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Chapter 30 - Cruising Systems

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The height can likewise be calculated by summing the weighted heights and dividing by the sum of the weights. The average height for species B is:

$$Mean\ HT = \frac{\sum^{n}(HT \times F_t)}{\sum^{n} F_t} = \frac{657.5}{11.6} = 61.1$$

35.5 - Exhibit 02

Cruise Summary for Simple Point Sample

	Species			All
	A	B	C	Species
ft^3/acre	1007.01	916.77	177.85	2101.63
Total ft^3	18126.18	16501.86	3201.30	37829.34
Trees/acre	37.1	61.0	11.6	109.7
Arithmetic Mean Diameter	12.4	10.5	11.2	11.2
Quadratic Mean Diameter	12.6	10.7	11.2	11.4
Mean Height	67.1	58.4	56.7	61.1

35.6 - Application

Point sampling is an effective system to use in clear-cutting situations where large areas are to be cruised and where there is a range of tree sizes.

36 - 3P Sampling

36.1 - 3P Sampling Method

3P sampling is a form of variable probability sampling.

36.11 - Operational Features

3P cruising involves visiting each tree to be sold on a timber sale. Tree volume or value (KPI) is estimated and the estimate is compared with a random number. If the estimated volume or value (KPI) is equal to or greater than the random number, measure the tree as a sample tree. Other tree variables that are closely correlated with tree volume such as DBH^2, (DBH^2 x HT), or $(.5 \times DBH)^2$ may also be used as KPI.

The larger the predicted volume or value (KPI), the greater chance a tree has of being selected as a sample unit. This chance or probability is proportional to KPI, hence, probability proportional to prediction or 3P. A tree of KPI 10, for example, has twice the chance of being selected as a tree of KPI 5. Therefore, the larger or more valuable trees are favored for sample tree selection.

36.12 - Statistical Features

The variable of interest in 3P sampling is the Measured/Predicted (M/P) ratio. This ratio is determined for each sample tree by dividing the measured volume or value for the tree by the predicted volume or value. The coefficient of variation of the M/P ratio is low, usually 35 percent or less. Therefore, few sample trees are needed to achieve standard errors of 5 percent or less. The M/P ratio, being sample based, is subject to sampling error.

36.2 - Field Procedures

A crew consisting of a tallier and two to four markers may provide the most efficient field setup.

As each tree is marked, the cruiser estimates and calls out the predicted volume or value (KPI). The tallier records the KPI and compares it to a random number. If the KPI equals or exceeds the random number, the tree qualifies as a 3P sample tree.

KPI must never exceed the largest number in the random number list, which is termed "K". Any KPI larger than K must be reduced to K or the tree and all others like it must be 100 percent cruised. Trees 100 percent cruised are termed "sure measures".

Sometimes minor or very high value species are 100 percent cruised. Such a situation might occur in a 3P cruise when it is known that only a few trees of a certain species will be marked during the sale and sampling will be inappropriate.

Once a cruiser has called the KPI and the subject tree qualifies as a sample, do not change the KPI. Sometimes a cruiser makes a poor estimate and upon measuring the sample tree, may be tempted to alter the estimate. Do not alter the estimate; to do so would bias the cruise.

Good bookkeeping technique is important in 3P cruising. Tally KPI's by stratum during the cruise so that later summarization involves no guess work.

Collect all cruise data at a day's end, particularly the KPI tally. A good KPI recording technique is to cut the random number lists into two-, three-, or four-column strips depending on the number of species in the sale. Label all strips as to sampling stratum before or immediately after cutting to insure no future mixups. Number strips within stratum consecutively for accountability purposes. As each KPI is called, record it beside a random number.

A solitary cruiser doing the complete cruising job (including sample selection and tallying) must keep the random numbers masked to avoid any chance of influencing the KPI. One number should be revealed at a time and then only after the KPI has been determined. A cheap masking device is a 35mm film cassette. Cut the random number lists into strips, by columns, splice with see-through tape, and spool onto the cassette arbor. Numbers can be revealed one at a time through the felt-tipped cassette opening.

36.3 - Calculating Sample Size and Preparing 3P Random Number Lists

Use the following optimum allocation method to calculate the number of 3P sample trees:

1. Specify the 3P sampling objective for the sale as a whole.

2. Subdivide the sale population into sampling components. The purpose of this is to reduce the coefficient of variation (CV) within the sampling strata.

3. Calculate the coefficient of variation (CV) by sample group and a weighted CV for the sale.

4. Calculate the number of 3P sample trees needed by sample group.

For example, CV for the measured/predicted (M/P) ratio is estimated to be 35 percent for sample group 1 and 25 percent for sample group 2. These estimates are based on past experience. Desired total cruise error is E_T = 10%. Calculate the CV fraction as follows:

Sample Group	Species	Estimated Volume	(a) Percent of Vol.	(b) Estimated CV %	CV Fraction (a) x (b)
1	A	50000	.526	35	18.41
2	B and C	45000	.474	25	11.85
		95000	1.000		31.00 Weighted CV

Calculate the number of 3P sample trees for the sale as a whole from the formula:

$$n_{3p} = \frac{t^2 \; CV^2}{E_T^2}$$

Where:

n_{3p} = Number of 3P sample trees
t = Student t. (t = 2 for 95% confidence level)
CV = Coefficient of variation of the M/P ratio in percent.
E_T = Sampling error.

Calculate the total number of trees needed for the sale as a whole:

$$n_{3p} = \frac{2^2\ 31^2}{10^2} = 39$$

Allocate the 39 sample trees to sample groups or stratum as follows:

$$n_j = \frac{(CV\ Fraction)\ (n_{3p})}{Weighted\ CV}$$

$$n_1 = \frac{(18.4)\ (39)}{31} = 24\ Sample\ Trees\ (3P)$$

$$n_2 = \frac{(11.9)\ (39)}{31} = 15\ Sample\ Trees\ (3P)$$

Where:

n_j = the number of 3P sample trees in j^{th} sample group.

In any kind of sampling procedure, sample size is set to meet some statistical accuracy standards. In 3P cruising, because of low variance associated with the M/P ratio, small sampling errors are achieved with small sample sizes. If the coefficient of variation of the M/P ratio were 20 percent (not uncommon in 3P cruising), the sampling error at two standard errors for a sample size of 100 trees would be only 4 percent.

Ordinarily, for a one-stratum sale, sample size need not exceed 75 to 100 trees. This will ensure a maximum 8 percent sampling error at the 95 percent confidence level, even if the CV is as high as 35 percent.

When a sale is comprised of more than one stratum, or sample group, sample size for each stratum should be set high enough to adequately represent stratum grade characteristics. Thirty sample trees is the minimum that should be considered for any one stratum or sample group; however, even in multi-stratum sales, the sample size generally should not exceed 150 trees.

In this example, it was estimated that a total of 39 sample trees would be needed, distributed by species as shown, to represent adequately each stratum and to achieve the desired sampling error overall.

In sample group 2, 15 trees are estimated to be an adequate sample of both species although this should be increased to the minimum of 30 trees as described above. This gives the convenience of one random number list for both species. Tally the KPI's by species, if volume estimates are desired by species.

Produce random number lists to meet the needs of the individual sale. A random number list is needed for each stratum, or sample group, in the sale. A stratum, or sample group, is ordinarily comprised of one species, but can include two or more species. The lists are ordinarily produced by a computer (Grosenbaugh, 1965; Mesavage, 1971). However, 3P random number lists may be prepared from published lists of random digits or by any unbiased random number generator, such as in a handheld programmable calculator.

Random number lists are not needed when a programable data recorder, programed to generate random numbers, is used to collect the data.

Exhibit 01 illustrates the data needed to prepare a 3P random number list.

36.3 - Exhibit 01

Information Needed to Prepare Random Numbers

Sample Group Number (j)	(1) Species	(2) Scaling Factor	(3) Largest Expected K_j	(4) Est. Vol. ft^3 (ΣKPI_j)	(5) 3P Sample Trees (n_j)	(6) KZ
1	A	10	90*	5000*	24	208*
2	B, C	1	100	45000	15	3000

*These values have been adjusted by the scaling factor of 10. For example, in sample group 1 the largest expected k_j is 900 and the total estimated volume is 50,000 ft^3.

The items listed in exhibit 01 are:

1. <u>Species</u>. List all species within the stratum. Use this information in labeling the random number list.

2. <u>Scaling Factor</u>. When cruising large timber it may be more efficient, without any loss in accuracy, to make predictions in tens of cubic feet. In these cases, use the scaling factor to express the units for the random numbers, KPI's, K, and KZ.

   In this example, K for Species A is 90 with a scaling factor of 10. This means that the random numbers are produced in terms of tens of cubic feet and that no Species A tree larger than 900 cubic feet is expected on the sale. The Species B and C are not scaled and have a K of 100. More precision in the volume estimates is desired in this stratum because the trees are generally small. Random numbers for Species B and C are produced in terms of cubic feet, and the largest expected tree in the stratum is 100 cubic feet.

3. K. The term K is associated with the volume of the largest tree in the stratum; the scaling factor (item 2) is applied to this value. A prediction (KPI) cannot exceed this value. Thus, set K at least as high as the expected volume of the largest tree. When a tree is encountered that is larger than expected (that is, larger than K) do one of two things: 1) 100 percent cruise the tree (Sure-Measured and not a part of the 3P cruise), or 2) record KPI being equal to K. Avoid either alternative by setting K artificially high. Setting K high will not affect the sampling rate.

4. Est. Vol. Make a preliminary estimate of stratum net volume or an advance estimate of stratum sum KPI. In the example, it is shown in cubic feet; the scaling factor (item 2) is applied to this value. Use this information to estimate KZ. If KPI's are to be in terms of some other variable, such as DBH squared, make the advance estimate of stratum sum KPI in terms of DBH squared.

5. 3P Sample Trees. This is the sample size estimated to be needed to satisfy sampling error standards.

6. KZ. KZ is the sampling rate and is equal to Estimated Stratum Volume (Item 4) divided by number of 3P Sample Trees (Item 5); the scaling factor is applied to this value. It is not necessary that item 4, estimated net stratum volume, be very accurate (±30 percent perhaps).

   If a preliminary estimate of stratum volume is low, the actual number of 3P sample trees will tend to exceed the presale estimate of Item (5), number of 3P sample trees. Conversely, if the preliminary estimate of the sum of KPI is high, the actual number of 3P sample trees will tend to be less than predicted. Sample error may also be lower or higher, respectively, than originally planned.

   On the average, one sample tree can be expected for each group of tree estimates (KPI's) totaling KZ, the sampling rate (Item 6). The probability of any tree being selected as a sample unit is equal to KPI/KZ.

36.4 - Calculating Sampling Statistics

36.41 - Sample Expansion

As with any sampling scheme, calculate the tree factor (also called 3P weight), or the number of trees represented by each sample tree. Calculate the tree factor as:

$$F_t = \frac{\sum^{n} KPI}{n_{3p} \times KPI}$$

Where:

F_t = Tree factor for i^{th} tree
KPI = Estimated volume for the i^{th} sample tree
n_{3p} = Number of selected 3P sample trees
n = Number of trees visited

Calculate the volume factor by multiplying the tree factor, or number of trees, by the measured volume, or:

$$F_V = F_t \times MV$$

where:

F_v = Volume factor for i^{th} tree
MV = Measured volume of i^{th} tree
F_t = Tree factor for i^{th} tree

Estimate the total volume by summing the volume factors for each strata, then summing the strata sums. That is:

$$SV_T = \sum^{s}\left(\sum^{n_{3p}} F_v\right)$$

where:

SV_T = Total estimated sale volume
F_v = Volume factor for i^{th} tree in j^{th} stratum
n_{3p} = Number of Selected 3P sample trees in j^{th} stratum
s = Number of strata

When the sample includes only one stratum, ignore the sum over strata, reducing the formula to:

$$SV_T = \sum^{n_{3p}} F_v$$

where:

SV_T = Total estimated sale volume
F_v = Volume factor for i^{th} tree
n_{3p} = Number of selected 3P sample trees

Although the previous formulas are used to expand the sample, it is instructive to rearrange the formula. In this algebraically equivalent formula, the strata (or sale) volume can be calculated by multiplying the sum of the estimated volumes of all trees visited by the average measured to predicted (estimated) ratio (M/P ratio). That is:

$$SV_T = \sum^{s}\left(\left(\sum^{n} KPI\right) \times \bar{R}\right)$$

where:

$\bar{R}$ = Mean M/P ratio of the j^{th} strata

$$= \frac{\sum^{n_{3p}}\left(\frac{MV}{KPI}\right)}{n_{3p}}$$

SV_T = Total estimated sale volume
KPI = Estimated tree volume of i^{th} tree in j^{th} stratum
MV = Measured tree volume of i^{th} tree in j^{th} stratum
n = Number of trees visited in j^{th} stratum
n_{3p} = Number of selected 3P-sample trees in j^{th} stratum
MV/KPI = Ratio of the measured to estimated volume of the i^{th} 3P sample tree in jth stratum
s = Number of strata.

Again, when a sample has only one strata, the summation over strata essentially disappears.

The M/P ratio, the variable of interest, is an adjustment factor to be applied to the predicted sale volume or sum of KPI's. The ratio is sample-based and is, therefore, subject to sampling error.

The tree factor, or 3P weight, can be multiplied by tree characteristics other than measured volume to develop other timber sale information.. Average tree height and quadratic mean diameter are examples. Use the 3P weight to develop various cross tabulations such as distribution of sale volume by tree size, log grade, log size and so on.

The calculations for a one stratum timber sale are shown in exhibit 01.

36.41 - Exhibit 01

3P Volumes

Tree No.	KPI	MV	M/P Ratio (R)	M/P Ratio^2 (R^2)	DBH	HT	Tree Fact (F_t)	Vol Fact (F_v)	Exp DBH (DBHxF_t)	Exp DBH^2 (DBH^{2}xF_t)	Exp HT (HTxF_t)
1	11										
*2	23	24.2	1.05	1.10	11	74	2.22	53.7	24.4	268.6	164.3
3	30										
*4	25	26.9	1.08	1.17	13	73	2.04	54.9	26.5	344.8	148.9
5	4										
6	25										
*7	40	43.8	1.10	1.21	15	88	1.28	56.1	19.2	288.0	112.6
8	22										
9	11										
*10	10	10.1	1.01	1.02	9	61	5.11	51.6	46.0	413.9	311.7
11	28										
12	7										
*13	42	41.4	0.99	0.98	16	67	1.22	50.5	19.5	312.3	81.7
14	22										
*15	34	30.9	0.91	0.83	13	74	1.50	46.4	19.5	253.5	111.0
*16	66	66.3	1.00	1.00	17	92	0.77	51.1	13.1	222.5	70.8
17	19										
18	9										
19	14										
*20	33	31.8	0.96	0.92	13	77	1.55	49.3	20.2	262.0	119.4
21	12										
22	34										
*23	17	16.8	0.99	0.98	11	63	3.01	50.6	33.1	364.2	189.6
*24	32	32.4	1.01	1.02	14	69	1.60	51.8	22.4	313.6	110.4
25	48										
*26	28	27.8	0.99	0.98	12	74	1.83	50.9	22.0	263.5	135.4
27	15										
*28	9	10.7	1.19	1.42	9	63	5.68	60.8	51.1	460.1	357.8
*29	14	14.2	1.01	1.02	11	56	3.65	51.8	40.2	441.7	204.4
30	24										
*31	23	21.6	0.94	0.88	12	70	2.22	48.0	26.6	319.7	155.4
32	40										
*33	18	17.7	0.98	0.96	12	57	2.84	50.3	34.1	409.0	161.9
34	20										
*35	17	17.3	1.02	1.04	10	65	3.01	52.1	30.1	301.0	195.7
*36	25	26.9	1.08	1.17	13	73	2.04	54.9	26.5	344.8	148.9
37	4										
38	25										
*39	40	43.8	1.10	1.21	15	88	1.28	56.1	19.2	288.0	112.6
Total	920		18.41	18.91			42.85	940.9	493.7	5871	2893
Count	39	18									

Note: There are three species (A, B, C); * = 3P measured sample tree. Estimated Total Number of Trees = 43, although there were only 39 trees in actual sample.
Expanded Total Volume SV_T = 940.9 Cubic Feet

Calculate the total volume alternately as:

Est. Total Vol. = Sum KPI x Average Ratio
= 920 x (18.41/18)
= 941 Ft^3

36.42 - Sampling Error

The standard error of a 3P cruise is the standard error of the M/P ratio. Calculate the sampling error of a 3P cruise using the following formula:

$$E = \frac{SE}{\bar{R}} \times 100 \times t$$

Where:

E = Sampling error in percent

$$SE = \sqrt{\frac{\sum^{n_{3P}} R^2 - \frac{\left(\sum^{n_{3P}} R\right)^2}{n_{3P}}}{(n_{3P}-1)\, n_{3P}}}$$

$$\bar{R} = \frac{\sum^{n_{3P}} R}{n_{3P}}$$

R = M/P ratio for the i^{th} 3P sample tree.
n_{3p} = Number of 3P sample trees.
t = 2 (95% confidence interval).

The sampling error calculation in this example is for the one stratum timber sale shown in section 36.41, exhibit 01.

$$SE = \sqrt{\frac{\sum^{n_{3P}} R^2 - \frac{\left(\sum^{n_{3P}} R\right)^2}{n_{3P}}}{(n_{3P}-1)\, n_{3P}}}$$

$$= \sqrt{\frac{18.91 - \frac{(18.41)^2}{18}}{(18-1) \times 18}}$$

$$= \sqrt{\frac{.0807}{307}}$$

$$= 0.0162$$

$$\bar{R} = \frac{18.41}{18} = 1.02$$

$$E = \frac{0.0162}{1.02} \times 100 \times 2 = 3.2$$

36.5 - Additional Population Characteristics

Calculate the additional population characteristics using the tabulated data from section 36.41 exhibit 01.

$$\text{Mean Diameter} = \frac{\sum^{n_{3p}} (DBH \times F_t)}{\sum^{n_{3p}} F_t}$$

$$= \frac{493.7}{42.85} = 11.5 \text{ inches}$$

$$\text{Quadratic Mean Diameter} = \sqrt{\frac{\sum^{n_{3p}} (DBH^2 \times F_t)}{\sum^{n_{3p}} F_t}}$$

$$= \sqrt{\frac{5871}{42.85}} = 11.7 \text{ inches}$$

$$\text{Mean Height} = \frac{\sum^{n_{3p}} (HT \times F_t)}{\sum^{n_{3p}} F_t}$$

$$= \frac{2893}{42.85} = 67.5 \text{ feet}$$

36.6 - Application

3P cruising is probably most efficient in partial cut situations where the coefficient of variation of individual marked tree volumes or dollar value is large. 3P cruising fits well with many road right-of-way cruising jobs.

37 - Two-Stage Sampling

There are several methods of two-stage sampling. Common variants are discussed in this section.

37.1 - Sample-Tree with 3P Subsamplinq Method

Sample tree with 3P is a two-stage sampling method. The first stage is an equal probability sample where the probability of a tree being selected as a sampling unit is proportional to the tree frequency in the population. Trees are selected randomly or systematically and KPI is estimated for each first stage sample tree.

The second stage is a 3P sample of the trees selected with equal probability in the first stage. Select 3P sample trees by comparing the volume prediction (KPI) for a sample tree with a random number. If the KPI is equal to or greater than the random number, measure as a 3P sample tree.

37.12 - Operational Features

The operational features of sample-tree cruising are described in section 33 and the operational features for 3P sampling are described in section 36. The 3P procedure for sample-tree 3P is the same, except that KPI is estimated and recorded only for the sample trees selected in the first stage.

37.13 - Statistical Features

Sample-tree 3P sampling includes equal probability sampling at the first stage and variable probability (3P) sampling at the second stage. The variable of interest at the first stage is generally volume per tree, while the variable of interest at the second stage is the measured to predicted (M/P) ratio.

There are two sources of statistical error in a sample-tree 3P sample: the estimate of the stratum sum KPI and the estimate of the stratum mean M/P ratio. The population parameter for which sampling error is estimated in the first stage is mean tree KPI. In the second or 3P stage, sampling error is estimated for the mean M/P ratio which is ordinarily measured tree volume divided by estimated tree volume in sample-tree 3P cruising.

37.14 - Calculating Sample Size

Calculate combined sampling error in order to determine sample size for sample-tree 3P cruising. Determine sample size (number of trees) for the sample-tree sample and for the 3P sample to satisfy the desired sampling error for the stratum.

The combined sampling error includes the sample tree (first stage) error and the 3P (second stage) error.

If both the sample tree and 3P portions of the combined error are known, the formula for calculating the sampling error of a sample-tree 3P cruise is:

$$E_T = \sqrt{E_{st}^2 + E_{3p}^2}$$

Where:

E_T = Combined percent sampling error (Total error)
E_{st} = Percent sampling error for the sample-tree sample.
E_{3p} = Percent sampling error for the 3P sample.

More commonly, the cruise is being designed to meet a specific combined error. In this case, the desired combined error is known. One of the errors (sample-Tree or 3P) needs to be estimated, with the remaining error being determined using the following example where sample tree error is being estimated.

$$E_{st} = \sqrt{E_T^2 - E_{3p}^2}$$

Generally, 3P error is the more easily estimated of the two errors and can be determined through the use of past local cruises and experience.

Given:

Target combined sampling error of 20% (95% Confidence level)
Estimated 3P sampling error of 12% (95% Confidence level)

Determine sample tree sampling error:

$$E_{st} = \sqrt{E_T^2 - E_{3p}^2}$$

$$= \sqrt{20^2 - 12^2}$$

$$= \sqrt{256}$$

$$= 16.0\%$$

Note: Desired combined sampling error must be higher than the estimated 3P error.

To determine the number of samples needed to attain the errors identified for each stage of the sample tree with 3P cruise, estimate the coefficient of variation between sample trees in the first stage, and between predicted and actual measurements in the second stage. This estimate can be based on preliminary sample data, past cruises in the area in similar types, or inventory data.

First Stage

Assumed Coefficient of Variation = 80%
t = 2
Sampling error = 16%

$$n_{st} = \frac{(t^2)\ (CV_{st})^2}{E_{st}^2}$$

$$= \frac{(2^2)\ (80^2)}{16^2}$$

$$= 100 \text{ trees}$$

Second Stage

Assumed Coefficient of Variation = 25%
t = 2
Sampling error = 12%

$$n_{3p} = \frac{(t^2)\ (CV_{3p})^2}{E_{3p}^2}$$

$$= \frac{(2^2)\ (25^2)}{12^2}$$

$$= 18 \text{ trees}$$

37.15 - Producing Random Number List

Random numbers for sample-tree with 3P cruises are ordinarily in terms of per tree volume.

Use the following information to prepare a random number list:

1. Estimate of the sum of the KPI's (estimated tree volumes) for all the first stage sample trees (trees for which a KPI will be made).
2. Estimate of the largest tree volume in the population (K).
3. Estimate of the total number of first stage sample trees.
4. Number of 3P sample trees desired.
5. Sampling rate (KZ) (see below).
6. Number of first stage sample trees n_{st} = 100
7. Number of 3P sample trees n_{3p} = 18

In addition, estimates of largest volume (K = 80 ft^3 in example) and average KPI (22 ft^3 in the example) are used. This information can come from precruise data, inventory data, prior cruises, and personal knowledge of the area. For example:

Estimated Sum KPI = 22 x 100 = 2,200 Ft^3.

$$\text{Sampling rate (KZ)} = \frac{\text{Sum KPI}}{\text{No. of 3P sample trees}}$$

$$= \frac{2{,}200}{18}$$

= 122.2 (rounded to next whole number)

= 123

For this example, the set of random numbers would have the following characteristics:

K = 80 ft^3.

KZ = 123 or one 3P sample tree per 123 Ft^3.

37.16 - Calculating Sampling Statistics

37.16a - Sample Expansion

The expansion of the sample-tree 3P sample trees to the stratum level involves calculating the combined sample tree weight or frequency considering both the sample-tree sample and the 3P sample.

Compute the tree factor, or estimated frequency (F), for a sample-tree 3P sample from the following formula:

$$F_t = F_{t(st)} \times F_{t(3p)}$$

Where:

F_t = Tree factor; frequency or number of trees at the j^{th} stratum level for the i^{th} sample-tree 3P sample tree.

$$F_{t(st)} = \frac{\text{Number of trees tallied}}{\text{Number of first stage sample trees}}$$

$$F_{t(3p)} = \text{3P frequency} = \frac{\sum^{n} KPI}{n_{3p} \times KPI}$$

$\sum^{n} KPI$ = Sum of estimated volumes of all first-stage sample trees.
n_{3p} = Number of 3P sampled trees
KPI = Estimated volume (KPI) of the i^{th} 3P sample tree
n = Number of first-stage sample trees.

The estimated expanded volume at the stratum level of an individual sample-tree 3P sample tree is computed from the following formula:

$$F_v = F_t \times MV$$

Where: F_v = Estimated stratum volume for the i^{th} 3P sample tree in the j^{th} stratum.
MV = Measured volume of the i^{th} 3P sample tree in the j^{th} stratum.
F_t = Tree factor for i^{th} tree

Calculate estimated total stratum volume by summing all sample-tree 3P sample tree volumes.

Calculations for a one-stratum example are shown in exhibit 01,
given:
428 trees tallied.
39 trees in sample-tree sample
10 acres in stratum.

The sample tree expansion factor for all trees is:

$$F_{st} = \frac{428}{39} = 10.97$$

The 3P expansion for each 3P tree is:

$$F_{t(3p)} = \frac{\sum^{n} KPI}{n_{3p} \times KPI}$$

The resulting tree factor is:

$$F_t = F_{t(st)} \times F_{t(3p)}$$

37.16a - Exhibit 01

Calculation of Volume, Sample Tree with 3P Subsample Exp

Tree No.	KPI	KPI^2	MV	M/P Ratio (R)	M/P $Ratio^2$ (R^2)	DBH	HT	3P Wt ($F_{t(3p)}$)	Tree Fact (F_t)	Exp Vol ($MVxF_t$)	Exp DBH ($DBHxF_t$)	Exp DBH^2 (DBH^2xF_t)	Exp HT ($HTxF_t$)
1	11	121											
*2	23	529	24.2	1.05	1.10	11	74	2.22	24.35	589.4	267.9	2946.8	1802.2
3	30	900											
*4	25	625	26.9	1.08	1.17	13	73	2.04	22.38	602.0	290.9	3782.0	1633.7
5	4	16											
6	25	625											
*7	40	1600	43.8	1.10	1.21	15	88	1.28	14.04	615.0	210.6	3159.4	1235.7
8	22	484											
9	11	121											
*10	10	100	10.1	1.01	1.02	9	61	5.11	56.06	566.2	504.5	4540.6	3419.5
11	28	784											
12	7	49											
*13	42	1764	41.4	0.99	0.98	16	67	1.22	13.38	554.1	214.1	3426.2	896.7
14	22	484											
*15	34	1156	30.9	0.91	0.83	13	74	1.50	16.46	508.5	213.9	2780.9	1217.7
*16	66	4356	66.3	1.00	1.00	17	92	0.77	8.45	560.0	143.6	2441.2	777.1
17	19	361											
18	9	81											
19	14	196											
*20	33	1089	31.8	0.96	0.92	13	77	1.55	17.00	540.7	221.0	2873.6	1309.3
21	12	144											
22	34	1156											
*23	17	289	16.8	0.99	0.98	11	63	3.01	33.02	554.7	363.2	3995.4	2080.2
*24	32	1024	32.4	1.01	1.02	14	69	1.60	17.55	568.7	245.7	3440.2	1211.1
25	48	2304											
*26	28	784	27.8	0.99	0.98	12	74	1.83	20.08	558.1	240.9	2890.8	1485.6
27	15	225											
*28	9	81	10.7	1.19	1.42	9	63	5.68	62.31	666.7	560.8	5047.1	3925.5
*29	14	196	14.2	1.01	1.02	11	56	3.65	40.04	568.6	440.4	4844.9	2242.3
30	24	576											
*31	23	529	21.6	0.94	0.88	12	70	2.22	24.35	526.0	292.2	3506.9	1704.7
32	40	1600											
*33	18	324	17.7	0.98	0.96	12	57	2.84	31.15	551.4	373.9	4486.3	1775.8
34	20	400											
*35	17	289	17.3	1.02	1.04	10	65	3.01	33.02	571.2	330.2	3302.0	2146.3
*36	25	625	26.9	1.08	1.17	13	73	2.04	22.38	602.0	290.9	3782.0	1633.7
37	4	16											
38	25	625											
*39	40	1600	43.8	1.10	1.21	15	88	1.28	14.04	615.0	210.6	3159.4	1235.7
Sum	920	28228		18.41	18.91			42.85	470.06	10318.3	5415.3	64406	31733
Count	39		18										

* = 3P sample tree
Estimated Number of Trees = 470
Estimated Total Volume = 10,318.3 ft^3
Estimated Volume per Acre = 10,318.3/10 = 1031.8 ft^3/ac

37.16b - Sampling Error

Calculate sampling error of a sample-tree 3P cruise from the following formula:

$$E_T = \sqrt{\left(\frac{SE_{st}}{\overline{KPI}} \times 100 \times t\right)^2 + \left(\frac{SE_{3p}}{\overline{R}} \times 100 \times t\right)^2}$$

Where: E_T = Stratum sampling error.

$SE_{st} = \sqrt{\dfrac{\sum^{n_{st}} KPI^2 - \dfrac{\left(\sum^{n_{st}} KPI\right)^2}{n_{st}}}{(n_{st} - 1) \times n_{st}}}$

KPI = Estimated volumes (KPI) of all first stage sample trees.
n_{st} = Number of first stage sample trees.
$\overline{KPI}$ = Mean KPI of first stage sample trees.

$SE_{3p} = \sqrt{\dfrac{\sum^{n_{3p}} R^2 - \dfrac{\left(\sum^{n_{3p}} R\right)^2}{n_{3p}}}{(n_{3p} - 1) \times n_{3p}}}$

R = M/P ratios for i^{th} 3P sample tree.
n_{3p} = Number of sample tree-3P sample trees.
$\overline{R}$ = Mean M/P ratio.
t = 2 (95 percent confidence)

The calculations use the values from section 37.16a, exhibit 01. The mean M/P ratio is 18.41/18 = 1.02. The standard error components are:

$$SE_{st} = \sqrt{\frac{\sum^{n_{st}} KPI^2 - \frac{\left(\sum^{n_{st}} KPI\right)^2}{n_{st}}}{(n_{st} - 1)(n_{st})}}$$

$$= \sqrt{\frac{28,228 - \frac{920^2}{39}}{(39-1)(39)}}$$

$$= 2.098$$

$$SE_{3p} = \sqrt{\frac{\sum^{n_{3p}} R^2 - \frac{\left(\sum^{n_{3p}} R\right)^2}{n_{3p}}}{(n_{3p} - 1)(n_{3p})}}$$

$$= \sqrt{\frac{18.91 - \frac{(18.41)^2}{18}}{(18-1)(18)}}$$

$$= 0.0162$$

The sampling errors are:

$$E_{st} = \frac{SE_{st}}{KPI} \times 100 \times t$$

$$= \frac{2.098}{23.59} \times 100 \times 2$$

$$= 17.8\%$$

$$E_{sp} = \frac{SE_{3P}}{R} \times 100 \times t$$

$$= \frac{0.0162}{1.023} \times 100 \times t$$

$$= 3.2\%$$

The total error is:

$$E_T = \sqrt{E_{st}^2 + E_{3p}^2}$$

$$= \sqrt{17.8^2 + 3.2^2}$$

$$= \sqrt{327.1} = 18.1\% \text{ (95\% } \textit{confidence level}\text{)}$$

37.17 - Additional Population Characteristics

Calculate the additional population characteristics using the tabulated data from section 37.16a, exhibit 01.

$$Mean\ Diameter = \frac{\sum^{n_{3p}} (DBH \times F_t)}{\sum^{n_{3p}} F_t}$$

$$= \frac{5415.3}{470.06}$$

$$= 11.5\ inches$$

$$Quadratic\ Mean\ Diameter = \sqrt{\frac{\sum^{n_{3p}} (DBH^2 \times F_t)}{\sum^{n_{3p}} F_t}}$$

$$= \sqrt{\frac{64406}{470.06}}$$

$$= 11.7\ inches$$

$$Mean\ Height = \frac{\sum^{n_{3p}} (HT \times F_t)}{\sum^{n_{3p}} F_t}$$

$$= \frac{31733}{470.06}$$

$$= 67.5\ feet$$

37.18 - Application

Sample tree with 3P subsampling is most efficient in partial cut situations where the coefficient of variation of individual marked tree volumes or values is large. This form of 3P cruising fits well with many road right-of-way cruising jobs.

37.2 - Fixed Area Plot With 3P Subsampling

37.21 - Fixed Area Plot With 3P Subsamplinq Method

Plot-3P is a two-stage sampling system. The first stage is a fixed area sample where the probability of a tree being selected as a sample unit is proportional to the tree frequency in the population. The second stage is a 3P sample of the sample trees on the fixed area plots.

In the second or 3P stage, estimate and record volume (KPI) of each sample tree on the fixed area plot. Select 3P sample trees by comparing the volume estimate (KPI) for a sample tree with a random number. If the KPI is equal to or greater than the random number, measure the tree as a sample tree.

37.22 - Operational Features

The operational features of the first stage fixed area plot cruising are described in section 34. The purpose of the plot sample in plot-3P cruising is to estimate the stratum sum of KPI's. KPI is in terms of tree volume.

Operational features for the second stage 3P sampling are described in section 36. The 3P procedure for plot-3P cruising is the same except that KPI is estimated and recorded only for the trees on the plot.

37.23 - Statistical Features

Plot-3P sampling includes equal probability sampling at the first stage and variable probability (3P) sampling at the second stage. The variable of interest at the first stage is generally volume per acre while the variable of interest at the second stage is the measured to predicted (M/P) ratio.

There are two sources of statistical error in plot-3P sampling: the estimate of the stratum sum KPI (estimated volume) and the estimate of the stratum mean M/P ratio. In the first stage, the population parameter for which sampling error is estimated is mean sum estimated volume (KPI) for each plot.

In the second or 3P stage, sampling error is estimated for the mean M/P ratio which is measured tree volume divided by estimated tree volume in plot-3P cruising. The sampling error for each stratum includes both sources of error.

37.24 - Calculating Sample Size

Combined sampling error must be calculated in order to determine sample size for fixed plot-3P cruising. Sample size is determined for both the sample fixed plot (number of plots) and for the 3P sample (number of trees) to satisfy the desired sampling error for the stratum.

The combined sampling error includes the fixed plot (first stage) error and the 3P (second stage) error.

If both the fixed plot and 3P portions of the combined error are known, use the following formula for calculating the sampling error of a fixed plot-3P cruise:

$$E_T = \sqrt{E_{fp}^2 + E_{3p}^2}$$

where:

E_T = Combined sampling in percent (Total error)
E_{fp} = Sampling error (in percent) of the fixed plot sample.
E_{3p}= Sampling error (in percent) of the 3P sample.

More commonly, the cruise is designed to meet a specific combined error. In this case the desired combined error is known. Estimate one of the errors (fixed plot or 3P) and determine the remaining error using the following, example where the fixed plot error is determined:

$$E_{fp} = \sqrt{E_T^2 - E_{3p}^2}$$

Note that the desired combined sampling error must be higher than either the fixed plot or 3P sampling error. 3P error is generally the more easily estimated of the two errors and can be determined through past local cruises and experience. Given the target combined sampling error of 20 percent at 95 percent confidence level, and a 3P estimated sampling error of 12 percent at a 95 percent confidence level, the fixed plot error is:

$$E_{fp} = \sqrt{E_T^2 - E_{3p}^2}$$

$$= \sqrt{20^2 - 12^2}$$

$$= \sqrt{256}$$

$$= 16.0\%$$

To determine the number of samples needed to attain the errors identified for each stage of the fixed plot-3P cruise, estimate the coefficient of variation between sample plots in the first stage, and between predicted and actual measurements in the second stage. This estimate may be based on reconnaissance data, past cruises in the area in similar types, or inventory data.

First Stage (Plots)
Assumed Coefficient
of Variation = 50%
Confidence Level = 95% (t=2)

$$n_{fp} = \frac{(CV_{fp}^2)\ (t^2)}{E_{fp}^2}$$

$$= \frac{(50^2)\ (2^2)}{16^2}$$

$$= 39\ points$$

Second Stage (Trees)
Assumed Coefficient
of Variation = 25%
Confidence Level = 95% (t=2)

$$n_{3p} = \frac{(CV_{3p}^2)\ (t^2)}{E_{3p}^2}$$

$$= \frac{(25^2)\ (2^2)}{12^2}$$

$$= 18\ trees$$

37.25 - Producing Random Number List

Random numbers for plot-3P cruises are in terms per tree of volume.

Use the following example to prepare a random number list:

1. Estimate of the sum of KPI's (estimated tree volumes) on all the sample plots.
2. Estimate of the largest tree volume in the population (K).
3. Estimate of the total number sample trees on all the plots.
4. Number of 3P sample trees desired.
5. Sampling rate (KZ).
6. Number of plots = 39
7. Number of 3P sample trees = 18

In addition, it is estimated:

Largest volume (K) = 80 ft^3.
Average KPI = 23.1 ft^3.
Average number of sample trees per plot = 4.3

Therefore:

Estimated Sum KPI = 39 plots x 4.3 trees per plot x 23.1 ft^3 per tree.
(Sum of estimated tree volumes on 39 plots) = 3,874 ft^3

$$\text{Sampling rate (KZ)} = \frac{\text{Estimated Sum KPI}}{\text{number of 3P sample trees}} = \frac{3{,}874}{18} = 215$$

Estimated total number of trees on all plots = 39 plots x 4.3 trees/plot
= 168

K and KZ are based on a combination of pre-cruise information, inventory data, and personal knowledge of the area. For this example, the set of random numbers would have the following characteristics:

K = 80 ft^3
KZ = 215 or one 3P sample tree per 215 ft^3

37.26 - Calculating Sampling Statistics

37.26a - Sample Expansion

The expansion of the plot-3P sample trees to the per acre and stratum level involves calculating the sample tree weight or frequency considering both the plot sample and the 3P sample.

For demonstration purposes in this section, 10 fixed plots with 18 3P sample trees are being expanded to derive an estimated volume of the area.

The tree factor, or estimated frequency per acre for an individual plot-3P sample tree is derived as the product of two component expansion factors. These components are identical to the tree factors developed previously for simple fixed area and simple 3P sampling. It is computed from the following formula:

$$F_t = F_{t(fp)} \times F_{t(3p)}$$

where: F_t = Tree factor for the i^{th} 3P sample tree (trees per acre)
$F_{t(fp)}$ = Tree factor for i^{th} tree to expand fixed plot component

$$= \frac{1}{Sz \times p}$$

$F_{t(3p)}$ = Tree factor for i^{th} tree to expand 32 sampling component

$$= \frac{\sum^{n} KPI}{n_{3p}\ KPI}$$

Sz = Plot size of fixed area plot
p = Number of fixed area plots
KPI = Estimated volume of i^{th} tree visited
n = Number of trees visited on all fixed plots
n_{3p} = Number of selected 3P sample trees

The volume factor, or estimated volume per acre for an individual plot-3P sample tree is computed from the following formula:

$$F_v = F_t \times MV$$

where:

F_v = volume factor, or estimated volume for i^{th} 3P sample (Vol. per acre)
MV = measured volume of the i^{th} 3P sample tree

Compute the estimated total tract stratum volume contribution by an individual 3P sample tree as the product of the volume per acre factor and the total acres:

$$SV = F_v \times Acres$$

where:

SV = Estimated sale volume for i^{th} 3P sample tree
Acres = Acres in the Tract

Calculate estimated total stratum volume per acre and total stratum volume by summing all plot-sample tree values. Exhibit 01 shows a two-stratum example given:

1. 10 one fifth acre plots, therefore $F_{t(fp)}$ = (1/(.2 x 10)) = 0.5

2. 18 acres in stratum

3. Volume measured in ft^3

4. Species group 1 = A

5. Species group 2 = B, C

37.26a - Exhibit 01

Calculations for a Two-Stratum Sale.

Plot No.	Tree No.	Sp	KPI	DBH	HT	MV	3P Wt ($F_{t(3p)}$)	Tree Fact (F_t)	Vol Fact (F_v)	Tract Vol (SV)
1	1	A	11							
*1	2	A	23	11	74	24.2	2.73	1.37	33.2	597.6
1	3	A	30							
*1	4	B	25	13	73	26.9	1.67	0.84	22.6	406.8
1	5	B	4							
1	6	B	25							
*1	7	B	40	15	88	43.8	1.05	0.53	23.2	417.6
2	1	A	22							
2	2	B	11							
*2	3	B	10	9	61	10.1	4.18	2.09	21.1	379.8
2	4	B	28							
2	5	B	7							
*2	6	A	42	16	67	41.4	1.49	0.75	31.1	559.8
3										
4	1	A	22							
*4	2	A	34	13	74	30.9	1.85	0.93	28.7	516.6
*4	3	A	66	17	92	66.3	0.95	0.48	31.8	572.4
4	4	B	19							
4	5	B	9							
4	6	C	14							
*5	1	A	33	13	77	31.8	1.90	0.95	30.2	543.6
5	2	C	12							
5	3	A	34							
*5	4	C	17	11	63	16.8	2.46	1.23	20.7	372.6
*6	5	A	32	14	69	32.4	1.96	0.98	31.8	572.4
6	6	A	48							
7										
*8	1	A	28	12	74	27.8	2.24	1.12	31.1	559.8
8	2	B	15							
*8	3	B	9	9	63	10.7	4.64	2.32	24.8	446.4
*8	4	B	14	11	56	14.2	2.99	1.50	21.3	383.4
8	5	B	24							
*8	6	B	23	12	70	21.6	1.82	0.91	19.7	354.6
9	1	A	40							
*9	2	C	18	12	57	17.7	2.32	1.16	20.5	369.0
9	3	A	20							
*9	4	A	17	10	65	17.3	3.69	1.85	32.0	576.0
*10	1	B	25	13	73	26.9	1.67	0.84	22.6	406.8
10	2	B	4							
10	3	B	25							
*10	4	B	40	15	88	43.8	1.05	0.53	23.2	417.6
Total			920				40.66	20.38	469.6	8452.8
Count			39			18				

* = 3P Sample Tree.

Item	Group 1	Group 2	All
KPI	502	418	920
n_{3p}	8	10	18
F_t	8.4	12.0	20.4
F_v	249.9	219.7	469.6
SV	4498	3955	8453

Calculate the stratum values:

Estimated Number Trees/Acre = $\left(\sum^{n_{3p}} F_t\right)$ = 20.38

Estimated Total Number of Trees = 366

Estimated Volume/Acre = $\left(\sum^{n_{3p}} F_v\right)$ = 469.6 ft^3

Estimated Total Volume = $\left(\sum^{n_{3p}} SV\right)$ = 8,453.0 ft^3

37.26b - Sampling Error

Use the following formulas to compute sampling error. Two sampling error components must be accounted for; the sampling error of the fixed plot and the sampling error of the 3P sub-sample. The formula for calculating the sampling error over all strata is:

$$E_T = \sqrt{\left(\frac{SE_{fp}}{\overline{PV}} \times 100 \times t\right)^2 + \left(\frac{SE_{3p}}{\overline{R}} \times 100 \times t\right)^2}$$

Where:

E_T = The stratum sampling error

SE_{fp} = Standard error of estimated plot volumes on fixed plots

$$= \sqrt{\frac{\sum^{p} PV^2 - \frac{\left(\sum^{p} PV\right)^2}{p}}{(p-1)\ (p)}}$$

PV = Sum of estimated volumes for j^{th} fixed plot

p = The number of fixed plots

PV = Mean estimated volume for all p plots

SE_{3p} = Standard error of 3P sample trees

$$= \sqrt{\frac{\sum^{n_{3p}} R^2 - \frac{\left(\sum^{n_{3p}} R\right)^2}{n_{3p}}}{(n_{3p}-1)\ (n_{3p})}}$$

R = M/P (measured/predicted) ratio for i^{th} 3P sample tree

n_{3p} = Number of 3P sample trees

R = Mean M/P ratio

t = 2 (95 percent confidence)

Sampling error tabulations in exhibit 01 use the data from section 37.26a, exhibit 01, and consider all the sampling was from one stratum.

37.26b - Exhibit 01

Sampling Error

Plot No.	Tree No.	KPI	Plot KPI	Plot KPI^2	Meas Vol (MV)	M/P Ratio (R)	M/P $Ratio^2$ (R^2)
1	1	11					
1	2	23			24.2	1.052	1.107
1	3	30					
1	4	25			26.9	1.076	1.158
1	5	4					
1	6	25					
1	7	40	158	24964	43.8	1.095	1.199
2	1	22					
2	2	11					
2	3	10			10.1	1.010	1.020
2	4	28					
2	5	7					
2	6	42	120	14400	41.4	0.986	0.972
3			0	0			
4	1	22					
4	2	34			30.9	0.909	0.826
4	3	66			66.3	1.005	1.010
4	4	19					
4	5	9					
4	6	14	164	26896			
5	1	33			31.8	0.964	0.929
5	2	12					
5	3	34					
5	4	17	96	9216	16.8	0.988	0.976
6	5	32			32.4	1.013	1.026
6	6	48	80	6400			
7			0	0			
8	1	28			27.8	0.993	0.986
8	2	15					
8	3	9			10.7	1.189	1.414
8	4	14			14.2	1.014	1.028
8	5	24					
8	6	23	113	12769	21.6	0.939	0.882
9	1	40					
9	2	18			17.7	0.983	0.966
9	3	20					
9	4	17	95	9025	17.3	1.018	1.036
10	1	25			26.9	1.076	1.158
10	2	4					
10	3	25					
10	4	40	94	8836	43.8	1.095	1.199
	Sum	920	920	112506		18.405	18.892
	Count	39	10		18		

The sampling error calculations using the above formulas, and the tabulations in exhibit 01 are:

$$SE_{fp} = \sqrt{\frac{112506 - \frac{(920)^2}{10}}{(10-1)\ (10)}}$$

$$= \sqrt{\frac{27,866}{90}}$$

$$= \sqrt{309.62}$$

$$= 17.596$$

$$SE_{3p} = \sqrt{\frac{18.892 - \frac{(18.405)^2}{18}}{(18\ -\ 1)\ (18)}}$$

$$= \sqrt{\frac{0.0729}{306}}$$

$$= \sqrt{0.00024}$$

$$= 0.0154$$

Therefore, the sampling error for the total is:

$$E_T = \sqrt{\left(\frac{17.596}{920/10} \times 100 \times 2\right)^2 + \left(\frac{0.0154}{18.405/18} \times 100 \times 2\right)^2}$$

$$= \sqrt{(38.25)^2 + (3.01)^2}$$

$$= 38.4\%\ (95\%\ \textit{confidence level})$$

37.27 - Additional Population Characteristics

As with other sampling schemes, additional population characteristics can be estimated. Exhibit 01 shows the tabulations used to calculate the estimated mean diameter and height.

37.27 - Exhibit 01

Data for Calculating Additional Population Characteristics

Plot No.	Tree No.	SP	KPI	DBH	DBH2	HT	Meas Vol (MV)	3P Wt ($F_{t(3p)}$)	Tree Fact F_t	Exp DBH ($DBHxF_t$)	Exp DBH^2 (DBH^2xF_t)	Exp HT ($HTxF_t$)
1	1	A	11									
1	2	A	23	11	121	74	24.2	2.73	1.37	15.1	165.8	101.4
1	3	A	30									
1	4	B	25	13	169	73	26.9	1.67	0.84	10.9	142.0	61.3
1	5	B	4									
1	6	B	25									
1	7	B	40	15	225	88	43.8	1.05	0.53	8.0	119.3	46.6
2	1	A	22									
2	2	B	11									
2	3	B	10	9	81	61	10.1	4.18	2.09	18.8	169.3	127.5
2	4	B	28									
2	5	B	7									
2	6	A	42	16	256	67	41.4	1.49	0.75	12.0	192.0	50.3
3												
4	1	A	22									
4	2	A	34	13	169	74	30.9	1.85	0.93	12.1	157.2	68.8
4	3	A	66	17	289	92	66.3	0.95	0.48	8.2	138.7	44.2
4	4	B	19									
4	5	B	9									
4	6	C	14									
5	1	A	33	13	169	77	31.8	1.90	0.95	12.4	160.6	73.2
5	2	C	12									
5	3	A	34									
5	4	C	17	11	121	63	16.8	2.46	1.23	13.5	148.8	77.5
6	5	A	32	14	196	69	32.4	1.96	0.98	13.7	192.1	67.6
6	6	A	48									
7												
8	1	A	28	12	144	74	27.8	2.24	1.12	13.4	161.3	82.9
8	2	B	15									
8	3	B	9	9	81	63	10.7	4.64	2.32	20.9	187.9	146.2
8	4	B	14	11	121	56	14.2	2.99	1.50	16.5	181.5	84.0
8	5	B	24									
8	6	B	23	12	144	70	21.6	1.82	0.91	10.9	131.0	63.7
9	1	A	40									
9	2	C	18	12	144	57	17.7	2.32	1.16	13.9	167.0	66.1
9	3	A	20									
9	4	A	17	10	100	65	17.3	3.69	1.85	18.5	185.0	120.3
10	1	B	25	13	169	73	26.9	1.67	0.84	10.9	142.0	61.3
10	2	B	4									
10	3	B	25									
10	4	B	40	15	225	88	43.8	1.05	0.53	8.0	119.3	46.6
	Sum		920						20.38	237.7	2860.8	1389.5
	Count		39				18					

Calculate the additional population characteristics using the tabulated data in exhibit 01:

$$\text{Arithemetic Mean Diameter} = \frac{\sum^{n_{3p}} (DBH \times F_t)}{\sum^{n_{3p}} F_t}$$

$$= \frac{237.7}{20.38}$$

$$= 11.7\ in.$$

$$\text{Quadradic Mean Diameter} = \sqrt{\frac{\sum^{n_{3p}} (DHB^2 \times F_t)}{\sum^{n_{3p}} F_t}}$$

$$= \sqrt{\frac{2860.8}{20.38}}$$

$$= 11.8\ in.$$

$$\text{Mean Height} = \frac{\sum^{n_{3p}} (HT \times F_t)}{\sum^{n_{3p}} F_t}$$

$$= \frac{1389.5}{20.38}$$

$$= 68.2\ ft.$$

37.28 - Application

The fixed plot-3P cruise system is applicable in uniformly distributed timber, especially where clearcutting is the prescription and large areas are involved.

37.3 - Point Sampling with 3P Subsampling

Point-3P is a two-stage sampling system. The first stage is a point sample where sample trees are selected in proportion to tree basal area. The second stage is a 3P sample of the point-sampled trees.

In the second or 3P stage, estimate and record height (KPI) of each point-sampled tree (in this example, height is estimated, rather than volume) by species. Select 3P sample trees by comparing the height prediction for a tree with a random number. If the height prediction is equal to or greater than the random number, measure the tree as a 3P sample tree.

37.31 - Operational Features

The operational features and field procedures of point sampling are described in section 35. The purpose of the point sample in point-3P cruising is to estimate the stratum sum of KPIs. KPI is in terms of units of tree height. Tree height can be estimated in terms of lineal feet of total height or merchantable height or in terms of logs or half logs to some top diameter. However, the height unit must be consistent within a stratum.

Operational features and field procedures for 3P sampling are described in section 36. The 3P procedure for point-3P cruising is the same, except that KPI is estimated and recorded for only the point sampled trees.

37.32 - Statistical Features

Point-3P sampling includes two forms of variable probability sampling; PPS (probability proportional to size) at the first stage, and 3P (probability proportional to prediction) at the second stage. The variable of interest at the first stage is generally volume per acre while the variable of interest at the second stage is the measured to predicted (M/P) ratio.

There are two sources of statistical error in point 3P sampling: the estimate of the sum of stratum KPI and the estimate of the M/P ratio. In the first stage, sampling error is estimated for mean sum units of tree height (KPI) for each point. In the second, or 3P stage, sampling error is estimated for the mean M/P ratio, which is measured tree volume divided by D^2H in point-3P cruising. The combined sampling error for the strata includes both sources of error.

37.33 - Calculating Sample Size

The formula for calculating the sampling error of a point-3P cruise is:

$$E_T = \sqrt{E_P^2 + E_{3P}^2}$$

Where:

E_T =Combined sampling error in percent.
E_p = Sampling error (in percent) of the point sample.
E_{3p} = Sampling error (in percent) of the 3P sample.

Determine sample size for the point sample (number of points) and for the 3P sample (number of trees) to satisfy the target combined sampling error specified for the stratum.

Given the desired combined sampling error, an estimate must also be made for either the point sampling error or the 3P sampling error in order to determine the remaining unknown sampling error. For example, given a desired combined sampling error of 15 percent and an estimated point sampling error of 12 percent, both at the 95 percent level of confidence, the 3P sampling error can be determined as follows:

$$E_{3p} = \sqrt{E_T^2 - E_P^2}$$
$$= \sqrt{15^2 - 12^2}$$
$$= \sqrt{81}$$
$$= 9\%$$

The number of sample units for each stage of the point-3P cruise is determined as follows:

First Stage (Points)

Assumed Coefficient of Variation = 14%
Confidence Level = 95% (t=2)
Sampling error = 12%

$$n = \frac{(CV_P^2)\ (t^2)}{E_P^2}$$
$$= \frac{(14^2)\ (2^2)}{12^2}$$

= 5.4 = 6 points

Second Stage (Trees)

Assumed Coefficient of Variation = 19%
Confidence Level = 95% (t=2)
Sampling error = 9%

$$n = \frac{(CV_{3P}^2)\ (t^2)}{E_{3P}^2}$$
$$= \frac{(19^2)\ (2^2)}{9^2}$$

= 17.8 = 18 trees

37.34 - Random Number List

Random numbers for point-3P cruises are in terms of units of tree height. For the example, lineal feet to a 6-inch top, outside bark is assumed. Other possibilities include lineal feet of total height, or number of logs or half logs to some top diameter.

The example information needed to prepare a random number list is:

1. Estimate of the sum of estimated tree heights on the sample points (sum KPI).

2. Estimate of the tallest tree in the population (K).

3. Estimate of the number of point sampled trees.

4. Number of 3P sample trees desired.

5. Sampling rate (KZ).

6. Number of points = 6

7. Number of 3P sample trees = 18

In addition, the estimates include:

Tallest tree (K) = 100 feet
Average tree height = 70.0 feet
Average number of sample trees per point = 3.9

Therefore:

Estimate of sum KPI = 6 points x 3.9 trees per point x 70.0 feet per tree = 1,638 linear feet

Sampling Rate (KZ) $= \frac{1{,}638}{18 \text{ 3P samples}} = 91$

Estimated Number of point sampled trees = 6 x 3.9 = 24

For this example, the set of random numbers would have the following characteristics:

K = 100 feet (to a 6-inch top, DOB).
KZ = 91 or one 3P sample tree per 91 lineal feet of estimated tree height.

37.35 - Calculating Sampling Statistics

37.35a - Sample Expansion

The expansion of point-3P sample trees to the per acre and stratum level involves calculating the sample tree weight or frequency considering both the point sample and the 3P sample.

Compute the estimated frequency per acre for an individual point-3P sample tree from the following formula:

$$F_t = F_{t(p)} \times F_{t(3p)}$$

Where:

F_t = Tree factor or frequency or number of trees per acre for the i^{th} point-3P sample tree.

$F_{t(p)}$ = Point sample frequency of i^{th} tree

$$= \frac{Basal\ Area\ Factor}{0.005454 \times DBH^2 \times p}$$

DBH = diameter of the i^{th} point-3P sample tree

$F_{t(3p)}$ = 3P Frequency = $\frac{\sum^{n} KPI}{n_{3p} \times KPI}$

KPI in numerator = Height estimate of i^{th} point-sampled tree.

n = Number of point-sampled trees.

n_{3p} = Number of point-3P sampled trees.

KPI in denominator = KPI of the i^{th} point-3P sampled tree.

p = Number of points

Compute the estimated volume per acre for an individual point 3P sample tree from the following formula:

$$F_v = F_t \times MV$$

Where:

F_v = Estimated volume per acre for the ith point 3P sample tree.
MV = Measured volume of the ith point-3P sample tree.

Compute estimated stratum volume for an individual point 3P sample tree as follows:

$$SV = F_v \times Acres$$

Where:

SV = Stratum volume for the ith point-3P sample tree.
Acres = Acres in the stratum.

Calculate estimated stratum volume per acre and total stratum volume by summing all point-sample tree values.

The calculations for a timber sale of eighteen acres and ten sample points are shown in exhibit 01, given:

1. BAF = 20

2. Acres = 18

3. Volumes are in ft^3

4. Species group 1 = A

5. Species group 2 = B and C

37.35a - Exhibit 01

Volume Calculation: Point Sample 3P

Plot No.	Tree No.	SP	KPI	DBH	HT	Meas Vol (MV)	Point Wt ($F_{t(p)}$)	3P Wt ($F_{t(3p)}$)	Tree Fact (F_t)	Vol Fact (F_v)	Tract Vol (SV)
1	1	A	50								
1	2	A	75	11	74	24.2	3.03	1.83	5.54	134.1	2413.8
1	3	A	65						0.00		
1	4	B	75	13	73	26.9	2.17	1.90	4.12	110.8	1994.4
1	5	B	45						0.00		
1	6	B	60						0.00		
1	7	B	90	15	88	43.8	1.63	1.58	2.58	113.0	2034.0
2	1	A	60						0.00		
2	2	B	55						0.00		
2	3	B	60	9	61	10.1	4.53	2.38	10.78	108.9	1960.2
2	4	B	65						0.00		
2	5	B	45						0.00		
2	6	A	65	16	67	41.4	1.43	2.12	3.03	125.4	2257.2
3									0.00		
4	1	A	60						0.00		
4	2	A	75	13	74	30.9	2.17	1.83	3.97	122.7	2208.6
4	3	A	90	17	92	66.3	1.27	1.53	1.94	128.6	2314.8
4	4	B	65						0.00		
4	5	B	55						0.00		
4	6	C	55						0.00		
5	1	A	75	13	77	31.8	2.17	1.83	3.97	126.2	2271.6
5	2	C	50						0.00		
5	3	A	70						0.00		
5	4	C	65	11	63	16.8	3.03	2.19	6.64	111.6	2008.8
6	5	A	70	14	69	32.4	1.87	1.96	3.67	118.9	2140.2
6	6	A	70						0.00		
7									0.00		
8	1	A	75	12	74	27.8	2.55	1.83	4.67	129.8	2336.4
8	2	B	55						0.00		
8	3	B	65	9	63	10.7	4.53	2.19	9.92	106.1	1909.8
8	4	B	55	11	56	14.2	3.03	2.59	7.85	111.5	2007.0
8	5	B	65						0.00		
8	6	B	70	12	70	21.6	2.55	2.04	5.20	112.3	2021.4
9	1	A	70						0.00		
9	2	C	60	12	57	17.7	2.55	2.38	6.07	107.4	1933.2
9	3	A	65						0.00		
9	4	A	65	10	65	17.3	3.67	2.12	7.78	134.6	2422.8
10	1	B	75	13	73	26.9	2.17	1.90	4.12	110.8	1994.4
10	2	B	45						0.00		
10	3	B	60						0.00		
10	4	B	90	15	88	43.8	1.63	1.58	2.58	113.0	2034.0
	Total		2525						94.43	2125.7	38263
	Count		39			18					

Item	Group 1	Group 2	All
KPI	1100	1425	2525
n_{3p}	8	10	18
$F_{t(3p)}$	15.1	20.7	35.8
F_t	34.6	59.9	94.4
F_v	1020	1105	2126

37.35b - Sampling Error

Sampling error of a point 3P cruise can be calculated from the following formula:

$$E_T = \sqrt{E_p + E_{3p}}$$

$$= \sqrt{\left(\frac{SE_p}{\overline{PH}} \times 100 \times t\right)^2 + \left(\frac{SE_{3p}}{\overline{R}} \times 100 \times t\right)^2}$$

Where:

E_T = Stratum sampling error in percent.

$SE_p = \sqrt{\dfrac{\sum^{p} PH^2 - \dfrac{\left(\sum^{p} PH\right)^2}{p}}{(p-1)\ (p)}}$

PH = Sum of estimated tree heights of point sample trees on j^{th} plot.

p = Number of points.

$\overline{PH}$ = Mean of the sum of tree height over all points.

$SE_{3p} = \sqrt{\dfrac{\sum^{n_{3p}} R^2 - \dfrac{\left(\sum^{n_{3p}} R\right)^2}{n_{3p}}}{(n_{3p}-1)\ (n_{3p})}}$

R = M/P ratio for i^{th} point-3P sample tree = $\dfrac{\text{measured volume}}{(D^2H)}$

n_{3p} = Number of point-3P sample trees.

$\overline{R}$ = Mean M/P ratio.

Sampling error calculations use the data shown in exhibit 01 which was derived from the data shown in 35.35a exhibit 01.

37.35b - Exhibit 01

Sampling Error, One Stratum Sale

Plot No.	Tree No.	KPI	Plot KPI	Plot KPI^2	DBH	HT	Meas Vol (MV)	D^2H 1/ (DBH^2xH)	M/P 2/ Ratio (R)	M/P 3/ $Ratio^2$ (R^2)
1	1	50								
1	2	75			11	74	24.2	9075	0.0027	0.0000073
1	3	65								
1	4	75			13	73	26.9	12675	0.0021	0.0000044
1	5	45								
1	6	60								
1	7	90	460	211600	15	88	43.8	20250	0.0022	0.0000048
2	1	60								
2	2	55								
2	3	60			9	61	10.1	4860	0.0021	0.0000044
2	4	65								
2	5	45								
2	6	65	350	122500	16	67	41.4	16640	0.0025	0.0000063
3			0	0						
4	1	60								
4	2	75			13	74	30.9	12675	0.0024	0.0000058
4	3	90			17	92	66.3	26010	0.0025	0.0000063
4	4	65								
4	5	55								
4	6	55	400	160000						
5	1	75			13	77	31.8	12675	0.0025	0.0000063
5	2	50								
5	3	70								
5	4	65	260	67600	11	63	16.8	7865	0.0021	0.0000044
6	5	70			14	69	32.4	13720	0.0024	0.0000058
6	6	70	140	19600						
7			0	0						
8	1	75			12	74	27.8	10800	0.0026	0.0000068
8	2	55								
8	3	65			9	63	10.7	5265	0.0020	0.0000040
8	4	55			11	56	14.2	6655	0.0021	0.0000044
8	5	65								
8	6	70	385	148225	12	70	21.6	10080	0.0021	0.0000044
9	1	70								
9	2	60			12	57	17.7	8640	0.0020	0.0000040
9	3	65								
9	4	65	260	67600	10	65	17.3	6500	0.0027	0.0000073
10	1	75			13	73	26.9	12675	0.0021	0.0000044
10	2	45								
10	3	60								
10	4	90	270	72900	15	88	43.8	20250	0.0022	0.0000048
	Sum	2525	2525	870025					0.0413	0.0000959
	Count	39	10				18			

1/ D^2H = (Measured DBH)^2 x Estimated Height (KPI).
2/ M/P Ratio = Measured Tree Volume / D^2H = R.
3/ The covariance term is excluded because its effect on E is trivial.

Calculate the standard errors and sampling error:

$$SE_{3p} = \sqrt{\frac{\sum^{n_{3p}} R^2 - \frac{\left(\sum^{n_{3p}} R\right)^2}{n_{3p}}}{(n_{3p} - 1)\ (n_{3p})}}$$

$$= \sqrt{\frac{0.0000959 - \frac{0.0413}{18}}{(18 - 1)\ (18)}}$$

$$= 0.000061$$

$$E_{3p} = \frac{SE_{3p}}{\bar{R}} \times 100 \times t$$

$$= \frac{(0.000061)}{\left(\frac{0.0413}{18}\right)} \times 100 \times 2$$

$$= 5.3\%$$

$$SE_p = \sqrt{\frac{\sum^{p} PH^2 - \frac{\left(\sum^{p} PH\right)^2}{p}}{(p - 1)\ (p)}}$$

$$= \sqrt{\frac{870{,}025 - \frac{6{,}375{,}625}{10}}{(10 - 1)\ (10)}}$$

$$= 50.82$$

$$E_p = \frac{SE_p}{\overline{KPI}} \times 100 \times t$$

$$= \frac{50.82}{\left(\frac{2525}{10}\right)} \times 100 \times 2$$

$$= 40.3\%$$

$$E_T = \sqrt{E_p^2 + E_{3p}^2}$$

$$= \sqrt{40.3^2 + 5.3^2}$$

$$= 40.6\%\ (95\%\ \textit{confidence level})$$

37.36 - Additional Population Characteristics

Use the data from exhibit 01 to calculate the average stratum diameter and height.

37.36 - Exhibit 01

Plot Data for Calculating Population Characteristics for Point 3P

Plot No.	Tree No.	SP	KPI	DBH	DBH^2	HT	Point Wt ($F_{t(p)}$)	3P Wt ($F_{t(3p)}$)	Tree Fact (F_t)	Exp DBH ($DBHxF_t$)	Exp DBH^2 (DBH^2xF_t)	Exp HT ($HTxF_t$)
1	1	A	50									
1	2	A	75	11	121	74	3.03	1.83	5.54	60.9	670.3	410.0
1	3	A	65									
1	4	B	75	13	169	73	2.17	1.90	4.12	53.6	696.3	300.8
1	5	B	45									
1	6	B	60									
1	7	B	90	15	225	88	1.63	1.58	2.58	38.7	580.5	227.0
2	1	A	60									
2	2	B	55									
2	3	B	60	9	81	61	4.53	2.38	10.78	97.0	873.2	657.6
2	4	B	65									
2	5	B	45									
2	6	A	65	16	256	67	1.43	2.12	3.03	48.5	775.7	203.0
3												
4	1	A	60									
4	2	A	75	13	169	74	2.17	1.83	3.97	51.6	670.9	293.8
4	3	A	90	17	289	92	1.27	1.53	1.94	33.0	560.7	178.5
4	4	B	65									
4	5	B	55									
4	6	C	55									
5	1	A	75	13	169	77	2.17	1.83	3.97	51.6	670.9	305.7
5	2	C	50									
5	3	A	70									
	4	C	65	11	121	63	3.03	2.19	6.64	73.0	803.4	418.3
6	5	A	70	14	196	69	1.87	1.96	3.67	51.4	719.3	253.2
6	6	A	70									
7												
8	1	A	75	12	144	74	2.55	1.83	4.67	56.0	672.5	345.6
8	2	B	55									
8	3	B	65	9	81	63	4.53	2.19	9.92	89.3	803.5	625.0
8	4	B	55	11	121	56	3.03	2.59	7.85	86.4	949.9	439.6
8	5	B	65									
8	6	B	70	12	144	70	2.55	2.04	5.20	62.4	748.8	364.0
9	1	A	70									
9	2	C	60	12	144	57	2.55	2.38	6.07	72.8	874.1	346.0
9	3	A	65									
9	4	A	65	10	100	65	3.67	2.12	7.78	77.8	778.0	505.7
10	1	B	75	13	169	73	2.17	1.90	4.12	53.6	696.3	300.8
10	2	B	45									
10	3	B	60									
10	4	B	90	15	225	88	1.63	1.58	2.58	38.7	580.5	227.0
	Sum		2525						94.43	1096.3	13124.8	6401.6
	Count		39	18								

Calculate the additional population characteristics from the tabulated data in exhibit 01:

$$Arithemetic\ Mean\ Diameter = \frac{\sum^{n_{3p}} (DBH \times F_t)}{\sum^{n_{3p}} F_t}$$

$$= \frac{1096.3}{94.43}$$

$$= 11.6\ inches$$

$$Quadratic\ Mean\ Diameter = \sqrt{\frac{\sum^{n_{3p}} (DBH^2 \times F_t)}{\sum^{n_{3p}} F_t}}$$

$$= \sqrt{\frac{13124.8}{94.43}}$$

$$= 11.8\ inches$$

$$Mean\ Height = \frac{\sum^{n_{3p}} (HT \times F_t)}{\sum^{n_{3p}} F_t}$$

$$= \frac{6401.6}{94.43}$$

$$= 67.8\ feet$$

37.37 - Application

Point sampling with a 3P subsample is an effective method to use in clearcutting situations where large areas are to be cruised and where there is a small range of tree sizes.

37.4 - Point Count/Measure-Plot Method

In ratio double sampling, count sample trees at all points but measure volume only on some proportion of the points. Use the data from the points on which sample trees are measured to estimate a ratio of volume to basal area. Apply this ratio, called V-BAR, to the estimated basal area for the tract to arrive at an estimate of tract volume. There are two sources of statistical error in double sampling: estimated average basal area per acre and estimated average V-BAR.

37.41 - Operational Features

The operational and field procedures of point sampling are described in section 35. The purpose of the first stage point sample is to estimate strata basal area. The purpose of the second stage sample is to collect measured tree information for the development of volume to basal area ratios.

37.42 - Statistical Features

Ratio double sampling can be done with either a completely random or a systematic design. Determine the initial number of plots (or count plots), then select a subsample of these plots for measurement plots based on some proportion of the initial plots.

The advantage of ratio double sampling results from the time saved in counting the trees on some of the plots instead of measuring all the trees on all the plots. This advantage is reduced if the time saved is not appreciable or if the relationship between the volume per basal area ratio and the number of count trees is weakly correlated.

37.43 - Calculating Sample Size

Calculate the sample size as in Johnson (1965). An example for determining the sample size in ratio double sampling for the west side forests in Washington is:

$$n = k(1.732)\sqrt{r}$$

Where: n = Total number of count points plus measure points.

k = Number of measured points = $\left(\frac{CV^2}{E^2}\right)\left(\frac{0.433}{\sqrt{r}} + 0.25\right)$

r = Cost of measuring all trees on a point divided by cost of counting all trees on a point.

CV = Coefficient of variation for volume/basal area ratio (V-BAR).

E = Sampling error percent.

See the chapter 90 for tables giving the number of points needed to meet a specified sampling error, given the coefficient of variation and relative cost of measuring versus counting sample trees on a point. These tables are presented as examples and should be modified for local conditions.

37.44 - Calculating Sampling Statistics

37.44a - Sample Expansion

Two methods are commonly used in expanding point sample data: the factor method, (Beers and Miller, 1964) and the V-BAR method, (Beers and Miller, 1964; Dilworth and Bell, 1981). Both methods give identical results except for trivial rounding differences. Computations are given in this section for each method.

1. Factor Method. The factor expansion method involves calculating the sample tree weight or frequency using the proportion of the measured trees as a weighting factor. Compute the expansion factor for each tree from the following formula:

$$F_t = F_{t(p)} \times F_{t(c)}$$

Where:

F_t = Number of trees per acre for the i^{th} measured tree.
$F_{t(p)}$ = Tree expansion factor due to point sampling

$$= \frac{\text{Basal Area Factor}}{0.005454 \times DBH^2 \times p}$$

$F_{t(c)}$ = Frequency of count trees = n/k
n = Number of measured trees plus number of count trees.
k = Number of measured trees.
p = Number of plots established.

Compute the estimated volume per acre for the i^{th} individual measured tree from the following formula:

$$F_v = F_t \times MV$$

Where:

F_v = Estimated volume per acre for the i^{th} sample tree.
F_t = Number of trees for i^{th} measured tree.
MV = Measured volume for the i^{th} measured tree.

2. V-BAR Method. The "V-BAR method calculates the volume per acre by calculating the average volume to basal area ratio and multiplying it by average basal area per acre. Using the V-BAR method, calculate the volume per acre for each tree using the following formula:

$$F_v = VBAR \times \overline{BAPA}$$

Where:

F_v = Estimated volume per acre for i[th] measured tree.
VBAR = Volume to BA ratio of i[th] tree

$$= \frac{MV}{(0.005454 \times DBH^2)}$$

MV = Volume of the i[th] measured tree.
$\overline{BAPA}$ = Average BA per acre = (BAF x n/p) / k
BAF = Basal Area Factor
n = Number of count trees + number of measured trees.
p = Number of sample points.
k = Number of measured trees

Compute the estimated total volume for the individual measured trees for both methods as follows:

$$SV = F_v \times Acres.$$

Where:

SV = Total volume for the i[th] measured tree.
F_v = Volume factor for i[th] measured tree.
Acres = Number of acres in the stand.

In exhibits 01 - 02, sample trees are counted at all points and measured for volume at some fraction of the points, separated by species type.

Given:

1. number of count points = 5;
2. number of measure points = 5;
3. BAF = 20;
4. tract acres = 18.

37.44a - Exhibit 01

Measure/Count Data and Summary

Plot No.	Sp	Type	Count	DBH	HT	Net Vol (MV)
1	A	C	3.0			
	B	C	4.0			
2	A	M		12	59	21.0
	B	M		10	56	11.5
	B	M		9	61	10.1
	B	M		14	66	29.1
	B	M		9	43	6.7
	A	M		16	67	41.4
3		C	0.0			
4	A	M		12	61	21.0
	A	M		13	74	30.9
	A	M		17	92	66.3
	B	M		12	63	19.3
	B	M		9	54	8.8
	C	M		11	54	14.0
5	A	C	2.0			
	C	C	2.0			
6	A	M		14	69	32.4
	A	M		16	72	45.6
7		C	0.0			
8	A	M		12	74	27.8
	B	M		10	55	11.0
	B	M		9	63	10.7
	B	M		11	56	14.2
	B	M		13	63	24.2
	B	M		12	70	21.6
9	A	C	3.0			
	C	C	1.0			
10	B	M		13	73	26.9
	B	M		8	45	4.9
	B	M		13	61	23.0
	B	M		15	88	43.8

Item	Species A	Species B	Species C	All
n	16	19	4	39
k	8	15	1	24
Ft(c)	2.000	1.267	4.000	1.625
BAPA	4.000	2.533	8.000	3.250

Because this example is separated by species, n and k are the number of counted and measured trees by species. Calculate the expansion factors for each tree.

37.44a - Exhibit 02

Factor and V-BAR Calculation Example and Summary

Plot No.	Sp	Point Fact ($F_{t(p)}$)	Factor Count Fact ($F_{t(c)}$)	Factor Tree Fact (F_t)	Factor Vol Fact (F_v)	VBAR	VBAR BAPA	VBAR Vol Fact (F_v)
2	A	2.547	2.000	5.093	107.0	26.739	4.000	107.0
	B	3.667	1.267	4.650	53.5	21.085	2.533	53.4
	B	4.527	1.267	5.740	58.0	22.862	2.533	57.9
	B	1.871	1.267	2.370	69.0	27.222	2.533	69.0
	B	4.527	1.267	5.740	38.5	15.166	2.533	38.4
	A	1.432	2.000	2.860	118.4	29.651	4.000	118.6
4	A	2.547	2.000	5.090	106.9	26.739	4.000	107.0
	A	2.170	2.000	4.340	134.1	33.524	4.000	134.1
	A	1.269	2.000	2.540	168.4	42.063	4.000	168.3
	B	2.547	1.267	3.230	62.3	24.574	2.533	62.2
	B	4.527	1.267	5.740	50.5	19.920	2.533	50.5
	C	3.031	4.000	12.120	169.7	21.214	8.000	169.7
6	A	1.871	2.000	3.740	121.2	30.309	4.000	121.2
	A	1.432	2.000	2.860	130.4	32.660	4.000	130.6
8	A	2.547	2.000	5.090	141.5	35.397	4.000	141.6
	B	3.667	1.267	4.650	51.2	20.169	2.533	51.1
	B	4.527	1.267	5.740	61.4	24.221	2.533	61.4
	B	3.031	1.267	3.840	54.5	21.517	2.533	54.5
	B	2.170	1.267	2.750	66.6	26.255	2.533	66.5
	B	2.547	1.267	3.230	69.8	27.503	2.533	69.7
10	B	2.170	1.267	2.750	74.0	29.184	2.533	73.9
	B	5.730	1.267	7.260	35.6	14.038	2.533	35.6
	B	2.170	1.267	2.750	63.3	24.953	2.533	63.2
	B	1.630	1.267	2.060	90.2	35.692	2.533	90.4
Total				106.233	2096.0	632.7		2095.6

Item.	Species A	Species B	Species C	All Species
FACTOR				
Estimated Vol/acre (Sum F_v)	1027.9	898.4	169.7	2096.0
Estimated Trees/acre (Sum F_t)	31.6	62.5	12.1	106.2
Estimated total Vol (Sum SV)	18502.2	16171.2	3054.6	37728.0
V-BAR				
Estimated Vol/acre (Sum F_v)	1028.3	897.6	169.7	2095.6
Estimated BA/acre	32.0	38.0	8.0	78.0
Estimated total Vol (Sum SV)	18509.9	16156.7	3054.8	37721.5
No. Measured Trees (k)	8	15	1	24
No. Counted Trees (n-k)	8	4	3	15
No. Measured + Counted Trees (n)	16	19	4	39

When dealing with multiple species in double sampling do the following:

1. Collect data by species or species groups (tree counts and measured tree data) if definitive species data is needed for appraisal purposes.

2. Group a species weakly represented in measured point data, with another species as similar in form and value as is possible.

3. Disregard trivial differences between the sum of the species per acre volumes and the calculated tract average volume per acre. The difference is due to the difference among species in the ratio of measure to count trees.

37.44b - Sampling Error

When double sampling, use the following formulas (Johnson 1965), for calculating the standard error and the sampling error:

$$SE = \sqrt{\frac{\sum^{m} X^2 - \frac{(\sum^{m} X)^2}{m}}{p(m-1)} + \frac{\sum^{m} X^2 + \bar{R}^2 \sum^{m} W^2 - 2\bar{R}\sum^{m} XW}{m\ (m-1)} \left(\frac{P-m}{p}\right)}$$

Where:

X = Volume factors for a measured point (F_v or V-BAR)

$\bar{R}$ = Mean volume factor for measured trees = $\frac{\sum^{k} F_v}{k}$.

W = Tree count for a measured point.

p = Number of measured points + count points.

m = Number of measured points.

If V-BARS are used in lieu of volume factors, multiply the square root of the formula by the basal area factor used. The data in exhibit 01 is used in the example calculation.

37.44b - Exhibit 01

Data

Point	F_v	F^2_v	W	W^2	W x F_v
2	444.400	197491.36	6	36	2666.400
4	691.900	478725.61	6	36	4151.400
6	251.600	63302.56	2	4	503.200
8	445.000	198025.00	6	36	2670.000
10	263.100	69221.61	4	16	1052.400
Total	2096.000	1006766.14	24	128	11043.400

Given:

- $p = 10$
- $m = 5$
- $\bar{R} = 2096.000/24 = 87.333$
- $\bar{R}^2 = 7627.053$
- $(p-m)/p = 0.5$
- $\bar{x} = 2096.00/5 = 419.200$

Then, substituting component values in the formula:

$$SE = \sqrt{\frac{1006766.14-\frac{2096.000^2}{5}}{10(4)} + \frac{1006766.14 + 7627.053(128) - 2(87.333)(11043.400)}{5(4)}\left(\frac{10-5}{10}\right)}$$

$$= \sqrt{3203.074 + 2706.121(0.5)}$$

$$= \sqrt{4556.135}$$

$$= 67.499$$

then: $E = 100\left(\frac{t \times SE}{\bar{x}}\right) = 100\ (2)\left(\frac{67.499}{419.20}\right) = 32.204$

37.45 - Additional Population Characteristics

$$\text{Mean Diameter} = \frac{\sum^{k}(DBH \times F_t)}{\sum^{k} F_t}$$

$$= \frac{1208.9}{106.2} = 11.4$$

$$\text{Quadratic Mean Diameter} = \sqrt{\frac{\sum^{k}(DBH^2 \times F_t)}{\sum^{k} F_t}}$$

$$= \sqrt{\frac{14302.3}{106.2}} = 11.6$$

$$\text{Mean Height} = \frac{\sum^{k}(HT \times F_t)}{\sum^{k} F_t}$$

$$= \frac{6594.3}{106.2} = 62.1$$

37.46 - Application

Point sampling is an effective system to use in clear-cutting situations where large areas are to be cruised and where there is a range of tree sizes. This is a very cost effective method where the individual tree VBAR variance is low and the basal area variance per acre is high. The cost of taking count plots is much less than measure plots, and the sampling is where most of the variance occurs.

37.5 - Point Count/Measure-Tree Method

Point count/measure-tree is a ratio double sampling technique. In ratio double sampling, all sample trees are counted but only a portion of the trees are measured at each plot. The data from the sample trees that are measured is used to estimate a ratio of volume to basal area. This ratio, called V-BAR, is applied to the estimated basal area for the tract to arrive at an estimate of tract volume. There are two sources of statistical error in double sampling: estimated average basal area per acre and estimated average V-BAR.

37.51 - Operational Features

The operational and field procedures of point sampling are described in section 35. The purpose of the first stage point sample is to estimate strata basal area. The purpose of the second stage sample is to collect measured tree information for the development of volume to basal area ratios.

37.52 - Statistical Features

Ratio double sampling can be done with either a completely random or a systematic design. Determine the initial number of plots, then select a subsample of the trees on these plots for measurement.

The advantage of ratio double sampling results from the time saved in measuring only a portion of trees on the plots. This advantage is reduced if the time saved is not appreciable or if the relationship between the volume per basal area ratio and the number of count trees is weakly correlated.

37.53 - Calculating Sample Size

Calculate combined sampling error to determine sample size for point count/measure-tree cruising. Determine sample size for the point sample and the count/measure-tree sample to satisfy the desired sampling error for the stratum.

The combined sampling error includes the point sample error and the count/measure tree error (Bruce, 1961).

If both the point sample and count/measure-tree errors are known, use the following formula for calculating the sampling error:

$$E_T = \sqrt{E_p^2 + E_{cm}^2}$$

Where:

E_T = Combined percent sampling error.
E_p = Percent sampling error for the point sample.
E_{cm} = Percent sampling error for the count/measure-tree sample.

This formula assumes the two errors are independent, which is not quite true. However, the formula may be used to approximate the error, and it is useful in estimating number and approximate ratio of count/measure plots or trees. The cruise may be designed to meet a specific combined error. In this case the desired combined error is known. Estimate one of the errors and determine the remaining error using the following example where count/measure error is estimated and the point error is being determined:

$$E_p = \sqrt{E_T^2 - E_{cm}^2}$$

For example, given:

1. Target combined sampling error of 20 percent (95 percent confidence level)

2. Count/measure error is estimated to be 12 percent (95 percent confidence level).

Calculate the point error:

$$E_p = \sqrt{E_T^2 - E_{cm}^2}$$
$$= \sqrt{20^2 - 12^2}$$
$$= \sqrt{256}$$
$$= 16.0\%$$

Note: Desired combined sampling error must be higher than either component sampling error.

Determine the number of samples needed to attain the errors identified for each stage of the point count/measure tree cruise. Estimate the coefficient of variation between point/basal area in the first stage, and between volume/basal area ratio (V-BAR) in the second stage. Base these estimates on preliminary sample data, past cruises in the area in similar types, or inventory data.

Calculate the number of plots for the first stage sample and the number of trees for the second stage:

First Stage
Assumed Coefficient of Variation = 25%
Confidence level = 95% (t=2)
Sampling error = 16%

$$n_p = \frac{(t^2)\ (CV_p^2)}{E_p^2} = \frac{(2^2)\ (25^2)}{16^2} = 9.77 = 10 \textit{ plots}$$

Second Stage
Assumed Coefficient of Variation = 25.5%
Confidence level = 95% (t=2)
Sampling error = 12%

$$n_{cm} = \frac{(t^2)\ (CV_{cm}^2)}{E_{cm}^2} = \frac{(2^2)\ (25.5)^2}{12^2} = 18.06 = 18.0 \textit{ trees}$$

37.54 - Calculating Sampling Statistics

37.54a - Sample Expansion

Two methods are commonly used in expanding point sample data: the factor method, (Beers and Miller, 1964) and the V-BAR method, (Beers and Miller, 1964; Dilworth and Bell, 1981). Both methods give identical results except for trivial rounding differences. Computations are given in this section for each method.

1. Factor Method. The factor expansion method involves calculating the sample tree weight or frequency using the proportion of the measured trees as a weighting factor. Compute the expansion factor for each tree from the following formula:

$$F_t = F_{t(p)} \times F_{t(c)}$$

Where:

F_t = Tree factor; number of trees per acre for the i^{th} measured tree

$F_{t(p)}$ = Expansion of prism cruise = $\frac{\text{Basal Area Factor}}{0.005454 \times DBH^2 \times p}$

$F_{t(c)}$ = Expansion of count; frequency of measured trees = $\frac{n}{k}$

n = Number of measured trees plus number of count trees.

k = Number of measured trees.

p = Number of sample points.

The estimated volume per acre for an individual measured tree is computed from the following formula:

$$F_v = F_t \times MV$$

Where:

F_v = Volume factor; estimated volume per acre for i^{th} measured tree.
F_t = Tree factor; estimated trees per acre for i^{th} measured tree
MV = Measured volume for the i^{th} measured tree.

2. V-BAR Method. The V-BAR method calculates the volume per acre by calculating the average volume to-basal area ratio and multiplying it by average basal area per--acre. The volume per acre using the V-BAR method is calculated using the following formula:

$$F_v = VBAR \times \overline{BAPA}$$

Where:

F_v = Volume factor; estimated volume per acre for the i^{th} measured tree.
VBAR = Volume to basal area ratio of the i^{th} measured tree

$$= \frac{MV}{0.005454 \times DBH^2}$$

MV = Measured volume of the i^{th} measured tree
k = Number of measured trees
$\overline{BAPA}$ = average BA per acre = $\frac{BAF \times n}{pk}$
BAF = Basal Area Factor.
n = Number of count trees plus number of measured trees.
p = Number of sample points.
k = Number of measured trees.

The estimated total volume for the individual measured trees is computed for both methods as follows:

$$SV = F_v \times Acres.$$

Where:

SV = Total volume for the i^{th} measured tree.
F_v = Volume factor.
Acres = Number of acres in the stand.

In the following example (ex. 01 - 02), all sample trees are counted at each point, but only a portion of the trees are measured for volume at each point.

Given:

1. Number of points = 10
2. BAF = 20
3. tract acres = 18.

37.54a - Exhibit 01

Stand Data for Point Count/Measure-Tree Sample

Plot No.	Sp	Type	Count	DBH	HT	Net Vol
1	A	M		10	48	10.7
	A	C	2.0			
	B	M		13	73	26.9
	B	C	3.0			
2	A	M		12	59	21.0
	B	M		10	56	11.5
	B	C	3.0			
	A	C	1.0			
3		M	0.0			
4	A	M		12	61	21.0
	A	C	2.0			
	B	M		12	63	19.3
	B	C	1.0			
	C	M		11	54	14.0
5	A	M		13	77	31.8
	C	M		11	52	13.0
	A	C	1.0			
	C	C	1.0			
6	A	M		14	69	32.4
	A	C	1.0			
7		M	0.0			
8	A	M		12	74	27.8
	B	M		10	55	11.0
	B	C	4.0			
9	A	M		15	69	38.4
	C	M		12	57	17.7
	A	C	2.0			
10	B	M		13	73	26.9
	B	C	3.0			

Item	Species A	Species B	Species C	All Species
n	16	19	4	39
k	7	5	3	15
$F_{t(c)}$	2.286	3.800	1.333	2.600
BAPA	4.571	7.600	2.667	5.200

37.54a - Exhibit 02

Factor and V-BAR Calculation and Summary
for Point Count/Measure-Tree Example

				Factor			VBAR	
Plot No.	Sp	$F_{t(p)}$	$F_{t(c)}$	F_t	F_v	VBAR	BAPA	F_v
1	A	3.667	2.286	8.383	89.7	19.619	4.571	89.7
	B	2.170	3.800	8.245	221.8	29.184	7.600	221.8
2	A	2.547	2.286	5.821	122.2	26.739	4.571	122.2
	B	3.667	3.800	13.935	160.3	21.085	7.600	160.2
4	A	2.547	2.286	5.821	.122.2	26.739	4.571	122.2
	B	2.547	3.800	9.677	186.8	24.574	7.600	186.8
	C	3.031	1.333	4.040	56.6	21.214	2.667	56.6
5	A	2.170	2.286	4.960	157.7	34.500	4.571	157.7
	C	3.031	1.333	4.040	52.5	19.699	2.667	52.5
6	A	1.871	2.286	4.277	138.6	30.309	4.571	138.5
8	A	2.547	2.286	5.821	161.8	35.397	4.571	161.8
	B	3.667	3.800	13.935	153.3	20.169	7.600	153.3
9	A	1.630	2.286	3.726	143.1	31.292	4.571	143.0
	C	2.547	1.333	3.395	60.1	22.537	2.667	60.1
10.0	B	2.170	3.800	8.245	221.8	29.184	7.600	221.8
Total				104.321	2048.5	392.241		2048.3

FACTOR	A	B	C	All Species
Estimated Vol/acre (Sum F_v)	935.3	944.0	169.2	2048.5
Estimated Trees/acre (Sum F_t)	38.8	54.0	11.5	104.3
Estimated total Vol (Sum SV)	16835.4	16992.0	3045.6	36873.0
V-BAR				
Estimated Vol/acre (Sum F_v)	935.2	943.9	169.2	2048.3
Estimated total Vol (Sum SV)	16833.7	16990.0	3046.0	36869.7
No. Measured-Trees (k)	7	5	3	15
No. Counted Trees (n-k)	9	14	1	24
No. Measured+Counted Trees (n)	16	19	4	39

Do the following when dealing with multiple species .in double Sampling:

1. Collect data by species or species groups (tree counts and measure tree data) if definitive species data is needed for appraisal purposes.

2. When a species is weakly represented in measured point data, group with another species as similar in form and value as possible.

3. Disregard trivial differences between the sum of the species per acre volumes and the calculated tract average volume per acre. The difference is due to the difference among species in the ratio of measure to count trees.

37.54b - Sampling Error

When this form of double sampling is used, use the following formulas for calculating the standard error and the sampling error:

$$E_T = \sqrt{E_{cm}^2 + E_p^2}$$

$$= \sqrt{\left(\frac{SE_{cm}}{\overline{VBAR}} \times 100 \times t\right)^2 + \left(\frac{SE_p}{\bar{n}_p} \times 100 \times t\right)^2}$$

Where:

E_T = Stratum sampling error percent

E_{cm} = Stratum sampling error of tree VBAR

E_p = Stratum sampling error of point sampling

$$SE_p = \sqrt{\frac{\sum^{p} n_p^2 - \frac{\left(\sum^{p} n_p\right)^2}{p}}{(p-1)\ (p)}}$$

n_p = Number of prism trees on the j^{th} point

$$SE_{cm} = \sqrt{\frac{\sum^{k} VBAR^2 - \frac{\left(\sum^{k} VBAR\right)^2}{k}}{(k-1)\ (k)}}$$

VBAR = Volume to basal area ratio (V-BAR) for i^{th} measured tree

k = Number of measured trees

p = Number of points

$\overline{VBAR}$ = Mean volume to basal area ratio = $\frac{\sum^{k} VBAR}{k}$

$\bar{n}_p$ = Mean number of trees per plot = $\frac{\sum^{p} n_p}{p}$

For example, given the data in exhibit 01, calculate the error as shown.

37.54b - Exhibit 01

Derivations

Point	VBAR	$VBAR^2$	n_p	n_p^2
1	19.619	384.905	7	49
	29.184	851.706		
2	26.739	714.974	6	36
	21.085	444.577		
4	26.739	714.974	6	36
	24.574	603.881		
	21.214	450.034		
5	34.500	1190.250	4	16
	19.699	388.051		
6	30.309	918.635	2	4
8	35.397	1252.948	6	36
	20.169	406.789		
9	31.292	979.189	4	16
	22.537	507.916		
10	29.184	851.706	4	16
Total	392.241	10660.535	39	209

Calculate the sampling statistics:

$$SE_p = \sqrt{\frac{209 - \left(\frac{39^2}{10}\right)}{(10-1)(10)}} = \sqrt{0.632} = 0.795$$

$$\bar{n}_p = \frac{39}{10} = 3.9$$

$$SE_{cm} = \sqrt{\frac{10660.535 - \left(\frac{392.241^2}{15}\right)}{(15-1)(15)}} = \sqrt{1.922} = 1.387$$

$$\overline{VBAR} = \frac{392.241}{15} = 26.149$$

Calculate the sampling error:

$$E = \sqrt{\left(\frac{0.795}{3.9} \times 100 \times 2\right)^2 + \left(\frac{1.387}{26.149} \times 100 \times 2\right)^2}$$

$$= \sqrt{40.769^2 + 10.608^2} = \sqrt{1,774.641}$$

$$= 42.1\% \text{ (95\% confidence level)}$$

37.55 - Additional Population Characteristics

$$Mean\ Diameter = \frac{\sum^{k}(DBH \times F_t)}{\sum^{k} F_t}$$

$$= \frac{1212.4}{104.321}$$

$$= 11.6\ inches$$

$$Quadratic\ Mean\ Diameter = \sqrt{\frac{\sum^{k}(DBH^2 \times F_t)}{\sum^{k} F_t}}$$

$$= \sqrt{\frac{14301.7}{104.321}} = 11.7\ inches$$

$$Mean\ Height = \frac{\sum^{k}(HT \times F_t)}{\sum^{k} F_t}$$

$$= \frac{6552.1}{104.321} = 62.8\ feet$$

37.56 - Application

Point sampling is an effective system to use in clear-cutting situations where large areas are to be cruised and where there is a range of tree sizes.

38 - Combining Strata

Combining information from multiple strata is needed to calculate total statistics and quantities for the sale population.

38.1 - Combining Sampling Statistics

The purpose of combining sampling statistics is to combine the volumes and sampling errors of different stands or strata into a total volume and sampling error for the entire sale.

A timber sale is comprised of several strata, each having individual volume and variation. Sample each stratum separately, using the sampling technique that will best estimate those individual stratum characteristics. This is, in effect, stratifying the sale into strata of like characteristics, which can be combined to give an estimate of the total sale population.

Determine the total volume and the sampling error for each stratum using one of the sampling methods discussed earlier. Calculate the estimated total volume by summing the individual stratum volumes. Calculate the sampling error for the sale population using the following formula:

$$E_T = \frac{\sqrt{\sum^{n}(V \times E)^2}}{V_T}$$

Where:

E_T = Sampling error for the sale population in percent
V = Estimated total volume for the j^{th} stratum
E = Sampling error percent for the j^{th} stratum
n = Number of strata in sale

V_T = Estimated total volume for the sale = $\sum^{n} V$

After expansion, the equation is:

$$E_T = \frac{\sqrt{(V_1E_1)^2 + (V_2E_2)^2 + \ldots\ldots\ldots + (V_nE_n)^2}}{V_1 + V_2 + \ldots\ldots\ldots + V_n}$$

Note: E_T will be smaller than the individual values of the sampling Errors (E) most of the time. The reason for this is, by combining the Individual samples, the overall sample size increases, which will decrease the overall sampling error. As the sample size, n, increases, the sample error, E, decreases, and as n approaches the population size (N), the sample error (E) approaches 0.

38.11 - Same Cruising Technique

Use the following example to calculate total error for two strata sampled using the same cruising technique (Sample tree, sec.33).

1. Stratum 1:

   Total Volume (V_1) = 4663 ft^3

   % Sampling Error (1_T) = 20.4%

   V_1 x E_1 = 95125.2

2. Stratum 2:

   Total Volume (V_2) = 5111 ft^3

   % Sampling Error (E_2) = 23.1%

   V_2 x E_2 = 118064.1

Estimated Total Volume (V_T) = 4663 + 5111 = 9774

Substituting into the formula:

$$E_T = \frac{\sqrt{(95125.2)^2 + (118064.1)^2}}{9774}$$

$$= 15.5\%$$

Note: Only one error term can come from each stratum. If the stratum was substratified, then stratifications within each stratum must be combined to get a single estimate of total volume and error for that stratum before it can be combined with volumes and errors from other strata.

38.12 - Different Cruising Techniques, Two Strata

Use the following example to calculate total sampling error for two strata sampled using different tree cruising techniques. In the example, the first stratum was cruised using point sampling (sec. 35.4), and the second stratum was cruised using point-count/measure-plot sampling (sec. 37.5). For the two strata:

1. Stratum 1: (Point Sampling)

   V_1 = 37,820.9

   E_1 = 39.8%

   V_1 x E_1 = 1505271.82

2. Stratum 2: (Point-Count/Measure-Plot)

   V_2 = 37,728.0

   E_2 = 22.1%

   V_2 x E_2 = 833788.8

Estimated Total Volume (V_T) = 37,820.9 + 37,728.0 = 75,548.9

Substituting into the formula:

$$E_T = \frac{\sqrt{(1505271.82)^2 + (833788.8)^2}}{75548.9} = 22.8\%$$

Note: When combining volumes (and sampling errors for these volumes), all volumes must be in the same units. Cubic foot volumes cannot be combined directly with board foot volumes. Likewise, the errors based on cubic foot volumes cannot be combined directly to errors based on board foot errors.

38.13 - Different Cruising Techniques, Three Strata

In the following example, three strata, using different cruising techniques, were sampled. The first stratum was sampled using sample tree; the second using point sampling; and the third using point count/measure plot. The following statistics illustrate the method of calculations.

1. Stratum 1. (Sample Tree)

   V_1 = 9774.0

   E_1 = 15.5%

   V_1 x E_1 = 151,497.0

2. Stratum 2. (Point)

   V_2 = 37,820.9

   E_2 = 39.8%

   V_2 x E_2 = 1,505,271.82

3. Stratum 3. (Point Count/Measure-Plot)

   V_3 = 37,728.0

   E_3 = 22.1%

   V_3 x E_3 = 833,788.8

   Estimated Total Volume = 9744.0 + 37820.9 + 37728.0 = 85,292.9

$$E_T = \frac{\sqrt{(151,497.0)^2 + (1505271.2)^2 + (833,788.8)^2}}{85,292.9}$$

$$= 20.3\%$$

38.2 - Combining Additional Population Characteristics

Use the following formula for estimating mean diameters and height for the sale population:

$$\text{Mean Diameter} = \frac{\sum^{n} (DBH \times ET)}{\sum^{n} ET}$$

$$\text{Quadratic Mean Diameter} = \sqrt{\frac{\sum^{n} (DBH^2 \times ET)}{\sum^{n} ET}}$$

$$\text{Mean Height} = \frac{\sum^{n} (HT \times ET)}{\sum^{n} ET}$$

Where:

ET = Estimated total number of trees each sample tree represents for j^{th} stratum.

n = number of strata being combined

For plot based samples (Fixed Plot, Variable Plot)

ET = F_t x number of acres

For tree based samples (Sample Tree, 3p)

ET = F_t

38.21 - Additional Characteristic, Two Strata

In the following example, two strata, using two different cruising techniques, were sampled. The first stratum was cruised using point sampling (sec. 35.4); the second stratum was cruised using point-count/measure-plot sampling (sec. 37.5). The following example illustrates calculation of the statistics:

1. Stratum 1. (Point)

   Number of acres = 18

$$\sum^{n} F_t = 109.7$$

$$\sum^{n} (DBH \times ET) = \sum^{n} (DBH \times F_t \times acres) = 22{,}134.61$$

$$\sum^{n} (DBH^2 \times ET) = \sum^{n} (DBH^2 \times F_t \times acres) = 257{,}425.7$$

$$\sum^{n} (HT^2 \times ET) = \sum^{n} (HT \times F_t \times acres) = 120{,}644.8$$

$$\sum^{n} ET = \sum^{n} (F_t \times acres) = 1974.6$$

2. Stratum 2. (Sample Tree)

$$\sum^{n} F_t = 428.0$$

$$\sum^{n} (DBH \times ET) = \sum^{n} (DBH \times F_t) = 5117.5$$

$$\sum n(DBH \times ET) = \sum (DBH^2 \times F_t) = 63{,}298.5$$

$$\sum^{n} (HT \times ET) = \sum^{n} (HT \times F_t) = 27{,}379.5$$

$$\sum^{n} (ET) = \sum^{n} F_t = 428.0$$

$$Mean\ Diameter = \frac{(22134.61 + 5117.5)}{(1974.6) + (428.0)}$$

$$= 11.3\ inches$$

$$Quadratic\ Mean\ Diameters = \sqrt{\frac{(257425.7 + 63298.5)}{(1974.6) + (428.0)}}$$

$$= 11.6\ inches$$

$$Mean\ Height = \frac{(120644.8 + 27379.5)}{(1974.6) + (428.0)}$$

$$= 61.6\ feet$$

38.3 - Applications of Combined Statistics

Use combined statistics to determine the final sale characteristics after all strata in a sale have been cruised.