Forest Service Handbook
National Headquarters - Washington Office
Washington, DC

Forest Service Handbook 2409.12a – Timber Volume Estimator Handbook
Chapter 30 - Analysis of Data

Amendment: 2409.12a-1993-1

Effective date: December 23, 1993

Duration: This amendment is effective until superseded or removed.

Approved by: Jack Ward Thomas, Chief

Date approved:

Responsible Staff:

Last Change: None

Superseded Document(s):

Digest: Following is an explanation of the changes throughout the directive by section.

2409.12a: Establishes new Timber Volume Estimator Handbook that provides Service-wide standards and instructions for preparation of equations or tables used to estimate the timber content of trees.

Table of Contents

30.2 - Objective

To describe the derivation and verification of approved stem profile estimators for use by Forest Service mensurationists to ensure accurate and consistent volume estimators used in all timber management functions.

30.3 - Policy

Construct volume estimators in a form that can be used to produce the volume of a tree from ground to tip, as well as to any specified merchantable height or diameter. Product estimates, such as Scribner board feet either shall be a by-product of the total cubic volume or shall be developed from estimates of segment diameter and heights. After developing new volume estimators, implement them in all functions of timber management including cruising, stand exams, and yield tables for forest planning.

31 - Estimator Types and Uses

Consider the types of estimator available for the timber estimation and the potential uses of each within a Region or Station.

31.1 - Stem Profile Equation

For most situations, select a stem profile equation. Consider both the advantages and disadvantages of stem profile methods.

1. Advantages.

   a. The equations provide great flexibility in volume calculation.

   b. Volume can be calculated to any desired standard of merchantability.

   c. Primary products related to log length and diameter can be estimated from diameters calculated without the need for separate product estimators.

   d. Stem volumes generated by the integral of a profile equation may be more accurate than those predicted by a direct volume estimator.

2. Disadvantages.

   a. Stem profile equations are complex in their derivation and use and are likely to require use of nonlinear least squares (NLLS) regression techniques.

   b. Introduction of independent variables other than diameter breast height and height will likely require more work and more computer time than fitting a volume equation.

c. Stem profile equations require a higher degree of training and perceptiveness than fitting volume equations.

31.2 - Direct Volume Estimator

Prepare direct volume estimators only when it is impractical to use stem profile equations. Prepare them in the form of volume tables, alignment charts, or volume equations. Consider both advantages and disadvantages of direct volume estimators.

1. Advantages.

a. They are less expensive to derive than stem profile equations.

b. Direct volume estimators can be prepared with less skill and effort than stem profile equations.

2. Disadvantages.

a. Direct volume estimators are characteristically loss flexible in use than stem profile equations.

b. Separate direct estimators for different potential products are necessary.

c. When a change in merchantability occurs, the estimator may become obsolete.

31.3 - Product Estimators

Use potential product estimators only to estimate product yield which is very frequently misclassified as volume. Do not use them to estimate volume in the tree, only quantity of product expected to be manufactured from the tree. For example, board feet of sawn lumber is commonly estimated. Consider both advantages and disadvantages of product estimators.

1. Advantages.

a. They are simpler and less expensive to derive when only one product needs to be estimated.

2. Disadvantages.

a. If the assumptions change because of changing technology, the estimator must be redone using the basic tree measurements.

b. Direct potential product estimators have all of the disadvantages of direct volume estimators.

c. The product estimator may become obsolete with changes in manufacturing technology.

d. Assumptions about the manufacturing process are implicit, undocumented, and unknown to the eventual user of the estimator.

31.4 - Selecting the Method Used

Review advantages and disadvantages outlined in sections 31.1-31.3. Choose a method by evaluating the effort required to obtain a stem profile equation and the effort required to prepare a set of merchantable volume and potential product equations that will produce the needed information. Evaluate the expected breadth of application of the results and the duration of application. If only a limited use for a short time is expected, and if having a usable estimator very promptly is important, choose a direct estimator. However, recognize that short cuts taken out of a sense of urgency may lead to inferior analyses that must be done again at a later time. If the volume estimators derived are likely to be used well into the future, use stem profile methodology.

32 - Preparation of Data

32.1 - Data Evaluation

Check documentation and the adequacy of the data (Ch. 10). Prepare a summary containing the location, dates, and other conditions of measurement supplemented with a detailed report describing the protocol of measurement. Check the species, age classes, and range of heights and diameters of the trees measured to determine suitability of the data base for the proposed volume estimator. Evaluate the number and kinds of abnormal trees (32.22). Prepare simple tables showing frequency of tree data by four or five diameter classes, height classes, and other classes for use in later volume estimation. For a regression analysis, ensure a rectangular (not normal) distribution of tree sizes and sizes and form, if a measure of form is to be included in the estimator. As a first estimate of sample size, use samples of 200 or more trees.

32.2 - Field Data Edit

Edit computer data files for accuracy of transcriptions from field sheets. Check for missing records and delete the tree if key elements of the data are missing. If the initial computer edit program is available, rerun it to verify that the data have been corrected after the preliminary edit.

32.21 - Reasonableness Edits

Use a computer edit program that checks for completeness of records and reasonableness of entries. Ensure it verifies all header entries are complete. Verify that the individual measurements form ascending or descending series and the steps are fairly uniform. If the measurements do not form a series, check them to ensure they are due to bumps, hollows or other stem abnormalities. Establish standards for rejection of pairs of measurements and of entire

trees. Use higher standards for measurements of independent variables than for measurements of dependent variables. Also, use higher standards for measurements that may be repeated in a growth study.

32.22 - Abnormal Trees

Evaluate forking or other common abnormalities. Recognize that rejection of trees with abnormalities means that the volume estimator will not correctly estimate trees with similar abnormalities. Trees might not be rejected for species where an abnormality such as forking is common but they should be rejected if the abnormality is uncommon. For example, broken tops may be rare in second-growth stands but nearly universal in old-growth stands. In such cases, reject broken top trees for second growth estimators and not for old growth estimators. Develop appropriate standards for rejection of abnormal trees in all projects.

Estimate volumes of abnormal trees separately if a large enough sample is measured. If their volumes are consistently and significantly different from other trees of the same species and the abnormality is prevalent in significant parts of the area or interest, prepare separate volume estimators.

32.23 - Graphic Edits

Use individual tree plots to quickly identify pairs of measurements that appear to be unusual, and to identify trees with broken tops or other abnormalities. Use this kind of editing when it is more effective than defining limits and flagging the measurements exceeding them. Identify all unusual and unforeseen conditions that are indicated.

Use scatter diagrams for groups of trees to suggest the average shape for the group, and flag the presence of individual trees that deviate widely from this average. Use changes in average shapes among groups of trees to suggest functions to be tested in developing the volume estimator. Use the simple diagrams to identify transformations of basic measurements that have straight line relations with other measured variables. Use X-Y scatter diagrams both to edit data and as an early step in data analysis.

32.3 - Preparing Data for Analysis

Prepare data for analysis at the time edits are made.

32.31 - Field Computation

Convert dendrometer and relaskop data to diameters, heights, and distances in the field, and correct any problems before leaving the instrument setup point (sec. 14.47).

32.32 - Average Diameters

If more than a single diameter is recorded at any point other than breast height, average them and save at least two decimals. These may help compensate for out-of-roundness at different heights

above the ground. Assess the degree of out-of roundness before deciding the usefulness and accuracy of using two breast height diameters and then specify how to record. Identify the reasons for saving two breast height diameters, such as standing trees measured with diameter tapes and felled trees measured with calipers.

32.33 - Bark Thickness

If for any reason the recorded bark thickness is not the sum of two measurements, clearly indicate this in the header information. Use bark thickness to generate inside-bark or outside-bark diameters from the measured diameter.

32.34 - Relative Diameter

Determine relative diameters by dividing the inside bark diameters recorded at each measurement point by an inside bark basal diameter, most commonly diameter breast height (dbh). If possible, do not use outside bark measurement because bark ratio is not constant at all measurement points along the bole. Use relative diameter measured outside bark only when it is not practical to get inside bark measurements.

32.35 - Relative Height

Express relative height as the distance from tip of a tree to height of measurement point divided by either total height or by total height minus breast height. Either makes zero relative height coincide with zero relative diameter, and the latter makes the 1.0 points coincide. Use this convention for relative height to simplify equations in the volume estimator.

32.4 - Working Files

Maintain at least two kinds of working files. In the first file known as the profile working file, include the record of tree data (heights, diameters, and bark-thicknesses at various levels in the tree) (sec 32.41). In the second, include the record of section and tree volumes (sec 32.42).

32.41 - Profile Working Files

In profile working files, include one line of data for each height of measurement (elevation). Include about 10 spaces for tree and plot identification (including tree species) in each line. Follow this with tree dbh and tree height as a minimum and optionally, additional tree measurement data. Put any plot or location data that is to be used in the analysis next in the sequence. Record the elevation number, starting with zero for ground level to ensure that the elevation number of the top of each section coincides with the section number; "one" usually is the stump, making breast height elevation usually "two" or "three."

Follow the elevation numbers with the cumulative heights in feet, the diameters outside bark, the double bark thickness and the diameter inside bark. Lastly enter the relative heights and diameters (both outside and inside bark). Use a separate column with indicators of first and last measurement of each tree when there are variable numbers of lines of data for trees.

32.42 - Section Volume Files

Calculate section volumes using the profile working file. Use procedures that avoid bias such as the following:

1. Since few trees are cylindrical, conduct a simple study to find a multiplying factor such as 1.3 or 1.5 that removes the bias of the textbook assumptions. Often stem profiles from the top of the stump to an inflection point 15 percent to 25 percent of the height of the tree are concave. Calculation of the volume of stem segments in this zone as frustra of cones or paraboloids (FSH 2409.11a, sec. 11.1) can give biased estimates. Keep the measurements close together to keep this bias at one percent or less.

2. Tree tips are usually assumed to be cones but are often somewhat more convex. Unless a more accurate multiplier has been determined in the field, assign volumes halfway between a cone (K/3) and a paraboloid (K/2) to reduce potential bias for tips of trees. Add a column of calculated volumes for each segment to the profile working file.

32.43 - Tree Volume Files

Prepare a separate file of tree volumes. Enter tree information similar to that in the profile working files, as well as any other information that may be used as an independent variable. In addition to stem volumes from stump to various merchantable tops, include stump volume and tip volumes above the various merchantable tops.

32.44 - Temporary Files

If temporary files are created in generating the working files, label these clearly and delete them when no longer needed. Consider the usefulness of preserving intermediate results as part of the working files before the temporary files are deleted. Store the temporary files with the data files, but in a manner that precludes confusing them with the permanent information.

33 - Stem Profile Equations

Stem profile equations are a mathematical expression relating diameter of a tree bole to height above ground at any point between ground level and the tree's tip. Recognize that the terms taper equation and taper function are often used interchangeably in scientific literature. Use of the term profile equation is recommended. Estimate parameters of the equation by regression. Use diameter, or a transformation of diameter, as the regression dependent variable. Use height to the point of prediction, or a transformation of such height, as the basic independent variable. Include other independent variables, if needed, to describe variation in tree form due to site conditions or stand density. Ordinarily, make the dependent regression variable the ratio of squared stem diameter to squared dbh, since scaling of diameters is necessary to accommodate trees of all sizes, and makes residual variance about regression homogeneous.

After estimating equation parameters, rearrange stem profile equations algebraically to predict squared stem diameter as a function of squared dbh and relative height. This facilitates computing stem volume by integrating the stem profile equation.

33.1 - General Form of a Stem Profile Model

Use the general form of a stem profile model which is:

$$d = f[D, h, H]$$

where D is tree dbh, H is total tree height, and d is the desired bole diameter at a distance h from the ground. Stated simply, bole diameter is a function of the dbh and relative height of that diameter. Use relative diameters and heights when fitting equations to reduce the effect of tree size on bole diameter predictions.

33.2 - Desirable Characteristics

From among the different model forms available (sec. 33.32), choose the best model based on desirable properties. The model should be flexible, accurate, and able to produce results consistent with expectations of tree shape.

33.21 - Model Flexibility

Plan for maximizing flexibility of the model to fit the wide variety of geometric shapes a tree bole may approximate. For example, the lower bole may be concave and the upper portion a convex paraboloid. Develop a flexible model that conforms to the varied bole shapes which may be concave near the base, convex in the middle, and conical or paraboloid at the top.

33.22 - Model Accuracy

Strive for a model that produces accurate diameter estimates all along the bole from the ground to the tip. Since models are less accurate at the base of the tree due to butt flair and at the tip due to bole irregularities, aim for acceptable accuracy from 2 percent of the total height to about 90 percent of the total height. Do not expend major effort in trying to fit the stump and top of the tree, since improvement in diameter estimates in these portions of the tree is not cost effective.

33.23 - Expectations

Regardless of equation complexity, ensure that the predictions yield results consistent with user expectations about tree profiles. Solve the profile function directly for the following three variables: 1) diameter (d) at a measured height (h); 2) height (h) to a point of known bole diameter (d); and 3) bole cubic-foot volume calculated from the stump to the point of bole merchantability. Use calculus to integrate the equation over the desired height limits and ensure that the model meets the following expectations:

1. DBH. At a height of 4.5 feet, the predicted outside bark bole diameter is expected to equal the measured dbh. Exact results at dbh, help maintain the model credibility with the user. If the profile equation does not naturally yield this result, condition the basic model to equal D when h = 4.5 or:

$$f(x) = D \text{ when } h = 4.5; \quad X = (H - 4.5)/H.$$

2. Tip of the tree. At the tip of the tree the diameter is expected to be zero. When h = H the profile equation should predict a zero diameter or:

$$f(x) = 0 \text{ when } h = H; \; X = (H - H)/H.$$

If the model does not produce the result, condition the base model to yield this result prior to fitting it to the data.

3. Smaller diameters at greater heights. The profile equation is expected to predict successively smaller diameters as the height increases up the bole. Although irregularities on the bole are common, do not try to predict convolutions on the bole, rather ensure the model predicts a smooth trend of decreasing diameters from the base to the tip.

33.24 - Other Considerations

In addition to equation flexibility, accuracy, and prediction expectation, consider the following items before deciding on a particular model form to use:

1. Use $Y = (d/D)^2$ or $Y = (d/D)$. When deriving a stem profile model make an early choice of the dependent variable, $Y = (d/D)$ or $Y = (d/D)^2$.

When possible, choose $(d/D)^2$ because cubic volume is a function of the mean cross-sectional area of the bole which, in turn, is directly related to $(d/D)^2$. Thus, obtain bole volume by directly integrating the function of $(d/D)^2$.

If a profile equation using $Y = (d/D)$ is chosen, square the entire function prior to integration to predict cubic volume. The squaring process may become a complex of algebraic manipulations, therefore, avoid this process when possible.

2. Integrable in closed form. If possible, choose a model that is integrable in closed form. If the equation cannot be integrated directly, then use a numerical integration technique to calculate volume. Integration requires many calculations per tree, which causes a higher computer cost. Do not, however, avoid profile functions because of this complexity; simply recognize the difficulty and possible increased expense.

3. Easi1y solved merchantable height. If possible, choose equations that can be algebraically solved for h. Profile function may be used to estimate an average merchantable height (h) given a known, dbh, tree height, and top diameter. Equations that must use numerical

methods to solve for h involve the same calculation and cost problems as discussed in item 2 above.

33.3 - Profile Model Examples

Choose profile models from the following examples of models that range from simple to complex. Generally, the simplest models are not suitable for most applications since they are not accurate over the full length of the bole. The more complex models often give like results over much of the bole length, and usually differ only at the ends of the bole.

33.31 - Variables

Variables unique to a particular model are defined with that model as it appears. The variables used for the models and equations are presented in Table 01.

33.31 - Table 01

Notation for Variables Used

D = dbh = Tree diameter breast high (4.5 feet).
H = Tree height in feet.
d = A diameter being estimated at some point h on the bole.
dob = diameter outside bark.
dib = diameter inside bark.
h = Height above the ground to d.
L = Lower limit of bole merchantability, feet above the ground.
U = Upper limit of bole merchantability, feet above the ground.
K = 0.005454 - converts diameter squared in square inches to square feet.
b = Coefficients estimated from a specific data set.

33.32 - Models

Select from among the 10 standard tree profile models presented in this section. They range from models of very simple form and poor accuracy to complex and accurate models. Exhibit 01 assigns numerical evaluations to some desirable model characteristics while designating others with a simple yes or no answer. Models which are rated as simple lack accuracy, while those that are rated accurate are the most complex.

No one model is best for all circumstances. Exhibits 02-12 illustrate the 10 models. Models in exhibits 08-12 usually give accurate results for a wide variety of species and geographic locations. when deriving stem profile equations for a particular species or area, test a number of model forms for suitability.

33.32 - Exhibit 01

A Tabular Evaluation of Models

Model	Simplicity	h=f(d)	Closed for V	Accu-racy	Flex-abil-ity	Reli-abil-ity	Upper bole d	Vol sets needed
Ormerod	5*	Yes	Yes	1	1	1	No	Yes
Behre hyperbola	5	Yes	Yes	1	1	1	No	Yes
Kozak et al.	4	Yes	Yes	2	2	2	No	No
Amidon	3	No	Yes	3	3	3	No	No
Demaerschalk compatible	3	Yes	Yes	2	2	2	No	Yes
Biging	2	Yes	Yes	--- not rated---			No	No
Demaerschatk and Kozak	1	No	No	4	5	4	Yes	No
Bruce et al.**	3	No	Yes	4	5	5	No	No
Schlaegel form class	2	No	Yes	5	5	5	Yes	No
Max - Burkhart**	2	Yes	Yes	5	5	5	No	No

*Each equation has been evaluated for a number of qualities. The numeric rating denotes the degree to which the quality is met with 5 being the highest level of attainment. Yes/no denotes whether or not the function possesses the quality.

**Both equations are highly regarded in application. Numerous applications are referenced in the literature.

33.32 - Exhibit 02

The Ormerod Model

1. Simple model. The simplest form of the Ormerod model is the generalized paraboloid.

$$d/D = X^b$$

where

$X = (H - h)/(H - 4.5)$. If $b < 1.0$, the bole shape will be parabolic. If $b > 1.0$, the shape will be neiloidal.

Solving for h given d:

$$h = H - [(H - 4.5)(d/D)^{1/b}].$$

Ormerod volume function:

$$V = \frac{KD^2(H-4.5)}{Z}\left[\left(\frac{(H-L)}{H-4.5}\right)^Z - \left(\frac{(H-U)}{H-4.5}\right)^Z\right]$$

where:

$$Z = 2b+1$$

2. Expanded Ormerod model. The simple form of the Ormerod model assumes one bole shape extending from the butt to the tip and is a poor profile model. Tree boles change shape from the butt to the tip. Hence, the model may be expanded as a step function, with separate equations fitted for each bole segment. This form of the model offers increased flexibility; however, a diameter (D_i) and height (H_i) must be measured on each segment, making it's use very expensive.

$$d_i = (D_i - C_i) X_i^{b_i} + C_i$$

where:

d_i = estimated diameter at height on segment i

D_i = measured diameter at height k on segment i

H_i = height to the top of segment i

C_i = diameter intercept of segment i

$$X_i = \frac{(H_i - h)}{(H_i - k)}$$

33.32 - Exhibit 03

The Behre Hyperbola

1. Profile model: A simple model useful where less accurate estimates are needed.

$$\frac{d}{B} = \frac{X}{b_0 + b_1 X}$$

where:

B = *A basal diameter in inches; may be d.b.h*

$X = (H-h)/H$

H = *The bole length in feet measured from the height of the basal diameter to the tip of the tree*

h = *Height above the basal diameter to d and a restriction that* $b_0 + b_1 = 1$

2. Solving for h given d:

$$h = H[1 - (b_0 d)/(B - b_1 d)]$$

The Behre Hyperbola

3. Volume function:

$$V = \frac{KB^2H}{b_1^3}\left\{b_1(X_b - X_t) + 2b_0 Ln\left[\frac{b_0 + b_1X_t}{b_0 + b_1X_b}\right] + b_0^2\left[\frac{1}{b_0 + b_1X_t} - \frac{1}{b_0 + b_1X_b}\right]\right\}$$

where:

$X_b = (H-L)/H$

$X_t = (H-U)/H$

and

Volume from the stump to the height of B is computed as a simple geometric solid

4. Considerations.

a. May be applicable in the Northwest.

b. Contains two coefficients, but uses essentially a single parameter, since $b_0 = 1 - b_1$, thus, $b_1 = (d - BX)/[d(1 - X)]$.

c. Fit the equation by finding b_1 for each pair of d and X and the average of the b's to get a single b_1. As b_1 increases from 0 to 1, the hyperbola represents trees of higher form class.

d. For use, good estimates of standing tree form class are required.

33.32 - Exhibit 04

The Kozak, et al. Model

1. Profile model. The taper model proposed by Kozak, et al., is a parabolic function which assumes the entire bole is shaped as a parabola. Butt swell is assumed to be a slight and unimportant component of tree volume. The model is:

$$\frac{d^2}{D^2} = b_1 \frac{(h-H)}{H} + b_2 \frac{(h^2-H^2)}{H^2}$$

2. Solving for h given d. The height h to a given top diameter d if D and H are known is given by:

$$h = \left\{ -b_1 - \left[b_1^2 - 4b_2 \left(b_0 - \frac{d^2}{D^2} \right) \right]^{1/2} \right\} (H/2b_2)$$

where:

$$b_0 = -b_1 - b_2$$

3. Kozak, et al. Volume function:

$$V = KD^2 \left[b_0 (U-L) \; \frac{b_1 (U^2-L^2)}{2H} + b_2 \frac{(U^3-L^3)}{3H^2} \right]$$

where:

$$b_0 = -b_1 - b_2$$

4. Considerations.

a. It may be useful where tree form closely approximates a parabola or where the target population consists of small diameter trees having minimum butt swell. In these situations, more complex models may not produce a better fit.

b. Do not use if precise volume estimates of individual trees are needed.

33.32 - Exhibit 05

The Amidon Model

1. Profile Model:

$$d = b_1 \frac{D(H-h)}{(H-4.5)} + b_2 \frac{(H^2-h^2)(h-4.5)}{H^2}$$

2. Solving for h given d. Use an interactive approximation procedure to predict a merchantable height given a merchantable top diameter. No analytical solution exists.

a. The Newton-Raphson iteration. Use the simple iterative approximation, the Newton-Raphson iteration. Set the profile equation equal to the diameter to be solved and find the respective height by iteratively guessing the solution. Use a five step process:

(1) Make an initial guess at the supposed height h_0;

(2) Solve the profile equation for h_0; compute $f(h_0)$;

(3) Find the first derivative in h of the profile equation and solve f or h_0; compute $f'(h_0)$;

(4) Compute the new height $h_1 = h_0 - f(h_0)/f'(h_0)$;

(5) Set $h_0 = h_1$ and repeat steps 1 through 4 until the correction factor $C = -f(h_0)/f'(h_0)$ ceases to change by some previously specified value, say 0.002, 0.1, 0.5 or maybe 1.0 foot.

b. Derivative in h of Amidon Model:

$$f'(h) = b_2 \frac{(H^2-3h^2+9h)}{H^2} - \frac{b_1 D}{(H-4.5)}$$

The Amidon Model

3. Amidon Volume Function:

$$V = K\Big[Z_0^2(U-L) + Z_0Z_1(U^2-L^2) + (Z_1^2+2Z_0Z_1)(U^3-L^3)/3$$
$$+ (Z_0Z_3+Z_1Z_2)(U^4-L^4)/2 + (Z_2^2+2Z_1Z_3)(U^5-L^5)/5$$
$$+ (Z_2Z_3)(U^6-L^6)/3 + Z_3^2(U^7-L^7)/7\Big]$$

where:

$$Z_0 = \frac{b_1DH}{2(H-4.5)} - \frac{4.5b_2}{2}$$

$$Z_1 = \frac{b_2}{2} - \frac{b_1D}{2(H-4.5)}$$

$$Z_2 = \frac{4.5b_2}{2H^2}$$

$$Z_3 = -\frac{b_2}{2H^2}$$

4. Considerations.

a. For five California conifers this model accurately predicted bole diameters of and ranked first over five other models of proven accuracy including the Max and Burkhart, Bruce, et al., Kozak, et al., and Demaerschalk exhibited in section 33.32.

b. Fit the model inside bark for given H and D outside the bark. So when $h = 4.5$, the predicted diameter is $d = b_1D$. Thus, the double bark thickness at breast height is $B = D(1 - b_1)$.

33.32 - Exhibit 06

The Demaerschalk Model

1. Profile Model:

$$\frac{d^2}{D^2}=10^{2b_0}\left[D^{(2b_1-2)}\right](H-h)^{2b_2}H^{2b_3}$$

2. Solving for h given d. The height h to a given top diameter d if D and H are known is given by:

$$h = H-\left[10^{-b_0}dD^{-b_1}H^{-b_3}\right]^{1/b_2}$$

3. Demaerschalk volume function:

$$V = K(10)^{2b_0}D^{2b_1}H^{2b_3}(X_1^Z-X_2^Z)$$

where:

$$Z = (2b_2+1)$$

$$X_1 = H-L$$

$$X_2 = H-U$$

If a tree volume equation of the form:

$$\ln(V) = a + b(\ln(D)) + c(\ln(H))$$

already exists, then convert it into a logarithmic taper equation of the form:

$$\ln(d) = b_0 + b_1(\ln(D)) + b_2[\ln(H - h)] + b_3(\ln(H)),$$

where:

a, b, and c are the volume equation coefficients and

$b_0 = (10^a pc/K)^{1/2}$;

$b_1 = b/2$;

$b_2 = (pc - 1)/2$;

$b_3 = (1 - p)c/2$;

p = a "free" parameter which provides the compatibility between the volume equation and the profile equation.

33.32 - Exhibit 06 -- Continued

The Demaerschalk Model

Or derive and integrate an alternative profile function of the form:

$$\ln(d) = b_0 + b_1(\ln(D)) + b_2[\ln(H - h)] + b_3(\ln(H)),$$

providing a compatible volume equation:

$$\ln(V) = a + b\ln(D) + c\ln(H)$$

where:

$$a = \ln\left[\frac{K(10)^{2b_0}}{2b_2+1}\right]$$

$$b = 2b_1$$

$$c = 2b_2 + 2b_3 + 1$$

4. Considerations.

a. Use this model in situations where intermediate levels of accuracy and complexity meet the needs. For example, if the intent is to predict either total cubic foot volume or cubic foot volume to a pulpwood top, this is acceptable.

b. The profile equation is usually not accurate over the entire bole length for predicting bole diameter.

c. This model integrates mathematically to give the exact same volume estimate predicted from an already existing total stem volume equation, and demonstrates the concept of compatible profile and volume equations.

33.32 - Exhibit 07

The Biging Model

1. Profile model. The model is based on the integral form of the Chapman-Richards function. Biging's model is:

$$d = D\left\{b_1 + b_2 \ln\left[1 - \lambda\,(h/H)^{1/3}\right]\right\}$$

where:

$$\lambda = \left[1 - \exp(-b_1/b_2)\right]$$

2. Solving for h given d. To predict a merchantable height h to a given top diameter d:

$$h = H\left\{\left[1 - \exp\left(\frac{(d - b_1 D)}{b_2 D}\right)\right]/\lambda\right\}^3$$

33.32 - Exhibit 07 -- continued

The Biging Model

3. Biging Volume Function: Use the following formula:

$$
\begin{aligned}
V &= K\int_L^U d^2(h)\,dh \\
&= K_1 H(U1-L1) \\
&+ K_2 H\left[-\frac{3}{\lambda^3}\right]\left[q\ln(q) - q - q^2 - \ln(q) + \frac{q^2}{2} + \frac{q^3}{3}\ln(q) - \frac{q^3}{9}\right]\Bigg|_{L1}^{U1} \\
&+ K_3 H\left[-\frac{3}{\lambda^3}\right]\left[\frac{q^3\ln^2(q)}{3} - q^2\ln^2(q) + q\ln^2(q) - \frac{2}{9}q^3\ln(q)\right. \\
&\left. + q^2\ln(q) - 2q\ln(q) + \frac{2}{27}q^3 - \frac{q^2}{2} + 2q\right]\Bigg|_{L2}^{U2}
\end{aligned}
$$

where:

L = *height to a stem base point; could be stump*

U = *height to a top point in the upper stem*

h = *height to a point between L and U*

$\lambda = [1-\exp(-b_1/b_2)]$

$q = [1-\lambda(h/H)^{1/3}]$

$L1 = L/H$

$L2 = [1-\lambda(L/H)^{1/3}]$

$U1 = U/H$

$U2 = [1-\lambda(U/H)^{1/3}]$

$K1 = Kb_1^2D^2$

$K2 = K(2b_1b_2D^2)$

$K3 = Kb_2^2D^2$

The Biging Model

4. Considerations.

a. Use this model in situations where need for intermediate accuracy exists.

b. The Biging equation is complex.

c. Fit this model to bole diameters inside the bark even though dbh is measured outside the bark. Thus, when h=H, then d=O, and when h=O, then d=b_1(dbh). Interpret coefficient b_1 to be the ratio of dib at the base of the tree to dbh. The equation may be fitted to bole diameters outside the bark, and if so, interpret the coefficient b_1 to be the ratio of bole dob at the stem base to dbh.

d. This model has applicability for ponderosa pine, Douglas-fir, white fir, red fir, sugar pine, and incense cedar. It equaled the Max-Burkhart model for prediction accuracy over the range of the collected data. The Biging model may be a good alternative to the Max-Burkhart model for some applications

33.32 - Exhibit 08

The Demaerschalk and Kozak Dual-Equation Model

1. The profile model. Use two models for different portions Of the tree. Since a single profile equation does not adequately describe bole profile from ground to tip, use one model to describe upper bole profile and another to describe lower bole profile.

These profile equations predict tree profile inside bark since this is the usual variable of interest when predicting tree volume. Since dbh inside bark (DIB) is not measured directly, predict it from the data using the model:

$$DIB = b_0 + b_1 DBH + b_2 (DBH)^2.$$

Likewise, predict the diameter inside bark at the inflection point (DI) from:

$$DI = C_0 + c_1 DIB + c_2 (DIB)^2.$$

a. The model for the bole from the tree tip down to the inflection point is:

$$\frac{d}{DI} = \left[\left[\frac{h/H}{R} \right]^{b_1} b_2^{\left(1 - \frac{h/H}{R}\right)} \right]$$

where:

h = the distance from the tree tip to d

DI = diameter inside bark at the inflection point

R = distance of the inflection point from the tip relative to H

The Demaerschalk and Kozak Dual-Equation Model

b. The bottom model from the inflection point to the ground is:

$$\frac{d}{DI} = \left[b_3 - (b_3 - 1)\left[\frac{1-h/H}{I}\right]^{b_4}\right]$$

where:

$I = 1 - R$, the relative height of the inflection point from ground level

$$b_3 = \frac{\frac{DIB}{DI} - \left[\frac{1 - \frac{H-4.5}{H}}{I}\right]^{b_4}}{1 - \left[\frac{1 - \frac{H-4.5}{H}}{I}\right]^{b_4}}$$

2. Solving for h given d, D, and H.

a. For the bottom model.

$$h = H\left[1 - I\left[\frac{b_3 - d/DI}{b_3 - 1}\right]^{1/b_4}\right]$$

where:

h = the distance from the top

b. For the top model. Solve for h using an iterative procedure, such as Newton-Raphson, since no analytical solution exists. The first derivative of the top model with respect to h is:

$$F'\left[f\left[\frac{d}{DI}\right]\right] = \left[\frac{b_2}{HR^{b_1}}\right]\left[b_1 h^{(b_1-1)} b_2^{-\left(\frac{h}{RH}\right)} - \left[\frac{h^{b_1}}{HR}\right] b_2^{-\left(\frac{h}{RH}\right)} \ln(b_2)\right]$$

The Demaerschalk and Kozak Dual-Equation Model

3. Computing cubic volume. Obtain cubic volumes for the bottom model by squaring the model and integrating over desired limits. Obtain the top model volume by finding the top and bottom diameters of small (0.5 foot) segments using the derived taper equation, computing the segment volumes using Smalian's equation, then summing the segment volumes. Add the top volume and bottom volume to get tree volume.

To make volume computations uniform for the whole tree, solve both models by summing small segments. Therefore, no integral form of the bottom is presented.

4. Considerations.

a. The models join at the inflection point, the point on the bole where the shape of the bole changes from the neiloidal base to the parabolic top. For each species group the inflection point differs. But, for all species it ranges from 20 to 25 percent of the tree height measured from ground level.

b. Condition the models so the predicted diameter is 0 at the tree tip, equals dbh at 4.5 feet, both are smooth and equal DI at the inflection point.

c. Since the bole inflection point cannot be visually determined, determine it for each species group prior to fitting the equations. That point is constant for all trees in that group. Determine R before fitting the equations.

d. The bottom model has two coefficients and two restrictive conditions; therefore, the bottom equation is unique for each tree. Coefficient b_4 is determined for each tree by iteration and b_3 is set equal to a function of b_4. Consider this to be a drawback in using this prediction system.

33.32 - Exhibit 09

The Bruce et al. Red Alder Model

1. Profile Model:

$$\frac{d^2}{D^2} = b_1 (X^{3/2}) (10^{-1})$$
$$+ b_2 (X^{3/2} - X^3) D (10^{-2})$$
$$+ b_3 (X^{3/2} - X^3) H (10^{-3})$$
$$+ b_4 (X^{3/2} - X^{32}) DH (10^{-5})$$
$$+ b_5 (X^{3/2} - X^{32}) H^{1/2} (10^{-3})$$
$$+ b_6 (X^{3/2} - X^{40}) H^2 (10^{-6})$$

where:

$$X = \left[\frac{(H-h)}{(H-4.5)} \right]$$

The Bruce et al. Red Alder Model

2. <u>Solving for h given d</u>. Since the profile model is a complex polynomial, a unique solution to estimate h given d, D, and H does not exist. Use an alternative approximation procedure (as for Amidon, 32.32, ex. 05) which, necessitates finding the first derivative of the profile function with respect to h:

$$
\begin{aligned}
f'(h) = & -b_1\left[\frac{3M^{1/2}}{2N^{3/2}}\right](10^{-1}) + b_2\left[\frac{3M^2}{N^3} - \frac{3M^{1/2}}{2N^{3/2}}\right]D(10^{-2}) \\
& + b_3\left[\frac{3M^2}{N^3} - \frac{3M^{1/2}}{2N^{3/2}}\right]H(10^{-3}) \\
& + b_4\left[\frac{32M^{31}}{N^{32}} - \frac{3M^{1/2}}{2N^{3/2}}\right]DH(10^{-5}) \\
& + b_5\left[\frac{32M^{31}}{N^{32}} - \frac{7M^{5/2}}{2N^{7/2}}\right]H^{1/2}(10^{-3}) \\
& + b_6\left[\frac{40M^{39}}{N^{40}} - \frac{3M^{1/2}}{2N^{3/2}}\right]H^2(10^{-6})
\end{aligned}
$$

where:

$M = (H-h)$

$N = (H-4.5)$

The Bruce et al. Red Alder Model

3. Bruce et al. Cubic Volume Function:

$$V = -KD^2(H-4.5)\left[\frac{2A}{5}(P^{5/2}-Q^{5/2}) + \frac{B}{4}(P^4-Q^4) + \frac{C}{33}(P^{33}-Q^{33}) + \frac{F}{41}(P^{41}-Q^{41})\right]$$

where:

$$P = [(H-U)/(H-4.5)]$$

$$Q = [(H-L)/(H-4.5)]$$

$$A = [b_1(10^{-1}) + b_2D(10^{-2}) + b_3H(10^{-3}) + b_4DH(10^{-5}) + b_5H^{1/2}(10^{-3}) + b_6H^2(10^{-6})$$

$$B = [b_2D(10^{-2}) + b_3H(10^{-3})]$$

$$C = [b_4DH(10^{-5}) + b_5H^{1/2}(10^{-3})]$$

$$F = [b_6H^2(10^{-6})]$$

4. Considerations. - Bruce et. al., were the first researchers to develop a profile equation to fit the butt swell at the base of the tree. They accomplished this with a general polynomial model using both fractional and high powers. They conditioned the model to predict a diameter equal to dbh at a height of 4.5 feet and one of zero at the tip of the tree (when h=H) . Also, they were the first to use d^2 as the predictor variable instead of d, reasoning that volume estimation is the main purpose of a profile equation and that cubic volume is proportional to the average cross-sectional area of the stem, which d^2 represents.

The Bruce et. al., model is known to be an accurate model for a wide range of species and tree sizes. The fractional powers and high powers they used were chosen to fit their basic model to the red alder data. For other sets of data, other powers should be investigated. To fit the Bruce et. al., model to other species data, test a range of both fractional powers and high powers, then select those powers that best fit the data.

33.32 - Exhibit 10

The Schlaegel Form-Class Model.

1. Profile Model. This form class model is developed from two separate models. The models are conditioned so they join and are equal at a bole height of 17.3 feet, the Girard form class measurement point. It is further conditioned to ensure that predicted diameter (d) equals dbh when the bole height (h) equals 4.5 feet; that d=Du, the form diameter, when h=17.3 feet; and that d=O when h=H.

After conditioning, the complete taper model is:

$$Yb = 1.0 - \frac{(D^2-Du^2)\,(Xd^P - Xh^P)}{D^2\,(Xd^P - Xu^P)} \qquad \text{for } Xu \le Xh \le 1.0,\ 0 \le h \le 17.3$$

$$Yt = (Xh/Xu) + b_2 Xh\,(Xh-Xu) \qquad \text{for } 0 \le Xh \le Xu,\ 17.3 \le h \le H$$

$$+ b_3 Xh\,(Xh^2-Xu^2) + b_4 Xh\,(Xh^3-Xu^3)$$

$$+ b_5 Xh\,(Xh^4-Xu^4)$$

where:

$Yb = d^2/D^2$, the lower bole model

$Yt = d^2/Du^2$, the upper bole model

Hu = the height of the form measurement point (Hu=17.3 feet)

$Xd = (H-4.5)/H$

$Xu = (H-17.3)/H$

$Xh = (H-h)/H$

Du = bole diameter (outside or inside bark) measured at Hu

2. Solving for h given d, D, Du, and H. The solution of the profile function for h given d depends on the magnitude of d. If d is greater than or equal to Du, then h must be less than or equal to 17.3 feet.

Thus:

$$h = H - H\left[Xd^P - \frac{(D^2-d^2)\,(Xd^P - Xu^P)}{(D^2-Du^2)}\right]^{1/P} \qquad \text{for } d \ge Du$$

33.32 - Exhibit 10 -- continued

The Schlaegel Form-Class Model.

If d is less than Du, then h must be between 17.3 feet and the tip of the tree, H. Because the top equation does not have a unique solution, use an iterative solution such as the Newton-Raphson technique. The first derivative of the top model Yt with respect to Xh is:

$$f'[Yt(Xh)] = Xu^{-1} + b_2[2Xh - Xu] + b_3[3Xh^2 - Xu^2] + b_4[4Xh^3 - Xu^3] + b_5[5Xh^4 - Xu^4]$$

3. Schlaegel Cubic Volume Function.

a. Solving for V when both the Lower and upper merchantability limits are greater than or equal to 17.3 feet:

$$V = KD^2H\left\{(Xb - Xt) - \left[\frac{(D^2 - Du^2)}{D^2(Xd^P - Xu^P)}\right]\left[Xd^P(Xb - Xt) - \left(\frac{Xb^{P+1} - Xt^{P+1}}{P+1}\right)\right]\right\}$$

where:

$$Xb = (H - L)/H$$

$$Xt = (H - U)/H$$

b. Solving for V when both the lower and upper merchantability limits are less than 17.3 feet:

$$\begin{aligned} V = KDu^2H\{&[(Xb^2 - Xt^2)/2Xu] \\ &+ b_2[2(Xb^3 - Xt^3) - 3Xu(Xb^2 - Xt^2)]/6 \\ &+ b_3[(Xb^4 - Xt^4) - 2Xu^2(Xb^2 - Xt^2)]/4 \\ &+ b_4[2(Xb^5 - Xt^5) - 5Xu^3(Xb^2 - Xt^2)]/10 \\ &+ b_5[(Xb^6 - Xt^6) - 3Xu^4(Xb^2 - Xt^2)]/6\} \end{aligned}$$

The Schlaegel Form-Class Model.

c. Solving for V when L < 17.3 feet and U > 17.3 feet. In this case find both the volume in the bottom of the tree (Vb) from L up to 17.3 feet, and in the top of the tree (Vt) from 17.3 feet up to U, then add Vb and Vt to estimate bole volume. First, find the volume in the bottom of the tree (Vb) by letting Xb = (H - L)/H and Xt = (H - 17.3)/H and solve for the volume to 17.3 feet using the equation as in 3a above. Next, find the volume in the top of the tree above 17.3 feet (Vt) by using equation as in 3b above, letting Xb = (H - 17.3)/H and Xt = (H - U)/H. Then bole volume between L and U is: V = Vb + Vt.

4. Considerations.

a. The form class profile model was developed for use in hardwoods and was developed and tested for willow oak and proved to be superior to the Max-Burkhart model in all categories of testing, whether by diameter classes, total height classes, relative height classes, or combinations of the three.

b. Even though it was developed for hardwoods, it is expected to perform well for conifers.

c. Use of this model requires an extra measurement, Du, at 17.3 feet. Estimating form diameter is an extra expense in timber cruising, but it provides a more accurate tree volume estimate. However, it reduces the variation between trees and the number of trees to be measured, which largely compensates for the cost of the extra diameter measurement.

d. In some situations, it may be sufficient to use a fixed form class for a species in a given location, or to set a form class for that species/location as a function of dbh, height, or both. In these cases, substitute local constants for Du, which negates the need to measure Du in the field.

e. The form class model allows an opportunity to greatly expand the usable range of an individual profile equation.

f. Trees having the same dbh and height have different volumes due to differences in bole shape (also known as taper or form). As form class changes by a single point (say 80 to 81), the volume changes by approximately 3 percent. Thus, much of the variation that exists in current volume and profile functions is due to assuming an average tree form.

g. Standard volume and profile equations apply to a limited geographic or physiographic area because of the changes in tree form that exist over large areas and

The Schlaegel Form-Class Model.

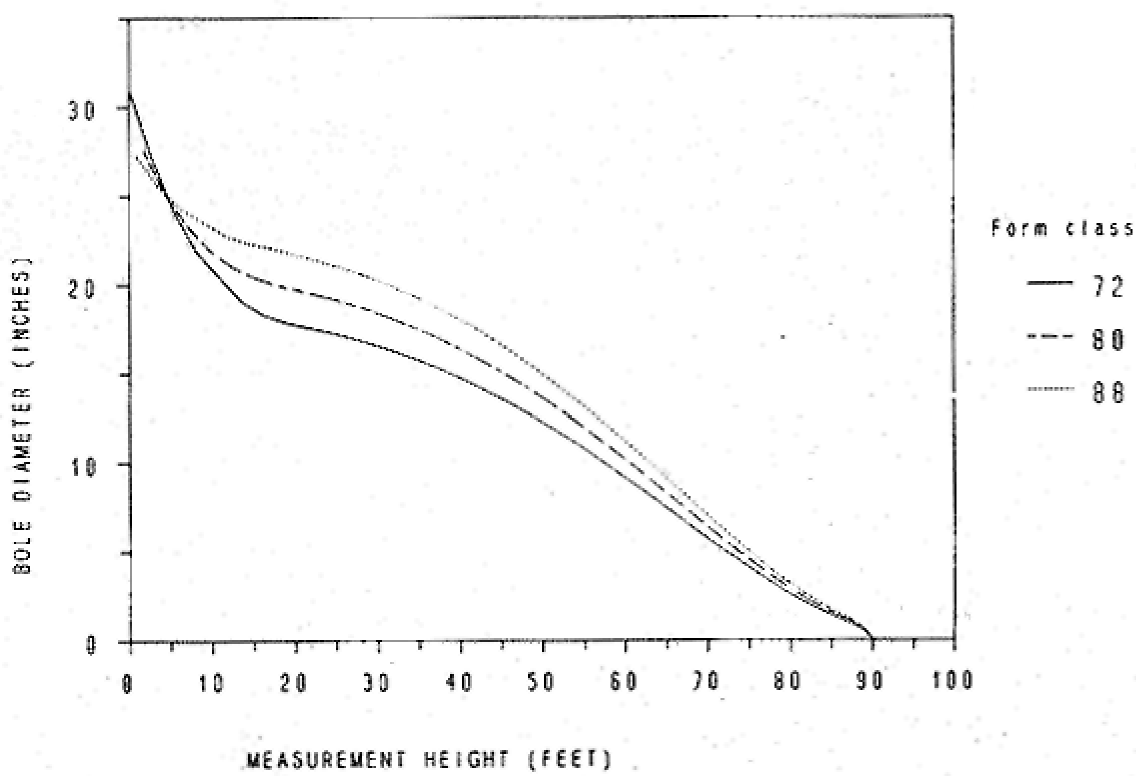
therefore, a number of volume and profile equations are developed for a single species to improve volume estimates in a localized area. A form class profile equation should he valid over a larger area.

h. A plot of a two-equation form class model is shown in exhibit 11.

33.32 - Exhibit 11

Plot of a two-equation form taper model with constant dbh and total height with different form diameters.

Plot of a two-equation form taper model with constant dbh and total height with different form diameters.



D = 25 inches

H = 90 feet

Du = 18, 20, & 22 inches

33.32 - Exhibit 12

The Max-Burkhart Model.

1. Profile Model:

$$\frac{d^2}{D^2} = b_1(X-1) + b_2(X^2-1)$$
$$+ b_3(a_1-X)^2 I_1 + b_4(a_2-X)^2 I_2$$

where:

$X = h/H$

a_i = *join points estimated from the data; a_2 is the lower point, and a_1 is the upper*

$I_1 = 1$ *when* $X \leq a_1$

$= 0$ *when* $X > a_1$

$I_2 = 1$ *when* $X \leq a_2$

$= 0$ *when* $X > a_2$

Thus, the taper equation in the bole below the bottom join point ($X \leq a_2$) is:

$$\frac{d^2}{D^2} = b_1(X-1) + b_2(X^2-1)$$
$$+ b_3(a_1-X)^2 + b_4(a_2-X)^2$$

In the middle portion of the bole above a_2 and below a_1, i.e. $a_2 < X \leq a_1$, the profile equation is:

$$\frac{d^2}{D^2} = b_1(X-1) + b_2(X^2-1) + b_3(a_1-X)^2$$

The top portion of the bole above a_1 ($a_1 < X$) is:

$$\frac{d^2}{D^2} = b_1(X-1) + b_2(X^2-1)$$

The Max-Burkhart Model.

2. Solving for h given d. Since the Max-Burkhart model consists of three joined 2nd degree polynomials, solve for h given d by solving the quadratic equation for one of the profile equation segments, depending on where d is located in relation to the diameters of the join points:

$$h = H[-B-(B^2-4AC)^{1/2}]/2A$$

where:

$$A = b_2+P_1b_3+P_2b_4$$

$$B = b_1-2P_1a_1b_3-2P_2a_2b_4$$

$$C = -b_1-b_2-\frac{d^2}{D^2}+P_1a_1^2b_3+P_2a_2^2b_4$$

$$P_1 = 1 \text{ if } d \geq d_1$$

$$= 0 \text{ if } d < d_1$$

$$P_2 = 1 \text{ if } d \geq d_2$$

$$= 0 \text{ if } d < d_2$$

$$d_1 = D[b_1(a_1-1)+b_2(a_1^2-1)]^{1/2}$$

= *estimated bole diameter at upper join point,* $(h=a_1H)$

$$d_2 = D[b_1(a_2-1)+b_2(a_2^2-1)+b_3(a_1-a_2)^2]^{1/2}$$

= *estimated bole diameter at lower join point,* $(h=a_2H)$

33.32 - Exhibit 12 -- continued

The Max-Burkhart Model.

3. <u>The Max-Burkhart Cubic Volume Function</u>. The Max-Burkhart equation is as follows:

$$V = KD^2H\left\{\frac{b_2}{3}(X^3-Y^3)+\frac{b_1}{2}(X^2-Y^2)-(b_1+b_2)(X-Y)\right.$$
$$-\frac{b_3}{3}\left[(a_1-X)^3I_1-(a_1-Y)^3J_1\right]$$
$$\left.-\frac{b_4}{3}\left[(a_2-X)^3I_2-(a_2-Y)^3J_2\right]\right\}$$

where:

$X = U/H$

$Y = L/H$

$I_1 = 1$ *if* $X \le a_1$

$= 0$ *if* $X > a_1$

$I_2 = 1$ *if* $X \le a_2$

$= 0$ *if* $X > a_2$

$J_1 = 1$ *if* $Y \le a_1$

$= 0$ *if* $Y > a_1$

$J_2 = 1$ *if* $Y \le a_2$

$= 0$ *if* $Y > a_2$

The Max-Burkhart Model.

4. Considerations.

a. Consider the Max-Burkhart model to be a very flexible, two-variable, general purpose model and typically, the model used as a basis for comparing newly developed models.

b. Consider this model where accuracy is deemed important. The Max-Burkhart profile model has been accepted as one of the most accurate and flexible two-variable models available.

c. The model is relatively complex, hard to understand, difficult to fit to the data, and hard to use.

d. The model consists of three parabolic equations connected at two locations called "join points". Condition the equations so that as the taper line is smooth and continuous from one parabolic equation to the next. The taper line is considered to be smooth at the join point if the slopes of the two equations are equal and continuous if equal at the join point. Also, condition the three parabolic equations to force the taper line through dbh and the tree tip.

e. Consult references on the method such as Jeff Martin (sec. 08).

34 - Direct Volume Estimation - Volume Equations

Prepare direct estimators that predict tree volume from dbh and height, from dbh alone, or from dbh, height, and some expression of tree form. If tarif tables are preferred, predict volumes from dbh and tarif number which is an index which incorporates the additional effects of both height and form. Use tree height as a predictor. Total height is appropriate for most conifers, or identify an appropriate merchantable height which is the Most Useful expression of height for some hardwoods. As with stem profile equations, estimate coefficients of volume equations by regression techniques from a sample of trees for which volume has been accurately measured. Design direct estimators to predict total stem volume or merchantable volume between the stump and the specified merchantable top limit.

34.1 - Tree Volume Models

consider total volume to be the cubic content of the tree. Recognize that board feet, cords, and other product measures are not volume. Ensure that when such terms are applied to unprocessed trees or logs they are a statement of product potential, not stated as a volume estimate.

A tree volume model relates measured dimensions of a tree stem to the volume contained in the stem. Recognize that a mathematical model is only an approximation to the actual geometric solid which is a tree stem. However, quite simple models are adequate to predict volume in many circumstances.

Apply the notation given in exhibit 01 for the direct volume estimation formulae in this section.

34.1 - Exhibit 01

Notation for Direct Volume Estimation - Volume Equations

V = total volume of a tree stem, from ground level to tip.

V_m = merchantable volume of a tree stem.

V_s = volume in the tree stump.

V_t = volume in the tree top above some stated merchantability limit.

D = tree diameter at breast height, outside bark.

D_s = stump diameter at cut surface.

D_m = merchantable top diameter limit.

D_u = an upper stem diameter, inside or outside bark as stated.

H = total tree height.

H_m = merchantable height.

L_t = length of unmerchantable top above Dm

$R_m = V_m/v$

$k = \pi/(4*144) = 0.005454154$

b_i = estimated coefficients

34.12 - Combined Variable Model

Use the Combined Variable Model only if the cylinder form factor is nearly the same for all trees regardless of their size. Recognize that this may not be true even if trees of all sizes had exactly the same shape, because breast height is not the same relative point on trees of different sizes. Consider this model when the range of tree sizes in the population is not great. However, consider that estimators based upon this model will, in theory, be biased in differing direction and magnitude at different points in the domain of D^2H.

The most common model used for total stem volume is the Combined Variable Model (Section 08, Spurr, 1952):

$$V = b_0 + b_1 D^2 H$$

Ensure that when estimated from data, b_0 is very small (theoretically, however, it should not equal zero) and b_1 is nearly proportional to the data set's mean ratio of actual tree volume to volume of a cylinder with the same diameter and height. This ratio is called the cylinder form factor (CFF). The constant of proportionality is k. Estimate the coefficients b_0 and b_1 using ordinary least squares (OLS) regression techniques.

Variance about the regression is not homogeneous and is correlated with tree size. Therefore, use observation weights proportional to $(D^2H)^{-n}$ when fitting this equation form. If such weighting is not done, estimates of parameters will be unbiased, but they will not be the best linear unbiased estimators (BLUE) and will not be minimum variance unbiased linear estimators.

The value of n which best describes residual variance may be as low as 0.5 or as large as 2. Use any value of n on the interval [1.5,2] which can be expected to give reasonably good results (better than n=O which implies equal observation weights). Do not use values of n outside (1.5,2) which are unlikely to be optimal.

Employ weighting functions depending upon the regression software used and user preference. For example, some regression software allows defining the value of a variable which is to be used as the observation weight. Some programs go further and scale the weights so that the sum of weights is equal to the number of observations in the sample. Others require the constant of proportionality to be adjusted by the user in order to meet this condition. If software being used does not have the facility for defining a weight variable, multiply both sides of the model by the square root of the weighting function. That is, multiply through by $(D^2H)^{-n/2}$. This results in a transformed model, but one with the same coefficients as the original model. Use the estimated coefficients, which should be nearly BLUE, in the original untransformed model.

Do not ordinarily construct a volume estimator based on the Combined Variable model alone. In most cases, construct one with one set of coefficients for trees no larger than an arbitrary size, and a different set for trees larger than that size to accommodate form factor differences between trees of different sizes. Such models have an abrupt change in the relationship of volume to dbh and height at the arbitrarily chosen size and are no longer widely recommended. When reasons

of expedience justify the practice, define the point at which the two regression lines intersect (the "join point") in terms of the variable D^2H, not in terms of D alone. Select the join point that, jointly with other coefficients, minimizes the sum of squared residuals. Do not select it separately and arbitrarily.

34.13 - Adjustments to the Combined Variable Model

Because the Combined Variable Model's constant form factor assumption is not applicable over a reasonable range of tree sizes, it may be desirable to add linear terms to the model which allow cylinder form factor to vary with tree size. Consider one of the many possible examples such as the Australian Model (Stoat, 1945):

$$V = b_0 + b_1D^2H + b_2DH$$

The cylinder form factor can vary with tree size. Do this by dividing the equation by kD^2H, thus:

$$CFF = V(kD^2H)^{-1} = b_0(kD^2H)^{-1} + b_1k^{-1} + b_2(kD)^{-1}$$

Since b_0 is very small, the first term right of the second equality has negligible effect on CFF. If $b_1 > 0$ and $b_2 > 0$, what remains of the expression describes a CFF which decreases with tree dbh (more rapidly at first then more slowly at larger dbh) asymptotic to a limiting CFF, b_1. This is consistent with the relation of form factor to tree size. However, the rate of change in CFF with change in dbh may not be well described by the reciprocal term for all of the dbh range. No integer value for the negative exponent of D may adequately describe the relationship between CFF and D over the entire range of D. In fact, no single exponent (whether integer or not) may be sufficient. Also, form factor is related to tree height as well as to dbh--perhaps to the ratio H/D. If poor results are obtained consider form factor models.

34.14 - Form Factor Models

In these models, calculate cylinder form factor (CFF) $CFF = V(kD^2H)^{-1}$ for each tree in the volume estimator sample. Use CFF as the dependent variable in deriving an appropriate form factor model and estimating coefficients. Estimate coefficient values by ordinary least squares (OLS) regression. Consider CFF as a dimensionless number which reduces concern for strict maintenance of cubic dimensionality in the model. Also, a model with volume as the dependent variable exhibits heteroskedasticity. Since CFF is nearly homoskedastic, do not use observation weights in estimating coefficients. An example of a model describing the relation of CFF to D and H is:

$$CFF = b_0 + b_1D^{-1} + b_2D^{-2} + b_3H + b_4H^2 + b_5(H/D) + b_6(H/D)^2$$

Volume is then estimated for any tree by:

$$V = k\,CFF\,D^2H$$

The CFF model in this example contains polynomial and polynomial-reciprocal terms, but some CFF equations have contained terms as high as the fifth degree. Terms with fractional exponents may be included, as well as those with integer exponents. Avoid any set of independent variables that exhibits strong multicollinearity which may generate a moment matrix too close to singular for sufficiently accurate inversion.

Use a stepwise regression procedure to avoid problems with a singular matrix since variable sets with excessive multicollinearity are not selected. Assess the terms that might be included in the model that are likely to have predictive value using analysis of variance from regression. Use a stepwise regression procedure that is included in most statistical packages. Set F-to-enter and F-to-delete values at 3.99 and 4.00, respectively. The programmed stepwise algorithm results in a fitted regression equation which contains those, and only those, variables significant at or beyond the 0.05 probability level.

When volume is the dependent variable, most models will appear to have a precise fit with coefficients of determination (R^2) in the range 0.95-0.98. However, when CFF is used as the dependent variable do not expect such precise fits. Expect values of R^2 in the range 0.20-0.40. The greater the range of tree sizes in the sample, the greater the variation in CFF, and thus more of it is explained by regression. Consider a coefficient of determination in the neighborhood of 0.40 to be a good fit. Do not compare precision of two fitted regressions, one using CFF as the dependent variable, and the other volume as the dependent variable since two different things are being compared. Compare the two, using volume residuals from the CFF estimator computed as:

$$\text{residual} = (V - k\ CFF\ D^2H)$$

where V = observed volume of an individual sample tree.

CFF = cylinder form factor predicted for the same tree by the fitted estimating equation.

Square and sum the residuals computed in this way over the sample to obtain a sum of squared residuals or other derived statistics. These are comparable to the sum of squared residuals obtained from fitting an equation with volume as the dependent variable.

Since summary measures of goodness-of-fit such as R^2 or the root mean square residual (RMS) seldom convey enough information by themselves to allow a thorough comparison among fitted equations or of a particular fit to an objective norm, use analysis of residuals described in section 35.

Polynomial-reciprocal models fit data quite well within the range of the sample, but often predict wildly outside the domain of independent variables represented by the sample data. Therefore, if the sample covers all tree sizes to be encountered in practice, expect no problem. Because this is not likely, extend the trend of the estimator and the CFF estimator beyond the sample data cautiously and only when it is a practical necessity. Use a linear equation rather than the polynomial-reciprocal CFF estimator outside the domain of independent variables represented by

the sample. Find coefficients of the linear equation by locating two suitable points graphically and effecting a simultaneous solution.

The polynomial-reciprocal model has a high degree of flexibility and may allow unrealistic and undesirable changes of slope an the CFF surface even within the range of the sample. Check for an abnormally formed tree included in the sample, a mistake in the observed values of the dependent or independent variables, or anything leading to an exceptionally large difference between observed and regression-predicted values. For this reason conduct an analysis of residuals to detect local irregularities in the regression surface.

34.15 - Allometric or Log-Linear Models

This model is similar to the Combined Variable Model:

$$V = b_0 + b_1 D^2 H$$

If b_0 is ignored because it is so small as to be truly negligible (for example, volume of a tree 4.5 feet tall), b_1 is left as the sole expression of tree form. Do not accept the idea of a constant cylinder form factor (CFF). If it appears the polynomial-reciprocal estimator for CFF may give problems, consider a different representation of CFF (or b_1) . The ratio of a tree's total height to its dbh must be related to its CFF. The height to diameter ratio (H/D) is the inverse of average taper between breast height and the tree's tip. If CFF were a simple multiple of H/D, such as CFF q(H/D), then a volume model could be written

$$V = k\ q\ (H/D) D^2 H = k\ q\ DH^2$$

This model is dimensionally correct. Tree volume is the cross-sectional area (kD^2) accumulated up the stem. The accumulation of kD^2 is a linear function of H. Consider a more flexible relationship between CFF and H/D, for example:

$$CFF = H^b / D^c$$

allows writing

$$V = k\ CFF\ D^2 H$$

$$= k(H^b/D^c) D^2 H$$

which by combining exponents becomes

$$V = k\ D^{2-c} H^{1+b}$$

or, letting $b_1 = 2-c$ and $b_2 = 1+b$,

$$V = k\ D^{b_1}\ H^{b_2}$$

Use one of two ways to fit this to sample data. Choose which to use based on the distribution of error associated with the model, not upon the fact that it is easier to fit a linear regression. If random error e is additive, write the model with error included as:

$$V = k\, D^{b_1}\, H^{b_2} + e$$

then estimate coefficients b_1 and b_2 by nonlinear regression. Do not make transformations by taking logarithms to render coefficients amenable to estimation by linear regression. Previous discussion in this section is based on assumption that error is additive. Do not assume the Combined Variable Model has additive error, and that the allometric model does not simply because the exponents of D and H differ slightly from 2 and 1, respectively. If the model with additive error is fitted in the form given, use observation weights proportional to $(D^2H)^{-n}$. Depending upon the nonlinear regression software used, weighting may need to be done by dividing the equation by D^2H, thus arriving at an allometric form factor model. Fit this allometric model to data, whether with form factor or volume as the dependent variable, and expect to find that b1 is not exactly equal to 2, nor is b_2 exactly equal to 1. Do not expect equality if the concept of form factor changing with tree size is accepted. It may also be that $b_1 + b_2$ is not exactly equal to 3; that is, that strict cubic dimensionality is not maintained. Expect the sum of exponents to be near 3 and the small difference may not matter. However, if strictly logical dimensionality is desired, maintain it by writing the model:

$$V = k\, b_1 D^{b_2}\, H^{(3-b_2)} + e$$

Recognize that the reduction in precision of fit suffered by insistence upon strict cubic dimensionality will be small, and a significant benefit may be much quicker convergence of the fitting algorithm. The sum-of-squares surface generated by an allometric volume model with independent exponents possesses a long trough in the space of the exponents. Expect repeated iterations of the algorithm to move along the bottom of this trough with small reduction in the sum of squares, but enough not to terminate the fitting process. Expect optimality to be reached very slowly. Reduce the parameter space by maintaining a strictly cubic model changes the sum-of-squares surface to one that exhibits a well defined minimum point rather than a long trough with little gradient.

If multiplicative error is suspected, then write the model including the error term as:

$$V = k\, b_1 D^{b_2}\, H^{b_3}\, e$$

then, make the model linear by taking logarithms resulting in:

$$\ln V = \ln(k\, b_1) + b_2 \ln D + b_3 \ln H + \ln(e)$$

If strict cubic dimensionality is to be maintained use the model:

$$(\ln V - b_3 \ln H) = \ln(k\ b_1) + b_2(\ln D - \ln H) + \ln(e)$$

Fit linear models by OLS techniques. This justifies logarithmic transformation in the majority of cases where it is used. No consideration is given to the distribution of error, and logarithmic transformation is assumed to have a self-weighting effect. This means that heteroskedasticity (probably strongly exhibited by the untransformed model) is reduced in the log-linear model. It will be even more reduced where the dependent variable is ($\ln V - b_3 \ln H$), but will not be eliminated. Recognize that this is not the only effect of transformation on the error distribution.

Recognize that it is not consistent to claim additive error for models with V as the dependent variable and then claim additive error for models which use lnV as the dependent variable. However, the difference in estimates resulting from difference in error distribution assumptions may not be very great in practical applications.

Do not overlook the bias attached to estimates of V of logarithmic transformation and make needed correction for bias. For additional reference see Bradu and Mundlak (1970). The necessary bias correction factor (BCF) is approximately:

$$BCF = \exp(S^2_{yx}/2)$$

where

S^2_{yx} = the variance of lnV residuals about the regression surface.

Although correction should be made in estimating tree volumes, it may not be essential. The absence of large errors resulting from this omission is due to the characteristically small root mean square residual of lnV. Expect the correction factor to change estimates by no more than five percent of predicted volume. Recognize that such small bias is usually obscured by errors of diverse provenances in the volume estimation process.

34.16 - Form Class or Form Quotient Estimators

Consider making volume estimators more precise by including a variable to take into account the different form of each individual tree, or of an identifiable group of trees which differ in form from the average. To do this, use a stem diameter as the additional variable D_U, measured at some point above breast height. Incorporate D_U in the model through the form quotient, D_U/D.

For the form quotient to have any meaning from one tree to another, measure D_u at the same height above ground, or the same relative height, on all trees. Evaluate defining this height differently for different kinds of form quotients to see if one is more useful than another. Measuring D_U at half the tree's total height yields Jonson's form quotient. Another approach is measuring D_U at one-fifth the tree's total height. The most familiar expression of tree form is Girard form class, which is $100*D_U/D$, with D_U measured inside bark at the top of the butt log,

17.3 feet above ground line. Use a device called the Wiant Wedge (FSH 2409.12) to make measurement of Girard form class possible without measuring D_U.

Use form class or form quotient as an independent variable with a form factor model. Consider form quotient one of the most important variables.

Weigh the value of precision gained by use of form class or form quotient estimators against the additional cost of using them. Additional cost comes from the need to obtain a measure of form for each tree to which the estimator is applied. This may mean measurement of an upper stem diameter, which is more expensive than measuring dbh. The use of an instrument such as the Wiant Wedge does not ensure that the estimate of form class obtained is the same as that obtained by direct measurement of D_u. Obtain form class or quotient for felled estimator sample trees by direct measurement of D_u. Evaluate form class for better or poorer results than a standard volume estimator based upon D and H alone by evaluating the following:

1. The difference between mean form class of trees in the cruise area and the mean in the sample from which the standard volume estimator was constructed. If they are very nearly the same, the standard volume estimator will be as good as the use of a form class estimator with even a good mean form class estimate.

2. The change in average form class from one tree size to another is implicit in the standard estimator. If a single average form class based on a sample of climbed, felled, or windthrown trees is used for trees of all sizes, expect the volume estimates for larger and smaller trees to be somewhat biased. If the form class sample covers a sufficient range of tree sizes, it may be possible to characterize the relation between tree size and form class to avoid such bias. However, this complicates use of a form class estimator and adds to its cost.

3. The accuracy with which the sample mean form class represents the mean form class in the cruise area. If this is poor, results of using a form class estimator will be poor.

34.17 - Tarif Volume Estimators

Tarif tables are a comprehensive set of local volume tables indexed by tarif number. Do not construct new tarif tables. To use existing tables, establish the appropriate tarif number for a particular stand and select the corresponding tarif table. Measure only the diameter breast height (dbh) of cruised trees to enter the local volume table. Carefully determine the tarif number. Fell and measure a sample of trees to determine which tarif table best predicts their volumes. If the stand is small, consider measuring all tree heights and using a standard volume estimator. Use access tables based on the height dbh relationship if they have been prepared. Using these access tables, take only a sample of height measurements to estimate tarif number.

Do not use a different tarif number for each dbh class in the same evenaged stand, since it is contradictory to the theory of tarif table construction and use.

34.2 - Merchantable Volume

Most uses for volume estimates require estimates of merchantable volume rather than total volume. Three approaches to modeling merchantable volume are described in the following sections.

34.21 - Simple Approach

To use this approach, use the same model as for total volume, substituting V_m as the dependent variable. Recognize that the approach takes no account of the logical relationships involved and may be inaccurate. Use this method only when other approaches do not meet the needs.

34.22 - Merchantable Volume Ratio Model

This method may be used to ensure that estimates of V_m are less than V. The merchantable volume estimator is then:

$$V_m = R_m V$$

where R_m is zero for trees of less than merchantable size, becomes immediately greater than zero as trees reach merchantable size, and from that point increases monotonically with tree size asymptotic to unity. As a practical matter, over the range of possible tree sizes, R_m will be somewhat below its theoretical asymptote. For each sample tree an observed value of R_m is V_m/V. Possible models relating R_m to tree dimensions are:

1. $R_m = 1 - q \exp(-\mathbf{XB})$
2. $R_m = 1 - b/D$
3. $R_m = 1 - b/D^2$
4. $R_m = 1 - b/(D^2H)$

where b and q are model coefficients, **XB** is a generalized function of D and/or H, linear with respect to **B**, that increases with tree size. Use other possible models that exist, but ensure the model used has the property of increasing monotonically with tree size and never exceeding unity. Example models 2 through 4 are ratio estimators. Estimate the coefficients, b, by regressions fitted through the origin with $(1 - R_m)$ as the dependent variable. The observation weights depend upon the nature of variance in $(1 - R_m)$ about regression.

In example model 1 above, estimate q and **B** by OLS with the log-linear model

$$\ln(1 - R_m) = \ln(q) - \mathbf{XB}$$

Find the best specification for **XB** by experimentation with various transformations and interactions of D and H. The only requirement is that it increase in magnitude with increasing tree size. Using a log-linear model may seem at odds with advice about error distribution given

in section 34.15. If error associated with all of the previously mentioned models is additive, then the logarithmic transformation is not appropriate. However, to partially address the problem, fit the log-linear equation as one step to arrive at a suitable transformation of the information about tree dimensions. Fit the equation:

$$R_m = b_0 + b_1 \exp(-\mathbf{XB})$$

by OLS techniques consistent with the assumption of additive error implicit in the other models suggested above. If $b_0 < 0.95$ or $b_0 > 1$ then b_0 should be restricted by fitting:

$$(1 - R_m) = -b_1 \exp(-\mathbf{XB})$$

Coefficients in the vector **B** do not have desirable statistical properties. However, the exp(**-XB**) is a fair transformation of D and H measurement, and is probably better than the reciprocals or squared reciprocals.

34.23 - Merchantable volume Difference Model

Consider this model if only merchantable volume needs to be estimated. Calculate merchantable volume as total volume less those portions of the tree which are unmerchantable. Use the algebraic relationship:

$$V_m = V - V_s - V_t$$

If a satisfactory estimator for total volume, V, already exists, model V_s and V_1 and estimate the model parameters. V_s is a function of D_s, stump height, and stump shape. Expect stump shape to be different for different sized trees. Fix stump height at some height related to utilization practices, such as one foot. Determine if a linear relationship between D^2 and stump volume exists. Look for the relationship between D^2 and D_s^2, stump height, and stump shape to be contained in the coefficient of D^2. Expect some linear effect of D in the stump volume model, so use, as a beginning:

$$V_s = b_0 + b_1 D + b_2 D^2$$

If the linear term is not useful, delete it.

Assume the top portion of most merchantable-sized trees closely resembles a cone. Then, volume of the top is nearly:

$$V_t = (1/3)\ k\ (D_m)^2 L_t$$

where L_t is length of the unmerchantable top.

In any particular application D_m is fixed, not a variable, so express L_t in terms of D and H. L_t is longer for smaller, younger trees than for older, larger ones. Expect that L_t is related to average taper of the tree. (H/D) is the inverse of the tree's average taper. If L_t is linearly related to (H/D) as:

$$L_t = c_0 + c_1(H/D)$$

then express nonmerchantable top volume as:

$$V_t = (1/3)\ k\ D_m^2[c_0 + c_1(H/D)]$$

or, after condensing constants as:

$$V_t = b_0D_m^2 + b_1D_m^2(H/D)$$

Expect that a simple reciprocal expression of D is not always sufficient. For many southwestern species, consider a negative exponent of 1.5. Call this exponent n for generality and combine total volume, stump volume, and top volume into one expression:

$$V_m = V - (b_0 + b_1D + b_2D^2) - (b_3D_m^2 + b_4D_m^2HD^{-n})$$

Estimate coefficients (except n) by moving V, a previously determined value, to the left side of the equation. Select the exponent n by experiment. Drop terms which turn out to have no statistical or practical effect from the estimating equation.

34.24 - Use of Merchantable Height

Determine if the expected users measure merchantable height of a tree. If so, decide how to model merchantable volume depending upon whether or not total height is measured also. If it is not, expect estimation of total volume to be more difficult. If both total and merchantable heights were measured, top volume might become:

$$V_t = (1/3)\ k\ D_m^2(H - H_m)$$

and the merchantable volume difference model is:

$$V_m = V - (b_0 + b_1D + b_2D^2) - b_3D_m^2(H - H_m)$$

If the merchantable volume ratio approach is used, expect the ratio V_m/V to be related to the ratio H_m/H, though not linearly. Starting with the relationship:

$$V_m/V = b_1\left(\frac{H_m}{H}\right)^{b_2}$$

and allow individual exponents for H_m and H:

$$V_m/V = b_1 H_m^{b_2} H^{b_3}$$

Estimate the exponents b_2 and b_3 by transformation to log-linear form. If error about the model above is multiplicative, estimate b_1 from its logarithm in the log-linear form fitted by OLS. otherwise, after obtaining values of the exponents from fitting the log-linear form b_1 and b_0 if it is expected to have a nonzero value, estimate by:

$$V_m/V = b_0 + b_1 H_m^{b_2} H^{b_3}$$

Restrict b_0 to zero if necessary. In this case, do not regard exponents as maximum likelihood estimators, or as possessing any other desirable statistical properties other than being better exponents for transforming H_m and H than an arbitrarily chosen integer would be. However, they are not arbitrary since they were derived from the available data. Count them for one degree of freedom in any assessment of precision that is undertaken.

34.25 - Use of Merchantable Top Diameter as a Predictor Variable

In the merchantable volume ratio model (sec. 34.22) and the merchantable volume difference model (sec. 34.23), assume that when V_m is volume to a certain top diameter D_m, then D_m is fixed in a particular fitting of the model to data. Treat D_m as a variable in fitting the model, thereby achieving a smooth and transitive relationship between merchantable volumes V_m associated with a range of top diameters D_m.

Take a sample large enough so that different observations of D_m and V_m are taken on different trees, and make no more than one observation of D_m and V_m on each sample tree. This avoids correlated residuals about the regression surface. Recognize that one of the necessary conditions for best linear unbiased estimators (BLUE) is that residuals be independent.

Expect several merchantable volumes V_m to be calculated to corresponding top diameters D_m on each tree in the sample. This results in highly correlated residuals about regression and violates the BLUE assumptions. This is a situation with time series and cross-section effects in the same sample. Use Generalized Least Squares (GLS), instead of OLS, to derive BLUE from such data. Meeting the BLUE assumptions may not be crucial since parameter estimates will be unbiased, but estimates of variances will be biased. Therefore, hypothesis tests will be invalid. This may not matter if the structure of a model is well known from theory. Tests of hypotheses, such as for differences between geographic subregions, even though appropriate, may not be needed and are not valid.

34.3 - Comparison of Direct Estimators with Stem Profile Estimators

Avoid the use of direct volume estimators except as a check of Stem profile estimators. Prepare and use stem profile estimators unless the Regional Forester approves the justification to use another method. Comparison may involve a set of volumes or estimators, especially when

several merchantability standards are involved and consistency of volume estimates between them must be maintained. Make necessary assumptions about the shape of the tree, and expect biases that are difficult to detect.

35 - Verification of Estimators

Verify estimators with the data on which they are based to eliminate (1) lapses in programming logic, (2) flaws in algorithms, and (3) bias in computations. Verify using one or more of the tests in the following sections. Do not confuse this process with validation which tests how well the estimator predicts using an independent data set.

35.1 - Use of Standard Statistical Measures

Check the standard statistical measures of goodness of fit reported by the computer regression programs used in developing an estimator. Use these in choosing the mathematical models, but consider them less useful in verifying the estimator program. Be aware that these measures often have no single narrow interpretation. Check other valid evidence including performance with other data sets, consistency of significance in subsets of the present data, and biological or mechanical reasonableness. Consider choice of the most appropriate model to be part of the verification process.

Many estimates involve more than a single step, one or more transformations, and sometimes fitting of non-linear regression. In these situations, make a single comparison of observed and estimated values of the dependent variable to produce an overall standard error of estimate. Where the same value is estimated by two or more methods, use these standard errors to help decide which Ls the preferred method.

35.2 - Location and Subsample Differences

Verify a system of equations by using them to prepare tables and graphs of the estimated values, and compare these with similar tables and graphs of average observed values. Carefully select class intervals for such comparisons. Use five to eight subdivisions of each range of sizes to get reasonable comparisons. Do not use too few subdivisions which may give poor definition of the estimated values and do not use too many which give poor definition of the observed values. Extend the estimates to one class above and below the observed range to check performance of the estimator outside of its intended range of applicability.

35.3 - Form Class and Merchantable Top

Use covariance analyses as the appropriate tests for form class and merchantable top. Test not only intercepts, but also coefficients of one or more independent variables. Use the tests to show which models are most sensitive to wild values (outliers). More robust models are preferable. Use these tests to determine the variation of trees within data sets from that among data sets, and thus, to forecast the total variation likely to be encountered when the estimator is used on an

independent data set. This goes beyond the usual verification procedures. Use the results to avoid future concerns for bias.

When data comes from two or more sources, use it as an opportunity to test differences among location or data source, as well as differences among trees within locations or data sets. observed differences may indicate (1) variation among locations, (2) a biased estimator, or (3) ability of the worker. Evaluate differences with a broad view of the inevitable variation among trees, stands, localities, and mensurationists.

35.4 - Goodness of Fit

Fit the profile equation to subsets of the data; small, medium and large trees. Do this in addition to the covariance analyses given in section 35.3. Use three, four, or five sub-sets depending on both the amount of data and range of sizes. Recognize that significant independent variables that describe tree size for the entire data set often lose their significance or behave strangely because of the narrow range in the subset. Other independent variables may increase in significance. Fit equations to subsets while developing the model and later to verify it. See chapter 60 for procedures.

35.5 - Predictor of Standard Tree Results

Make estimates of several standard mensurational measures, such as form class, merchantable height, and so forth. Compare direct estimates made of these measures with similar measures derived from profile equations. Profile equations usually are fit with the square of diameter as the dependent value; therefore, those measures that involve averages of diameter may appear to be biased. If these apparent biases are small and in the right direction, consider the profile equation to be verified.

35.6 - Comparing Profile and Direct Estimates

Verify volume estimators derived as integrals of profile equations by comparing their volume with that of direct estimates. Direct comparison of two estimates may show that they are different in some respects. Expect these differences to be statistically significant, but to reveal nothing about which is the better estimate. In comparing two estimators to the data set on which they were based, expect the standard errors of estimate to be nearly the same, and neither estimator to show any great deviation of the "observed minus estimated volumes" when plotted over the variables entering the final estimating equation. Rework profile equations that cannot pass this verification test.

36 - Primary Product Estimators

Recognize that the amount of primary product (lumber, plywood, pulpwood, and similar products) manufactured from a tree or a log is closely related to the volume it contains, but it is not the same thing. Expect estimates of primary product potential, especially of sawn lumber to be excessively complicated by "standards and definitions," by "adjustment factors," and by

"allowances" peculiar to log scaling rules. If such estimators are necessary, use the procedures in the following sections.

36.1 - Sawn Lumber

Use the board foot as the unit of measure to be estimated. Recognize that a board foot is based upon the lumber piece's nominal dimensions and that under modern size standards and after finishing, dimensions are reduced from the nominal dimensions. For example, a piece of 2 x 4 measures 1.5 inches by 3.5 inches in thickness and width while length is a full measure.

Estimate sawtimber quantities by log rules and understand the many assumptions that underlie each rule. Recognize that most existing log rules are out of date and do not describe modern markets and milling practices well. In almost all cases, the log rules underestimate the quantity of lumber sawn from a log or tree by a modern sawmill. Identify the excess of production over prior estimate as overrun.

Use the Scribner Decimal C log rule, Cubic log rule, or the International 1/4-inch rule authorized at 36 CFR 223.3 to assess product amount in the tree segments. Follow procedures in FSH 2409.11, National Forest Log Scaling Handbook, or FSH 2409.11a, Cubic Scaling Handbook. If the lumber estimator is based on the production experience of a particular sawmill and a sample of trees, the estimator would be useful only for the mill for which it was based. Use either stem profile equation estimators or direct estimators.

36.11 - Stem Profile Equation Estimators

Ordinarily, use a stem profile equation to predict diameter inside bark at specified points on the tree bole. Secure or prepare a program, or preferably a subroutine, and use it to calculate the log rule volume. For any subject tree, use the following steps:

1. Pass tree measurements to the board foot subroutine.

2. Mathematically divide the tree stem into logs and segments according to direction in FSH 2409.11a.

3. With the stem profile equation, calculate a scaling diameter for each log (identified in preceding step 2).

4. Apply either the Cubic, Scribner Decimal C, or the International 1/4-Inch rule to the scaling diameters and lengths to obtain a gross lumber scale for each log segment in the tree.

5. Sum the scale of individual log segments to obtain gross total scale for the whole tree.

6. Do not apply statistics to the results.

Use stem profile, when possible, since revised merchantability standards may be imposed on the calculation at any time. This method does not predict net volumes or defect, but it allows more accurate adjustment for defect than does a direct estimator.

36.12 - Direct Lumber Estimators

Use similar procedures in deriving direct estimators for board feet of lumber as for direct merchantable volume estimators (sec. 34). Fell, measure, and scale a sample of trees according to the appropriate scaling procedures and merchantability standards. Scale gross not net amounts. Adjust for defect by other means when the estimators are applied (FSH 2409.12). Construct a mathematical model to describe the relationship of lumber potential in the tree to tree measurements of dbh, height, and sometimes form. Height may be total height, merchantable height in feet or meters, or merchantable height in logs or logs and half logs. Use total height for conifers and for hardwoods with a predominantly central stem. In these trees, total height is almost as strongly correlated with lumber content as are merchantable height measures, and it is easier to measure accurately and unambiguously. In hardwoods with broad crowns, use merchantable heights. Use one of two basic models: (1) express board feet as a function of tree measurements, or (2) use the board foot/cubic foot ratio as the dependent variable.

Base direct estimators on a particular set of merchantability standards, and when merchantability changes, derive a different estimator. Avoid including merchantable top diameter as an independent variable.

36.12a - Board Foot Models

The general form of these models is:

$$BF = b_0 + b_1 D^2 H + b_2 DH + b_3 H + p(D,H)$$

where:

p(D,H) = such polynomial and interaction terms in D and H as are necessary to obtain a satisfactory fit over the domain of D and H.

If desirable, expand this type of model to include measures of form if any have been taken. Estimate parameters by ordinary least squares (OLS) regression. Board feet are related to volume, but the cubic dimension of this type of model is not accurate. The board foot/cubic foot ratio increases with tree size rather than remaining constant, and may lead to introduction of some higher power polynomial terms which detract from the estimator. variance about this regression surface is not homogeneous, consequently, use observation weights proportional to $(D^2H)^{-n}$. Do not expect the optimal value of the exponent, n, to be the same as might be optimal for fitting direct volume estimators to the same sample of trees.

36.12b - Board Foot/Cubic Foot Ratio Models

Because board feet is a poor dependent variable, it is often more practical to use the board foot/cubic foot ratio model:

$$R_b = f(D,H,F)$$

where:

$$R_b = BF/V_m$$

f(D,H,F) = A monotonically increasing function of D, H, and F (if measure of form is available) asymptotic to R_b*. Where R_b is some board foot/cubic foot ratio which can never be exceeded no matter how large the tree.

A possible form for f(D,H,F) is:

$$R_b = b_0 - b_1D^{-1} + b_2D^{-2} + b_3D^{-3}$$

where b_0 is R_b* and all coefficient estimates are positive in absolute value. If the statistical estimate of b_O is not satisfactory, determine R_b* exogenously with the fitted model then being:

$$R_b - R_b^* = -b_1D^{-1} - b_2D^{-2} - b_3D^{-3}$$

Hypothesize that R_b* increases with tree height, in which case the model becomes:

$$R_b = (a_0 + a_1H) - b_1D^{-1} - b_2D^{-2} - b_3D^{-3}$$

If any of the coefficients b_i seem to be related to tree height, substitute the appropriate function of H for b_i. Regardless of how the model is expanded, maintain the property of being monotonically increasing asymptotic to R_b* (which may vary with H). This may be more easily done with the model:

$$R_b = R_b^* [1 - b_1 \exp(-\mathbf{XB})]$$

where:

XB = a general increasing linear function of D and/or H such that **XB** ≥ 0 for all D and H.

For fitting this model, initially estimate R_b* exogenously. If the asymptote is a function of H, determine the relationship exogenously. Then estimate the vector of coefficients, **B**, by fitting:

$$\ln(1 - R_b/R_b^*) = \ln(b_1) - \mathbf{XB}$$

Do not regard the coefficients in **B** as having any particularly desirable statistical properties beyond exp(**-XB**) being a good transformation of D and H for describing the board foot/cubic foot ratio. Then fit:

$$R_b = a_0 - a_1 \exp(-\mathbf{XB})$$

where:

$$a_0 = R_b^*, \text{ and}$$

$$a_1 = b_1 R_b^*$$

If R_b* is a function of H, then substitute that function for a_0 and a_1. If a_0 differs greatly from the initial estimate of R_b*, then repeat the process using a_0 as the initial estimate, re-estimate **B**, then finally re-estimate a_0. A better alternative to this procedure is to use nonlinear regression software to estimate all coefficients including **B**.

Variance around this regression surface will most likely be nearly homoscedastic, so that observation weights are all unity.

To construct a board foot estimator using merchantable top diameter as an independent variable, scale trees to a variety of merchantable tops. To avoid correlated regression errors, scale each tree to only one merchantable top and cover a range of top diameters by selecting a large enough sample of trees. If this is not practical, scale the same trees to a variety of merchantable tops, and accept the effects of correlated errors. Fit estimators based on the same model separately for each different merchantable top. Examine and model the relationships between coefficient estimates and merchantable top. Substitute the resulting functional relationships between coefficients and merchantable top into the R_b model and estimate all coefficients simultaneously. Recognize that this process is not easy, that satisfactory results are elusive, and the result can be attained with better accuracy using stem profile equations.

36.13 - Estimation of Losses in Lumber Potential

Recognize that lumber potential of a tree may be severely reduced by volume loss, and also by frost cracks, ring shake, end checking, or other cracks that reduce lumber potential without volume loss.

Follow procedures for volume loss adjustment in scaling given in FSH 2409.11a or FSH 2409.11. Develop the information from cut logs. However, recognize that the estimates associated with whole-tree lumber potential estimators rely upon information taken by the user on standing trees as given in FSH 2409.12. Generally:

1. Identify the portion of the tree subject to the damaging agent,

2. Estimate the fraction of that portion of the stem which has lost its usefulness.

For example, common cruising deductions such as "30 percent of the butt log," or "half the second log" are applications of this procedure. Detailed instruction for field application are given in FSH 2409.12 and Region or Forest supplements to it.

To use stem profile equations to calculate scaling diameters along the tree bole, only record the lower and upper limits of the affected portion of the stem and the proportion of that stem segment which has lost its lumber potential. Divide the remaining unaffected stem into logs according to existing procedures and scale by applying the selected scale rule to calculated diameters.

With direct lumber potential estimators, use an approach similar to that outlined for stem profile equations (section 32). Estimate the average proportion of the tree's potential lumber log by log then estimate the defect in each log and obtain estimate of the proportion of potential lumber lost in the whole tree. Use shortcuts such as the table shown in FSH 2409.12, section 22.31a to make the process more manageable.

36.2 - Veneer

Estimate veneer in units of a square foot 3/8 inch thick. For example: a 4 feet by 8 feet sheet of 3/4 inch thick plywood contains 4 X 8 X (3/4)/(3/8) = 64 square feet. A 4 feet by 8 feet sheet of 1/4 inch thick plywood contains 4 X 8 X (1/4)/(3/8) = 21.3 square feet

When preparing the estimator, recognize that veneer potential is closely related to that part of the volume contained in the veneer bolt, and veneer is turned from bolts that are nominally 8 feet long. Veneer yield also differs with species, age, and condition of the timber. Estimate utilization factors from a mill production study. Exclude the volume not used for veneer for the following reasons:

1. The length of bolts contains a trim allowance and veneer from that portion of the bolt is usually scrap. Therefore, use a locally established trim allowance when dividing a tree stem into veneer bolts.

2. A core is gripped by the veneer lathe chucks and is not used for veneer. Identify the core size and do not include it in the veneer potential.

3. Sheets of veneer come from a cylinder of wood contained within the bolt with the diameter equal to the small end. Due to taper and surface irregularities, the first veneer turned from a fresh bolt comes from it in pieces (fishtails) of gradually increasing size. Some pieces are used and some are wasted. Exclude the unused portion.

4. A small part of the volume in the cylinder is also unused because it contains knots or other defects. Exclude the unused portion.

To derive an estimate of veneer potential, begin with the following notation:

> V_t = total volume of the bolt
> V_f = volume in the cylinder
> V_c = volume in the unusable core
> V_f = volume in the "fishtail" portion of the bolt
> d_i = diameter at large end of the bolt
> $d_{,}$ = diameter at small end of the bolt
> d_c = diameter of the unusable core
> l = nominal length of the bolt
> k = pi/(4*144) = 0.005454154
> P = expected veneer yield of the bolt

Then:

$$V_t = k\ l\ (d_l^2 + d_s^2)/2$$

$$V_r = k\ d_s^2 l$$

$$V_c = k\ d_c^2 l$$

and:

$$V_f = V_t - V_r$$

$$= k*1*[(d_l^2 + d_s^2)/2 - d_s^2]$$

$$= k*1*(d_l^2 - d_s^2)/2$$

There are 32 square feet of 3/8 inch thick veneer per cubic foot of volume. If U_f represents the utilization factor for fishtail material and U_f the utilization factor for cylinder material, then:

$$P = 32U_f V_f + 32U_r(V_r - V_c)$$

$$= 32U_f*k*1*(d_l^2 - d_s^2)/2 + 32U_r*k*1*(d_s^2 - d_c^2)$$

The utilization factors U_f and U_r differ from one veneer plant to another. The factors are linear in the equation. Assume a nominal bolt length of eight feet, and simplify the equation to:

$$P = 0.69813\ U_f\ (d_l^2 - d_s^2) + 1.39626\ U_r\ (d_s^2 - d_c^2)$$

Apply this equation to bolts, and to only bolts with $d_s \geq d_c$. Estimate the appropriate utilization factors and use a stem profile equation to divide the predicted stem into bolts. Estimate the diameters d_1 and d_s for each bolt and calculate its potential veneer outturn. Obtain total expected veneer outturn for the tree by summing up bolts.

If a direct estimator is desired, measure a sample of felled trees at the points where the trees would be bucked for bolt production. Calculate veneer content for the bolts that would be obtained thereby and sum over the whole merchantable stem. This constitutes an observation on the dependent variable, veneer content. Estimate it by a regression equation similar in form to a merchantable volume equation.

36.3 - Poles and Pilings

Utility poles and pilings are usually measured by length and sold by length. They are usually further classified by grades which are determined partly, though not entirely, by diameter. Determine the measurement and grading of poles and pilings in use locally. For example, specifications for piling in the Puget Sound area are different from specifications on the Mississippi Gulf Coast or the Atlantic Coast. Due to variation in standards from one geographic area to another, no standard methods are specified.

36.4 - Fiber Products

This category of products includes paper, pressed fiberboard or fiberboard products, fuel, or any manufactured product in which wood is reduced to chip or fiber form. The quantities of such products which can be produced are closely related to the volume of wood. Derive conversion coefficients relating to pounds of paper or square feet of chipboard to the volume of wood going into the production process and any waste from observation of the process. Use elementary statistical estimation techniques to determine product outturn for a given cubic unit of wood volume.

Use the procedures in the Cubic Scaling Handbook (FSH 2409.11a) to predict fiber content of logs. From the many defects described therein, deduct only for rots, voids and char. Use the cubic volume to establish relationships with other predictors such as weight.

Because it is often easier to weigh a quantity of wood on trucks than to measure its volume, weight may be used as a predictor variable for fiber products rather than volume. Ensure that variation deriving from changes in weight from such things as the fuel that is in the tank, mud stuck to the mud flaps and undercarriage, the tire chains and binder chains aboard, drivers, ice, snow, and so forth is accounted for, Include a method to account for the variable moisture content of the wood. For example, if wood remains for a long time in a deck and dries out before being weighed, the amount of product per pound will be greater than for freshly cut wood. Density of wood also varies by species. Prepare and use different weight-to-product conversion coefficients for each species or for groups of species with similar characteristics. If needed, do likewise for live (high moisture) and dead (low moisture) within a species or group. Establish a procedure to predict fiber content of logs when loads are of mixed characteristics.