

**Forest Service Handbook  
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**Forest Service Handbook 2409.12 – Timber Cruising Handbook  
Chapter 30 - Cruising Systems**

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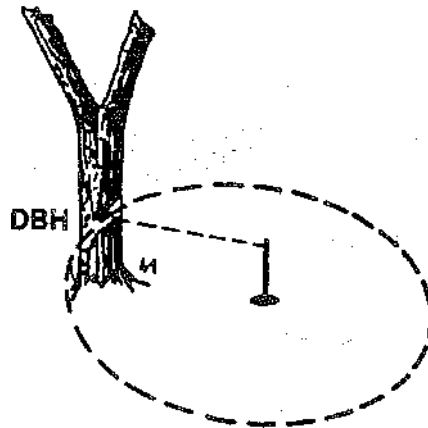
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## Table of Contents

34.3 - Calculating Sample Size.....	4
34.4 - Calculating Sampling Statistics .....	6
34.41 - Sample Expansion .....	6
34.42 - Sampling Error.....	8
34.5 - Additional Population Characteristics.....	9
34.6 - Application .....	11
35 - Point Sampling.....	12
35.1 - Point Sampling Method .....	12
35.11 - Operational Features .....	12
35.12 - Statistical Features .....	12
35.13 - Basal Area Factor.....	13
35.2 - Field Procedures for Point Sampling.....	14
35.21 - Sample Point Location and Monumentation of Points and Trees .....	14
35.21a - Angle Gauge Use .....	14
35.22 - Sample Tree Determination .....	18
35.22a - Assessing Borderline and Other Questionable Trees .....	18
35.3 - Calculating Sample Size.....	21
35.4 - Calculating Sampling Statistic.....	22
35.41 - Sample Expansion .....	22
35.42 - Sampling Error.....	26
35.5 - Additional Population Characteristics.....	27

1. Forked trees. If the plot boundary passes through the center of the tree at DBH, count the tree in (fig. 01).

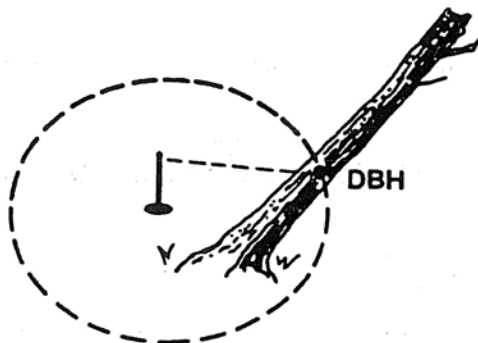


34.22 - Figure 01  
Forked Trees

Determine the plot radius to the face of the tree, corrected for slope, as follows:

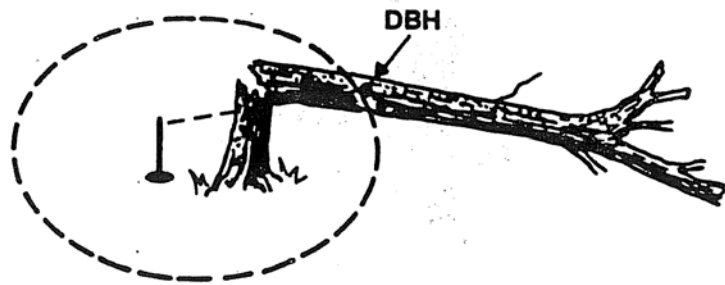
$$\text{Plot radius to tree face} = (\text{Plot radius (ft.)} - 0.5 \times \text{DBH (ft.)}) \times \text{correction for slope from DBH to top of center stake.}$$

2. Leaning trees. If the plot boundary passes through the center of the tree at DBH, count the tree in (fig. 02).



34.22 - Figure 02  
Leaning Trees

3. Downed trees. Locate breast height at 4-1/2 feet above the root collar, as tree lies on the ground. If this point lies on or inside the plot boundary, count the tree in (fig. 03).



34.22 - Figure 03  
Downed Trees

### 34.3 - Calculating Sample Size

Use the following method to calculate the number of fixed plots needed in the sample. To calculate sample size:

1. Specify the sampling error objective for the sale as a whole.
2. Subdivide (stratify) the sale population into sampling components. The purpose is to reduce the coefficient of variation within the sampling strata.
3. Calculate coefficient of variation by stratum and a weighted CV over all strata.
4. Calculate number of plots for the sale as a whole, then allocate by stratum.

For example, from a precruise, suppose the estimated values are as in exhibit 01.

### 34.3 - Exhibit 01

#### Data from Pre-cruise Analysis

<u>Stratum</u>	<u>Timber Condition</u>	<u>Est. MBF/acre</u>	<u>Acres</u>	<u>Est. MBF</u>	(a) <u>Pct Volume</u>	(b) <u>Est. CV%</u>	CV Fraction <u>(a)</u> <u>x (b)</u>
1	Mixed sizes	10	30	300	22	80	17.6
2	Even-aged	15	10	150	11	70	7.7
3	Scattered	5	20	100	8	90	7.2
4	Even-aged	20	40	800	59	60	<u>35.4</u>
			100	1350	100		67.9
							Weighted CV

Given:

Sale sampling error objective ( $E_T$ ) = 10%

Plot size: 1/5 acre

Weighted coefficient of variation: 67.9

Calculate the number of plots needed for sale as a whole ( $n_T$ ):

$$n_T = \frac{t^2 (\text{weighted CV})^2}{E_T^2}, \text{ where } t = 2 \text{ standard errors}$$

$$n_T = \frac{2^2 (67.9)^2}{10^2} = 184.4 \text{ or } 185 \text{ plots}$$

Allocate the number of plots by strata ( $n_j$ ):

$$n_j = \frac{(\text{CV Fraction}_j) (n_T)}{\text{weighted CV}}$$

	# Plots
$n_1 = 17.6 \times 185/67.9$	= 48
$n_2 = 7.7 \times 185/67.9$	= 21
$n_3 = 7.2 \times 185/67.9$	= 20
$n_4 = 35.4 \times 185/67.9$	= <u>96</u>
Total Sale	= 185

### 34.4 - Calculating Sampling Statistics

#### 34.41 - Sample Expansion

Exhibit 01 shows the basic procedure for expanding sample plot data. Data for only one strata is used; however, the procedure for combining strata calculating information about the sale as a whole is described in section 38.

The following information is given for the strata in this example:

Fixed Plot Size = 1/5 Acre  
Strata Area = 18 Acres  
Number of plots taken = 10

Compute the individual tree expansion factor or weight as:

$$F_t = \frac{1}{S_z \times p}$$

Where:

$F_t$  = Tree factor for  $i^{\text{th}}$  sample tree (trees per acre)  
 $S_z$  = Fixed area plot size (acres)  
 $p$  = Number of fixed area plots sampled

and for this example,

$$F_t = \frac{1}{0.2 \times 10} = 0.5$$

Since each sample tree represents  $F_t$  trees per acre, calculate the per acre volume by multiplying the trees per acre for a sample tree ( $F_t$ ) by the volume for that sample tree (MV). That is, the volume factor is:

$$F_v = F_t \times MV$$

Where:

$F_v$  = Volume factor for  $i^{\text{th}}$  tree (Volume per acre)  
 $F_t$  = Tree factor for  $i^{\text{th}}$  tree (a constant for fixed area; trees per acre)  
MV = Measured volume for  $i^{\text{th}}$  tree

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**34.41 - Exhibit 01**

**Expanded Volume**

Plot	Sp	DBH	HT	Net Vol (MV)	Plot Vol	Tree Fact (F <sub>t</sub> )	Exp Vol (MVxF <sub>t</sub> )	Exp Plot Vol	Exp Plot Vol <sup>2</sup>
1	A	10	48	10.7		0.5	5.4		
	A	11	74	24.2		0.5	12.1		
	A	13	68	29.0		0.5	14.5		
	B	13	73	26.9		0.5	13.5		
	B	8	45	4.9		0.5	2.5		
	B	13	61	23.0		0.5	11.5		
	B	15	88	43.8	162.5	0.5	21.9	81.4	6626.0
2	A	12	59	21.0		0.5	10.5		
	B	1.0	56	11.5		0.5	5.8		
	B	9	61	10.1		0.5	5.1		
	B	14	66	29.1		0.5	14.6		
	B	9	43	6.7		0.5	3.4		
	A	16	67	41.4	119.8	0.5	20.7	60.1	3612.0
3					0.0			0.0	0.0
4	A	12	61	21.0		0.5	10.5		
	A	13	74	30.9		0.5	15.5		
	A	17	92	66.3		0.5	33.2		
	B	12	63	19.3		0.5	.9.7		
	B	9	54	8.8		0.5	4.4		
	C	11	54	14.0	160.3	0.5	7.0	80.3	6448.1
5	A	13	77	31.8		0.5	15.9		
	C	11	52	13.0		0.5	6.5		
	A	14	70	33.4		0.5	16.7		
	C	11	63	16.8	95.0	0.5	8.4	47.5	2256.3
6	A	14	69	32.4		0.5	16.2		
	A	16	72	45.6	78.0	0.5	22.8	39.0	1521.0
7					0.0			0.0	0.0
8	A	12	74	27.8		0.5	13.9		
	B	10	55	11.0		0.5	5.5		
	B	9	63	10.7		0.5	5.4		
	B	11	56	14.2		0.5	7.1		
	B	13	63	24.2		0.5	12.1		
	B	12	70	21.6	109.5	0.5	10.8	54.8	3003.0
	A	15	69	38.4		0.5	19.2		
9	C	12	57	17.7		0.5	8.9		
	A	11	63	19.7		0.5	9.9		
	A	10	65	17.3	93.1	0.5	8.7	46.7	2180.9
10	B	13	73	26.9		0.5	13.5		
	B	8	45	4.9		0.5	2.5		
	B	13	61	23.0		0.5	11.5		
	B	15	88	43.8	98.6	0.5	21.9	49.4	2440.4
Sum				916.8	916.8	19.5	459.2	459.2	28087.7
Count		39							

$$\begin{aligned}\text{Strata Volume} &= \text{Strata Acres} \times (\text{Sum of Expanded ft}^3 \text{ Volume}) \\ &= 18 \times 459.2 \\ &= 8,265.6 \text{ ft}^3\end{aligned}$$

Calculate the stratum volume alternately as:

$$\begin{aligned}\text{Strata Vol.} &= \text{Strata ac.} \times \left( \sum^n MV \right) \times F_t \\ &= 18 \times 916.8 \times \frac{1}{0.2 \times 10} \\ &= 18 \times 916.8 \times .5 \\ &= 8251.2 \text{ ft}^3\end{aligned}$$

### 34.42 - Sampling Error

Calculate the sampling error for the strata using the data from section 34.41 - exhibit 01. The example calculations use the expanded volume per plot (x) to provide a consistent approach with other examples for point and two stage sampling presented in sections 35 and 37. For simple fixed plot sampling, where each tree has the same tree factor, the results are identical if the unexpanded volume per plot is used.

Compute the sample statistics for this example:

$$n = 10 \qquad \sum^n x = 459.2 \qquad \sum^n x^2 = 28087.7$$

$$\begin{aligned}\text{Mean} = \bar{X} &= \frac{\sum^n x}{n} \\ &= \frac{459.2}{10} = 45.9\end{aligned}$$

$$\begin{aligned}\text{Standard Deviation} = SD &= \sqrt{\frac{\sum^n x^2 - \frac{(\sum^n x)^2}{n}}{n-1}} \\ &= \sqrt{\frac{28087.7 - \frac{(459.2)^2}{10}}{10-1}} \\ &= 27.9\end{aligned}$$

$$\begin{aligned}\text{Coefficient of Variation} = CV &= \frac{SD}{\bar{X}} \times 100 \\ &= \frac{27.9}{45.9} \times 100 \\ &= 60.8\%\end{aligned}$$



Compute the standard error of the mean for the sample strata. The value of N used in the following equation is 90. Plots 1/5 acre in size were used in this example. Thus the total number of plots for the strata would be 5 plots per acre x 18 Acres = 90 potential plots. Compute the standard error:

$$\begin{aligned}
 N &= 90 \\
 SE &= \sqrt{\frac{\sum^n x^2 - \frac{(\sum^n x)^2}{n}}{n(n-1)} \left(1 - \frac{n}{N}\right)} \\
 &= \sqrt{\frac{28087.7 - \frac{459.2^2}{10}}{10(10-1)} \left(1 - \frac{10}{90}\right)} \\
 &= 8.3 \text{ ft}^3
 \end{aligned}$$

Compute the sample error:

$$\begin{aligned}
 E &= \frac{SE}{\bar{X}} \times 100 \times t \\
 &= \frac{8.3}{45.9} \times 100 \times 2 \\
 &= 36.2\%
 \end{aligned}$$

### 34.5 - Additional Population Characteristics

In addition to estimating the product volume, it may be necessary to make estimates of other population characteristics. In this example, determine the average diameter, the quadratic mean diameter, and the average height using the expanded plot information in exhibit 01.

As with the calculation for sampling error, the example calculations in this section use the expanded volumes to maintain consistency with the other examples. For fixed plot samples, the results are identical using the-Unexpanded plot volumes.

**34.5 - Exhibit 01**

## Expanded DBH, Quadratic DBH, and Height

Plot	Sp	DBH	DBH <sup>2</sup>	HT	Tree Fact (F <sub>t</sub> )	Exp DBH (DBH×F <sub>t</sub> )	Exp DBH2 (DBH <sup>2</sup> ×F <sub>t</sub> )	Exp HT (HT×F <sub>t</sub> )
1	A	10	100	48	0.5	5.0	50.0	24.0
	A	11	121	74	0.5	5.5	60.5	37.0
	A	13	169	68	0.5	6.5	84.5	34.0
	B	13	169	73	0.5	6.5	84.5	36.5
	B	8	64	45	0.5	4.0	32.0	22.5
	B	13	169	61	0.5	6.5	84.5	30.5
	B	15	225	88	0.5	7.5	112.5	44.0
2	A	12	144	59	0.5	6.0	72.0	29.5
	B	10	100	56	0.5	5.0	50.0	28.0
	B	9	81	61	0.5	4.5	40.5	30.5
	B	14	196	66	0.5	7.0	98.0	33.0
	B	9	81	43	0.5	4.5	40.5	21.5
4	A	16	256	67	0.5	8.0	128.0	33.5
	A	12	144	61	0.5	6.0	72.0	30.5
	A	13	169	74	0.5	6.5	84.5	37.0
	A	17	289	92	0.5	8.5	144.5	46.0
	B	12	144	63	0.5	6.0	72.0	31.5
	B	9	81	54	0.5	4.5	40.5	27.0
	C	11	121	54	0.5	5.5	60.5	27.0
5	A	13	169	77	0.5	6.5	84.5	38.5
	C	11	121	52	0.5	5.5	60.5	26.0
	A	14	196	70	0.5	7.0	98.0	35.0
	C	11	121	63	0.5	5.5	60.5	31.5
6	A	14	196	69	0.5	7.0	98.0	34.5
	A	16	256	72	0.5	8.0	128.0	36.0
8	A	12	144	74	0.5	6.0	72.0	37.0
	B	10	100	55	0.5	5.0	50.0	27.5
	B	9	81	63	0.5	4.5	40.5	31.5
	B	11	121	56	0.5	5.5	60.5	28.0
	B	13	169	63	0.5	6.5	84.5	31.5
	B	12	144	70	0.5	6.0	72.0	35.0
9	A	15	225	69	0.5	7.5	112.5	34.5
	C	12	144	57	0.5	6.0	72.0	28.5
	A	11	121	63	0.5	5.5	60.5	31.5
	A	10	100	65	0.5	5.0	50.0	32.5
10	B	13	169	73	0.5	6.5	84.5	36.5
	B	8	64	45	0.5	4.0	32.0	22.5
	B	13	169	61	0.5	6.5	84.5	30.5
	B	15	225	88	0.5	7.5	112.5	44.0
Sum					19.5	235.0	2929.0	1256.0
Count		39						

Calculate the additional population characteristics:

$$\begin{aligned} \text{Mean DBH} &= \frac{\sum^n (DBH \times F_t)}{\sum^n F_t} \\ &= \frac{235}{19.5} \\ &= 12.0 \text{ inches} \end{aligned}$$

$$\begin{aligned} \text{Quadratic Mean Diameter} &= \sqrt{\frac{\sum^n (DBH^2 \times F_t)}{\sum^n F_t}} \\ &= \sqrt{\frac{2929}{19.5}} \\ &= 12.2 \text{ inches} \end{aligned}$$

$$\begin{aligned} \text{Mean Height} &= \frac{\sum^n (HT \times F_t)}{\sum^n F_t} \\ &= \frac{1256.0}{19.5} \\ &= 64 \text{ feet} \end{aligned}$$

### 34.6 - Application

This method is applicable in timber of uniform distribution, especially where large areas are involved and clearcutting is the prescription.

## **35 - Point Sampling**

### **35.1 - Point Sampling Method**

Point sampling is an application of variable probability sampling.

#### **35.11 - Operational Features**

In this cruising method, establish a specified number of points in an unbiased manner over the tract of timber to be cruised. Visit each sample point and using an angle gauge, check each tree around the point, normally at breast height, as a possible sample tree. Select trees whose diameter is larger than the angle projected by the gauge, as sample trees.

#### **35.12 - Statistical Features**

The probability of a tree being selected as a sample unit in point sampling is proportional to its basal area. For example, a tree of 2 square feet of basal area has twice the chance of being selected as a tree of 1 square foot of basal area. To account for this variable probability in calculating number of trees per acre and other estimates, weight each sample tree inversely to its selection probability. Therefore, in sample expansion, a tree of 2 square feet basal area would count half as much as a tree of 1 square foot basal area. This type of variable probability sampling is referred to as sampling with probability proportional to size (PPS).

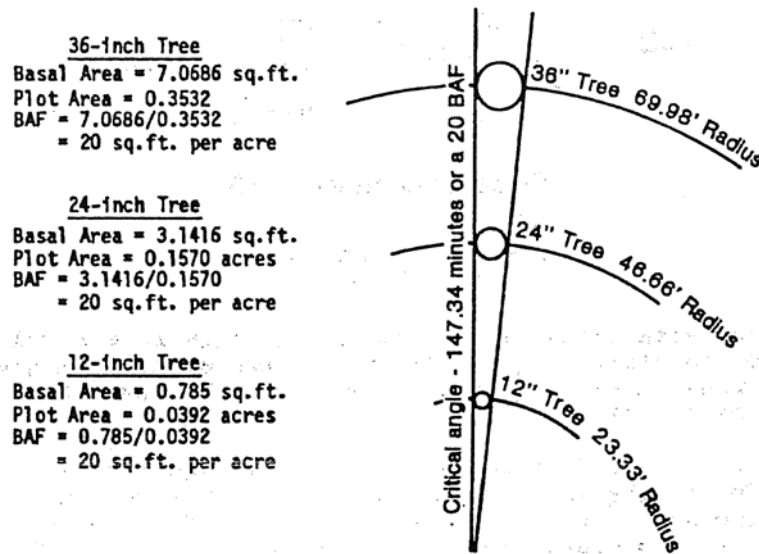
Timber cruising using, point sampling may be done by either simple point sampling or ratio double sampling. In either method, take a sample and systematically or randomly establish points over the area to be cruised. In simple point sampling, measure all point-sampled trees on all points. In ratio double sampling, count sample trees at all points but only measure on some of the points. The extension of simple point sampling to ratio double sampling is addressed in section 37.5.

The variable of interest for point sampling, subject to error, is volume or value per acre.

### 35.13 - Basal Area Factor

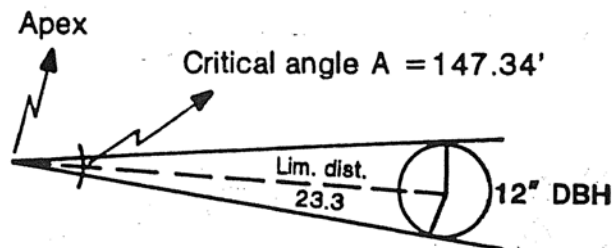
In point sampling, each sample tree, regardless of its diameter breast height (DBH), represents the same basal area per acre for a given critical angle. This constant is called the basal area factor (BAF) of the angle gauge (fig. 01). The rationale for this basic point sampling concept follows.

In fixed area sampling, when using circular plots, the plot radius is fixed for a plot of a given size. The plot radius for a fifth-acre plot is 52.7 feet, for example. Each tree on a fifth-acre plot, regardless of size is associated with a plot radius of 52.7 feet.



35.13 - Figure 01  
 Basal Area Factor Concept

In point sampling, each different tree diameter has a different plot radius associated with it and, therefore, a different plot size (fig. 01). The plot radius is the distance from the apex of the critical angle to the center of the tree subtending the critical angle (fig.02).



Basal area factor = 20

35.13 - Figure 02  
 Basal Factor 20

The circular plot area described by the plot radius is directly proportional to the basal area of the tree. For example, a tree of 2 square feet of basal area has a plot area twice the size of a tree of 1 square foot of basal area.

For a given critical angle, the basal area in square feet of a tree of any size, divided by the area of the trees plot in acres, is always equal to the same amount of basal area per acre. This angle gauge constant is called the basal area factor or BAF for short. Angle gauges are classified and referenced according to BAF.

## **35.2 - Field Procedures for Point Sampling**

### **35.21 - Sample Point Location and Monumentation of Points and Trees**

Select the location of the starting point at random if the sample points are to be located on a grid. A systematic sample with a random start can be considered one random sample out of an infinite number of random samples.

Clearly monument the starting point and sample point locations to afford relocation for check cruising. File documentation in the cruise record describing how the starting points and sample point locations can be found.

Mark sample point locations with a wooden stake or a wire pin. Label the mark so that a checker can be certain which point it is. If a wooden stake is used, mark the point on the stake with a nail or other mark, such as an "X", to provide a specific point from which to measure limiting distance.

To facilitate check cruising, number tally trees clockwise from north or from the direction of travel on the side of the tree facing the point.

#### **35.21a - Angle Gauge Use**

Use the appropriate size angle gauge, depending on the size and/or dispersion of the timber to be cruised.

The angle gauge used should have a basal area factor (BAF) that will select an average of 4 to 8 trees per point. Do not use a smaller BAF that results in more sample trees per point. This may not reduce the coefficient of variation but may result in missed trees. A larger BAF, on the other hand, will result in fewer sample trees per point but usually the coefficient of variation is increased. Use only one BAF for a particular stratum.

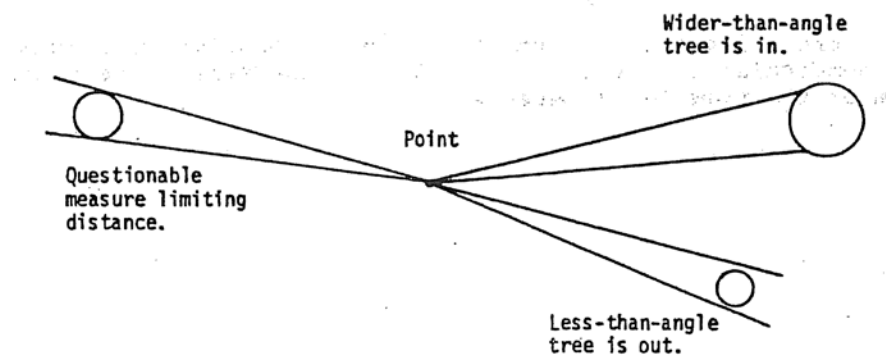
Angle gauges are made of various materials, such as metal, plastic, or glass, and differ in the way the critical angle is established.

Angle gauges made from metal or other lightweight material are cut at a specified width with a length of cord attached. The cord is marked so that the gauge is held a constant distance in front of the eye, usually about 18 to 20 inches. The width of the gauge and the length of the cord in combination establish the critical angle.

To use this type of angle gauge, the cruiser should stand with the eye over the point, viewing the width of the angle gauge in relation to tree DBH. A fixed angle is projected from the eye past the angle gauge. to' the tree at DBH. If the tree appears wider than the angle gauge, the tree is in. If it appears narrower, the tree is out. The angle gauge is made for use on a horizontal plane and does not work well on slopes.

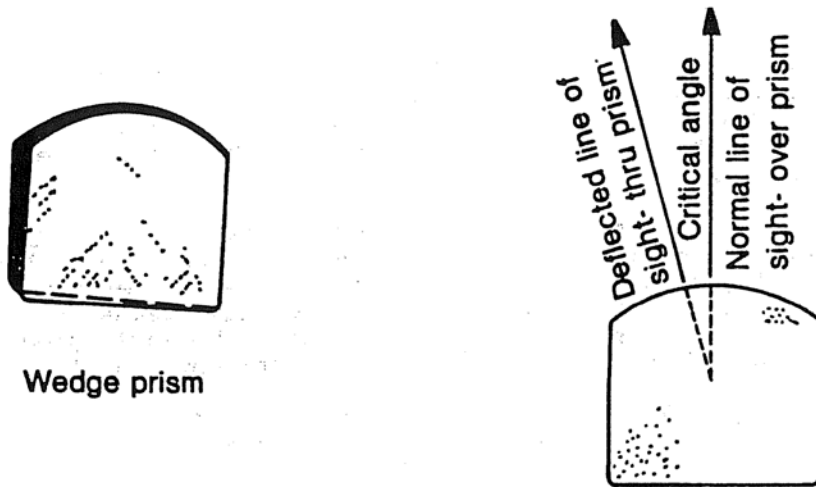
The relaskop, which is widely used for point sampling, and laser devices correct for slope and include a wide range of basal area factors.

A questionable tree (fig. 01) must be measured to determine if it is in or out (sec. 35.22a). These principles also apply to the relaskop.



35.21a - Figure 01  
Use of a Simple Angle Gauge

The wedge prism is an accurate angle gauge. The wedge prism is an optical instrument that bends light rays to establish the critical angle. The amount of critical angle is determined by the angle to which the prism is ground. Prisms are ground to specified units of refractive displacement called diopters, to produce exact basal area factors. A diopter is a right-angled deflection of 1 centimeter per 100 centimeters of distance from the object being viewed through the prism (fig. 02).

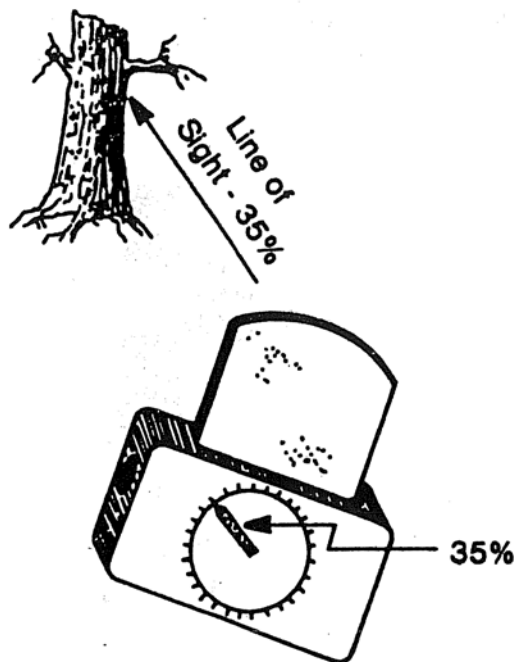


35.21a- Figure 02  
Wedge Prism

In use, keep the prism exactly over the point. Position it so that its face is perpendicular to the line of sight and the base is perpendicular to the stem of the tree being measured.

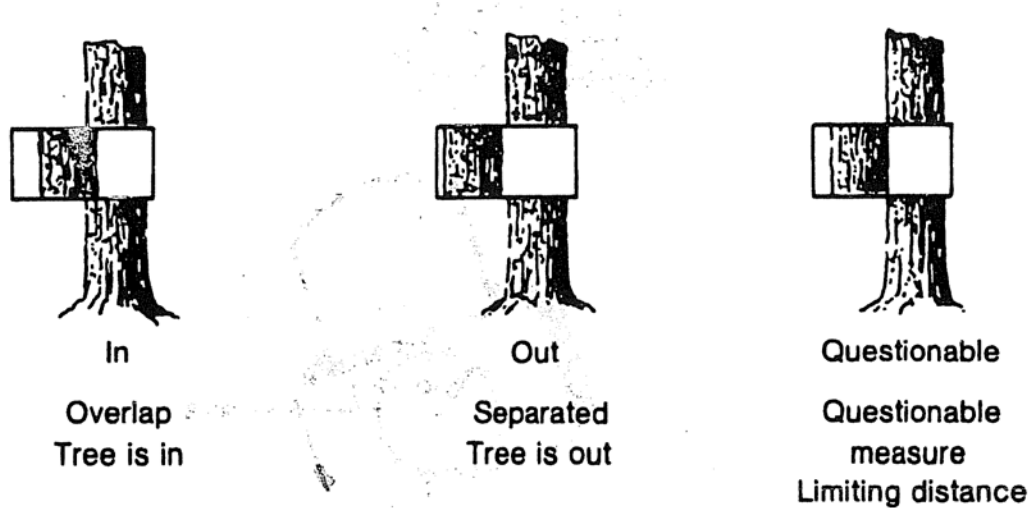


Where more than 10 percent slope is involved between the prism and the tree, rotate the prism around the line of sight by exactly the angle of slope between the prism and the tree's DBH (fig. 03).



35.21a - Figure 03  
Using A Clinometer to Correct for Slam

To determine in and out trees, view each tree through the prism. The portion of the tree viewed through the prism is deflected, creating a displaced image. If the displacement overlaps the tree, the tree is in. If the displacement is separated from the tree, the tree is out. If the displacement is questionable or appears borderline, take measurements to determine whether or not the tree is included in the sample (fig. 04). See section 35.22a for more details on borderline and questionable trees.



35.21a - Figure 04  
Use of a Prism in Point Sampling

### 35.22 - Sample Tree Determination

Select sample trees with the aid of an angle gauge with a fixed critical angle (sec. 35.1). Ordinarily, when using an angle gauge, trees are clearly either "in" or "out". However, it is not always possible to determine tree status with certainty. These are trees borderline to the plot radius, that is, too close to call when observed with the angle gauge. Another problem may exist when the tree DBH is masked by brush or other trees. Measure plot radius (limiting distance) for these trees to determine the in/out status.

#### 35.22a - Assessing Borderline and Other Questionable Trees

Measure limiting distance to borderline or other questionable trees to conclusively determine if the tree in question is a sample tree.

Limiting distance is another term for plot radius, which is the maximum distance a tree can be from the point and still be in (a sample tree). Limiting distance is a function of tree DBH and an angle gauge constant called the plot radius factor (PRF). The plot radius factor varies with the size of the angle gauge. For a BAF of 20, the PRF is 1.944.

Calculate limiting distance using the following formula:

Limiting Distance (ft.) = DBH x Plot Radius Factor

For example: DBH = 12 inches

PRF = 1.944

Limiting Distance = (12) (1.944)  
= 23.33 feet

The PRF may be determined for an angle gauge of any size from either of the two formulas below:

$$PRF = \frac{0.0833}{\sin A}$$

$$PRF = \frac{8.696}{\sqrt{BAF}}$$

Where:

PRF = Plot radius factor

A = Critical angle (in minutes) of the angle gauge.

BAF = Basal area factor of the angle gauge.

For example, consider an angle gauge with a critical angle of 147.34 minutes which is equal to a BAF of 20 square feet per acre (sec. 35.13 fig. 02).

$$\begin{aligned} PRF &= \frac{0.0833}{\sin A} \\ &= \frac{0.0833}{0.04285} \\ &= 1.944 \end{aligned}$$

$$\begin{aligned} PRF &= \frac{8.696}{\sqrt{BAF}} \\ &= \frac{8.696}{\sqrt{20}} \\ &= 1.944 \end{aligned}$$

Tree DBH (in inches) multiplied by 1.944 gives limiting distance (in feet) from the point to the center of the tree for BAF 20.

It is, however, more efficient and more accurate to measure limiting distance to the point from the face of the tree. Determine a corrected PRF to calculate limiting distance from the face of the tree instead of from the center of the tree.

Calculate a corrected PRF for a given size angle gauge as follows:

1. Calculate the limiting distance to the center of tree.

For a 12" tree and BAF 20:

Limiting Distance = PRF x DBH

Limiting Distance = 1.944 x 12 = 23.33 feet.

2. Calculate the corrected PRF to the face of the tree by using the following formula:

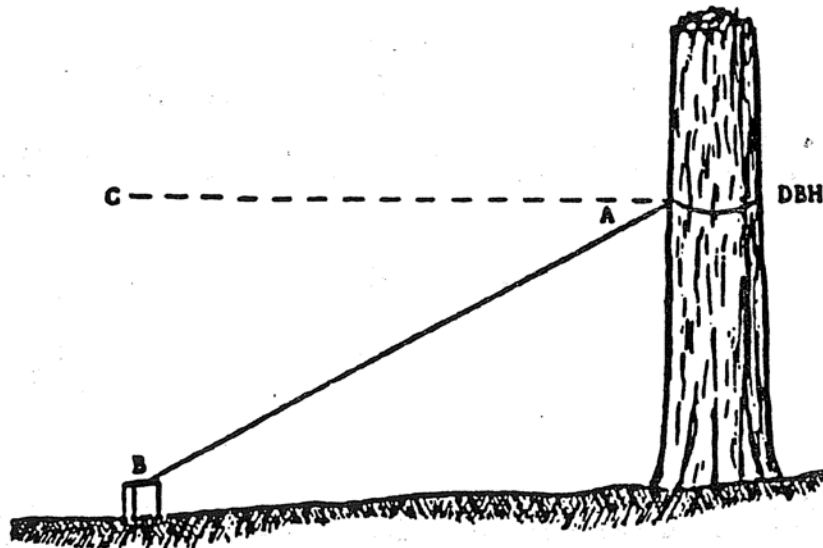
$$\text{Corrected PRF} = \frac{\text{Limiting Distance} - \text{Half DBH in Feet}}{\text{DBH in inches}}$$

For a 12" tree and BAF 20:

$$\text{Corrected PRF} = \frac{23.33' - 0.5'}{12"} = 1.902$$

Each tree DBH multiplied by 1.902 gives a set of limiting distances to the point from the face of the tree for an angle gauge of BAF 20. Limiting distance tables have been developed for various BAF's on this basis.

Use the following procedure to determine if a questionable tree is a sample tree:



**35.22a - Figure 01**  
**Determining if a Sample Tree is "IN"**

1. Measure the diameter to the tenth of an inch.
2. Determine the horizontal limiting distance from the face of the tree. Horizontal Limiting Distance = Corrected PRF x Diameter (DBH) (AC, fig. 01).
3. Determine the percent of slope from the face of the tree at DBH to the point center (AB, fig. 01).
4. If the slope is ten percent or more, correct the horizontal limiting distance to slope limiting distance. Multiply the horizontal limiting distance by the appropriate slope correction factor (sec. 14.21, exhibit 01).

5. Use a steel tape graduated in tenths of feet to measure the distance from the face of the tree at DBH to the point center (AB, fig. 01). These are two exact points that can be measured "from" and "to". If the tree is abnormal at DBH measure from the face where the diameter is taken. Refer to sections 14.12-14.12m for details.
6. If the measured distance is equal to or less than the slope limiting distance the tree is "IN" and is sampled. If no slope correction is needed the horizontal limiting distance is compared to the measured distance.

### 35.3 - Calculating Sample Size

The following is a method for calculating the number of point sample plots needed using the optimum allocation method. Calculate sample size as follows:

1. Specify the sampling error objective for the sale as a whole.
2. Subdivide (stratify) the sale population into sampling components. The purpose is to reduce the coefficient of variation within the sampling strata.
3. Calculate the coefficient of variation by stratum and a weighted CV over all strata.
4. Calculate the number of plots by stratum as follows:

#### 35.3 - Exhibit 01

Data from Pre-cruise Analysis

<u>Stratum</u>	<u>Timber Condition</u>	<u>Est MBF/Acre</u>	<u>Acres</u>	<u>Est. MBF</u>	(a) <u>Pct. Volume</u>	(b) <u>Est CV%</u>	<u>CV<sub>j</sub> Fraction</u>
					<u>e</u>	<u>n</u>	<u>n</u>
1	Scattered	2.8	18	50	4	90	3.6
2	Mixed sizes	10.0	35	350	26	80	20.8
3	Even-aged	15.0	10	150	11	70	7.7
4	Even-aged	20.0	<u>40</u>	<u>800</u>	<u>59</u>	60	<u>35.4</u>
Total			103	1350	100		67.5

Where:

$$\text{Stratum Percent Volume} = \frac{\text{Stratum Estimated MBF} \times 100}{\text{Sum of Estimated MBF}}$$

$$\text{CV}_j \text{ Fraction} = \frac{\text{Percent Volume} \times \text{Estimated CV}\%}{100}$$

Given:

Sale sampling error objective:  $E_T = 10\%$   
BAF: 20

Weighted CV: Sum of CV<sub>j</sub> Fractions, or 67.5

Calculate the number of plots needed for sale as a whole, n<sub>T</sub>:

$$n_T = \frac{t^2 (\text{weighted CV})^2}{E_T^2}, \text{ where } t = 2 \text{ standard errors}$$

$$n_T = \frac{2^2 (67.5)^2}{10^2} = 182.3 \text{ or } 183 \text{ Plots}$$

Allocate the number of plots by strata:

$$n_j = \frac{(\text{CV}_j \text{ fraction}) (n_T)}{\text{Weighted CV}}$$

$$\begin{aligned} n_1 &= 3.6 \times 183 / 67.5 = 10 \text{ PLOTS} \\ n_2 &= 20.8 \times 183 / 67.5 = 56 \text{ PLOTS} \\ n_3 &= 7.7 \times 183 / 67.5 = 21 \text{ PLOTS} \\ n_4 &= 35.4 \times 183 / 67.5 = \underline{96 \text{ PLOTS}} \\ \text{Sale Total} &= 183 \text{ PLOTS} \end{aligned}$$

### 35.4 - Calculating Sampling Statistic

#### 35.41 - Sample Expansion

Two methods are commonly used in expanding point sample data: (1) the factor method, Beers and Miller (1964), and (2) the V-BAR method, Beers and Miller (1964), Dilworth and Bell (1981). Both methods give identical results except for trivial rounding differences. The tree factor method has the advantage of being able to expand all tree characteristics like DBH, height, grade, or logs/MBE because it represents trees/acre and not just volume. Section 37.54a gives the details for the V-BAR method.

The factor method involves calculating a tree factor also called the sample tree weight or frequency based on the sampling method. For point sampling, calculate the factor using the basal area of the tree.

Two factors of interest are the tree factor ( $F_t$ ) and the volume factor ( $F_v$ ). The tree factor is the number of trees per acre represented by each sample tree and the volume factor is the volume per acre represented by each sample tree. Compute the factors as follows:

$$F_v = F_t \times MV$$

where:

$F_v$  = Volume factor for  $i^{\text{th}}$  tree (volume/acre)

$F_t$  = Tree factor for  $i^{\text{th}}$  tree (trees/acre) =  $\frac{BAF}{.005454 \times DBH^2 \times p}$

MV = Measured volume in  $i^{\text{th}}$  tree

BAF = Basal area factor

DBH = Diameter of  $i^{\text{th}}$  tree

p = Number of prism points taken

Compute the estimated total strata or tract volume contributed by each individual tree as follows:

$$SV = F_v \times \text{Acres}$$

where:

SV = Total volume for the  $i^{\text{th}}$  tree

Acres = Number of acres in the strata

In this method, measure all sample trees on all points. Exhibits 01 and 02 show the basic procedure for expanding point sample data and the plot by species cross-tabulation. The sample tree count, volume factors, and tree factors, and any other measured variables can be cross-tabulated by any other observed variable. The following information is given for the strata in the example:

Basal Area Factor (BAF) = 20

Strata Acres = 18

Number of plots taken = 10

Data for one strata (stratum 1) is shown. The procedure for combining strata and calculating information about the sale as a whole is described in section 38.

**35.41 - Exhibit 01**

## Simple Point Sample Data

Plot No.	Sp	DBH	Meas Vol (MV)	HT	Tree Fact (F <sub>t</sub> )	Vol Fact (F <sub>v</sub> )	Plot Vol	Plot Vol <sup>2</sup>
1	A	10	10.7	48	3.67	39.27		
1	A	11	24.2	74	3.03	73.33		
1	A	13	29.0	68	2.17	62.93		
1	B	13	26.9	73	2.17	58.37		
1	B	8	4.9	45	5.73	28.08		
1	B	13	23.0	61	2.17	49.91		
1	B	15	43.8	88	1.63	71.39	383.28	146903.56
2	A	12	21.0	59	2.55	53.55		
2	B	10	11.5	56	3.67	42.21		
2	B	9	10.1	61	4.53	45.75		
2	B	14	29.1	66	1.87	54.42		
2	B	9	6.7	43	4.53	30.35		
2	A	16	41.4	67	1.43	59.20	285.48	81498.83
3							0.00	0.00
4	A	12	21.0	61	2.55	53.55		
4	A	13	30.9	74	2.17	67.05		
4	A	17	66.3	92	1.27	84.20		
4	B	12	19.3	63	2.55	49.22		
4	B	9	8.8	54	4.53	39.86		
4	C	11	14.0	54	3.03	42.42	336.30	113097.69
5	A	13	31.8	77	2.17	69.01		
5	C	11	13.0	52	3.03	39.39		
5	A	14	33.4	70	1.87	62.46		
5	C	11	16.8	63	3.03	50.90	221.76	49177.50
6	A	14	32.4	69	1.87	60.59		
6	A	16	45.6	72	1.43	65.21	125.80	15825.64
7							0.00	0.00
8	A	12	27.8	74	2.55	70.89		
8	B	10	11.0	55	3.67	40.37		
8	B	9	10.7	63	4.53	48.47		
8	B	11	14.2	56	3.03	43.03		
8	B	13	24.2	63	2.17	52.51		
8	B	12	21.6	70	2.55	55.08	310.35	96317.12
9	A	15	38.4	69	1.63	62.59		
9	C	12	17.7	57	2.55	45.14		
9	A	11	19.7	63	3.03	59.69		
9	A	10	17.3	65	3.67	63.49	230.91	53319.43
10	B	13	26.9	73	2.17	58.37		
10	B	8	4.9	45	5.73	28.08		
10	B	13	23.0	61	2.17	49.91		
10	B	15	43.8	88	1.63	71.39	207.75	43160.06
Sum					109.73	2101.63	2101.63	599299.83
Count		39						



Forest Service Handbook 2409.12 – Timber Cruising Handbook  
Chapter 30 - Cruising Systems  
Amendment: 2409.12-1993-1  
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The sample tree count, volume factors, tree factors, and any other measured variables can be cross tabulated by any other observed variable. Exhibit 02 shows the plot by species cross tabulation.

### 35.41 - Exhibit 02

#### Sample Tree Count and Volume Factor Tabulations

Plot	<u>Count for Species</u>				<u>Volume for Species</u>			
	A	B	C	Sum	A	B	C	Sum
1	3	4	0	7	175.53	207.75	0.00	383.28
2	2	4	0	6	112.75	172.73	0.00	285.48
3	0	0	0	0	0.00	0.00	0.00	0.00
4	3	2	1	6	204.80	89.08	42.42	336.30
5	2	0	2	4	131.47	0.00	90.29	221.76
6	2	0	0	2	125.80	0.00	0.00	125.80
7	0	0	0	0	0.00	0.00	0.00	0.00
8	1	5	0	6	70.89	239.46	0.00	310.35
9	3	0	1	4	185.77	0.00	45.14	230.91
10	0	4	0	4	0.00	207.75	0.00	207.75
Sum	16	19	4	39	1007.01	916.77	177.85	2101.63

The resulting estimated values are:

$$\text{Estimated trees per acre} = 109.73$$

$$\text{Estimated volume per acre} = 2101.63 \text{ ft}^3$$

$$\begin{aligned} \text{Estimated Total Strata Volume (SV)} &= \left( \sum^n F_v \right) \times \text{acres} \\ &= 2101.63 \times 18 = 37,829.3 \end{aligned}$$

### 35.42 - Sampling Error

In simple point sampling where all sample trees are measured on all points, sampling error is computed from:

$$E = \frac{SE}{\bar{x}} \times 100 \times t$$

where:

$t = 2$  standard errors (95% confidence limit)

$$SE = \sqrt{\frac{\sum_{i=1}^n x^2 - \frac{(\sum_{i=1}^n x)^2}{n}}{n(n-1)}}$$

$\bar{x}$  = Average volume per point.

$x$  = Volume for  $i^{\text{th}}$  point.

$x^2$  = Volume squared for  $i^{\text{th}}$  point.

$n$  = Number of points.

To calculate the sampling error, use the tabulated plot sub totals and squares of plot sub totals from section 35.41 exhibit 01.

$$\bar{x} = \frac{2101.63}{10} = 210.16 \text{ ft}^3 \text{ per acre}$$

$$\begin{aligned} \text{Standard Error} = SE &= \sqrt{\frac{599,299.83 - \frac{(2101.63)^2}{10}}{10(9)}} \\ &= \sqrt{\frac{157,614.96}{90}} \\ &= 41.85 \end{aligned}$$

$$E = \frac{41.83}{210.16} \times 100 \times 2 = 39.8\%$$

Identical results are obtained using point sums of V-BARS instead of volume factors.

### 35.5 - Additional Population Characteristics

Use the following formulas and see exhibits 01 and 02 for examples of additional population characteristics.

$$\begin{aligned} \text{Arithmetic Mean Diameter} &= \frac{\text{Sum (DBH} \times \text{Tree Factor)}}{\text{Sum Tree Factors}} \\ &= \frac{\sum^n (\text{DBH} \times F_t)}{\sum^n F_t} \end{aligned}$$

$$\begin{aligned} \text{Quadratic Mean DBH} &= \sqrt{\frac{\text{Sum (DBH}^2 \times \text{Tree Factor)}}{\text{Sum Tree Factors}}} \\ &= \sqrt{\frac{\sum^n (\text{DBH}^2 \times F_t)}{\sum^n F_t}} \end{aligned}$$

$$\begin{aligned} \text{Mean Height} &= \frac{\text{Sum of (HT} \times \text{Tree Factor)}}{\text{Sum of Tree Factors}} \\ &= \frac{\sum^n (\text{HT} \times F_t)}{\sum^n F_t} \end{aligned}$$

Where:

DBH = Diameter of  $i^{\text{th}}$  tree

$F_t$  = Tree factor of  $i^{\text{th}}$  tree (Trees per acre)

HT = height of  $i^{\text{th}}$  tree

$n$  = number of sample trees from all points

**35.5 - Exhibit 01**

Cruise Data Necessary to Estimate Additional Population Characteristics.

Plot	Sp	DBH	DBH <sup>2</sup>	HT	Tree Fact (F <sub>t</sub> )	Exp DBH (DBH×F <sub>t</sub> )	Exp DBH2 (DBH <sup>2</sup> ×F <sub>t</sub> )	Exp HT (HT×F <sub>t</sub> )
1	A	10	100	48	3.67	36.7	367.0	176.2
1	A	11	121	74	3.03	33.3	366.6	224.2
1	A	13	169	68	2.17	28.2	366.7	147.6
1	B	13	169	73	2.17	28.2	366.7	158.4
1	B	8	64	45	5.73	45.8	366.7	257.9
1	8	13	169	61	2.17	28.2	366.7	132.4
1	B	15	225	88	1.63	24.5	366.8	143.4
2	A	12	144	59	2.55	30.6	367.2	150.5
2	8	10	100	56	3.67	36.7	367.0	205.5
2	B	9	81	61	4.53	40.8	366.9	276.3
2	B	14	196	66	1.87	26.2	366.5	123.4
2	B	9	81	43	4.53	40.8	366.9	194.8
2	A	16	256	67	1.43	22.9	366.1	95.8
3								
4	A	12	144	61	2.55	30.6	367.2	155.6
4	A	13	169	74	2.17	28.2	366.7	160.6
4	A	17	289	92	1.27	21.6	367.0	116.8
4	B	12	144	63	2.55	30.6	367.2	160.7
4	B	9	81	54	4.53	40.8	366.9	244.6
4	C	11	121	54	3.03	33.3	366.6	163.6
5	A	13	169	77	2.17	28.2	366.7	167.1
5	C	11	121	52	3.03	33.3	366.6	157.6
5	A	14	196	70	1.87	26.2	366.5	130.9
5	C	11	121	63	3.03	33.3	366.6	190.9
6	A	14	196	69	1.87	26.2	366.5	129.0
6	A	16	256	72	1.43	22.9	366.1	103.0
7								
8	A	12	144	74	2.55	30.6	367.2	188.7
8	B	10	100	55	3.67	36.7	367.0	201.9
8	B	9	81	63	4.53	40.8	366.9	285.4
8	B	11	121	56	3.03	33.3	366.6	169.7
8	B	13	169	63	2.17	28.2	366.7	136.7
8	B	12	144	70	2.55	30.6	367.2	178.5
9	A	15	225	69	1.63	24.5	366.8	112.5
9	C	12	144	57	2.55	30.6	367.2	145.4
9	A	11	121	63	3.03	33.3	366.6	190.9
9	A	10	100	65	3.67	36.7	367.0	238.6
10	R	13	169	73	2.17	28.2	366.7	158.4
10	B	8	64	45	5.73	45.8	366.7	257.9
10	B	13	169	61	2.17	28.2	366.7	132.4
10	B	15	225	88	1.63	24.5	366.8	143.4
Sum					109.73	1230.1	14304.5	6707.2
Count		39						

The estimated cubic feet per acre and total cubic feet for the 18 acre tract for the factor calculation method is shown in exhibit 02. Also shown are the estimated number of trees per acre and the estimated quadratic mean DBH. Intermediate totals are taken from the tables in section 35.41 exhibit 02 and section 35.5 exhibit 01.

These tabulated values may be calculated by species or for all species combined. The procedures are identical; however, only use the sums for the appropriate species.

The volume per acre is the sum of the volume factors for all the sample trees, or  $\sum^n F_v$ . For example, the volume per acre in species A can be calculated from section 35.41 exhibit 01, or 1007.01 ft<sup>3</sup>/acre. The total tract volume is the per acre volumes multiplied by the total acreage, or  $\left(\sum^n F_v\right) \times \text{Acres}$ . As an example, the tract volume in species B would be 916.77 x 18 = 16,501.86 ft<sup>3</sup>. The number of trees per acre is the sum of the tree factors, or  $\sum^n F_t$ . For species C, the sum of tree factors from exhibit 01 is 11.6 trees per acre.

The arithmetic mean diameter is the sum of the diameter times the tree factor divided by the sum of the tree factors:

$$\text{Mean Diameter} = \frac{\sum^n (DBH \times F_t)}{\sum^n F_t}$$

For species A, this is 460.7/37.1 = 12.4. The intermediate totals for calculating this and the remaining parameters can be derived from exhibit 01.

The quadratic mean diameter is the diameter of the tree of average basal area. Divide the basal area (or sum of diameters squared) by the number of trees and convert the average basal area back to a diameter. To calculate the quadratic diameter for the entire stand:

$$\text{Quadratic Mean Diameter} = \sqrt{\frac{\sum^n DBH^2 \times F_t}{\sum^n F_t}} = \sqrt{\frac{6707.2}{109.7}} = 11.4$$