be determined accurately whether borderline trees are in or out of the plot. Because there is less perimeter in a circular plot than a rectangular plot of the same area, there are fewer borderline trees.

2. Rectangular plots. With the steel tape on the centerline of the plot, the cruiser is able to rapidly and systematically progress through the plot. The rectangular plot allows cruiser to work continuously forward.

34.21 - Sample Plot Location and Monumentation of Plots and Trees

Plot size may vary between sampling strata. However, use only one plot size in each stratum. Gear plot size to tree density and select a plot size that will ensure an average of about 4 to 8 trees per plot. Consider stand variation and species composition in determining plot size.

Plot placement may follow various patterns, but locate plots in an unbiased manner. Select the location of the starting plot randomly if the sample plots are located on a grid.

Monument plot locations. Identify cruise starting points and plot locations in a way that make the locations highly visible in the field to enable a check cruiser to find them. File a written description with the cruise information to facilitate plot relocation.

Identify plot centers with either a wooden stake or wire pin. Number the tallied trees clockwise from north on the face of the tree toward the plot center.

34.22 - Establishing Plot Boundaries

Establish plot boundaries by measurement, not by pacing. Determine circular plot boundaries by measuring the radius from the marked plot center. Determine rectangular plot boundaries from the centerline defined by the chain stretched between the marked end-line centers. Exhibit 01 lists radii for a variety of circular plot sizes.

34.22 - Exhibit 01
Plot Radius Table

Acre	Feet Radius
1	117.8
1/2	83.3
1/3	68.0
1/4	58.9
1/5	52.7
1/10	37.2
1/20	26.3
1/25	23.5
1/40	18.6
1/50	16.7
1/100	11.8
1/300	6.8
1/500	5.3
1/1000	3.7

Determine if problem trees, such as forked, leaning, or down trees, are in or out, depending on the location of tree DBH in relation to the plot boundary. Use tree DBH as the point of reference rather than the base of the tree because the tree base is considered to be an ambiguous reference point, particularly in the case of uprooted trees. Consider the following examples: