Forest Service Handbook
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Forest Service Handbook 2409.12a – Timber Volume Estimator Handbook
Chapter 60 - Fitting

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Digest: Following is an explanation of the changes throughout the directive by section.

2409.12a: Establishes new Timber Volume Estimator Handbook that provides Service-wide standards and instructions for preparation of equations or tables used to estimate the timber content of trees.

Table of Contents

61 - Fitting Process

Use fitting to adapt the volume estimator to subsets of data (sec. 35.4).

61.1 - General Description - Least Squares Regression

Use the least squares regression procedure in fitting a stem profile equation to a set of stem measurements. Estimate the coefficients of the stem profile equation in the fitting process. Use the least squares approach to minimize the sum of squared differences between the actual value of the dependent variable and the value predicted by the equation. The minimized value is called the sum of squared error (SSE).

$$SSE = \sum^{n} (y - \hat{y})^2$$

where:

y = observed value of the dependent variable y

$\hat{y}$ = value of y predicted by equation

When a stem profile model is linear in the coefficients, use ordinary least squares (OLS) regression fitting procedure. When a model is nonlinear in some or all of the coefficients, use nonlinear least squares (NLLS) regression. Regard the minimum SSE obtained by fitting a particular model to data as the minimum for that model only. Consider fitting another, more flexible model which may result in a smaller SSE. Do not regard SSE or residual mean square (RMS) as the only goodness of fit criterion. Consult regression textbooks that describe procedures for analyzing residuals to detect any of several conditions which are not characteristic of well-fitted regression.

61.11 - Ordinary Least Squares (OLS) Regression

Use ordinary least squares (OLS) regression with simple or complex polynomial stem profile models which are linear in the coefficients. An example of a simple linear stem profile model is that of Kozak and others (sec. 08; sec. 33.32). This model is linear in the coefficients b_1 and b_2. A more complex linear model is that of Bruce and others (sec. 08; sec. 33.32). This more complex model is linear in all of its coefficients, b_1 to b_6. Use ordinary least squares for nonlinear models if they can be transformed into a linear form. For example, the simple nonlinear model of Ormerod, (sec. 33.32) which is nonlinear in the coefficient b_1 may be converted to a linear form using a logarithmic transformation.

All major statistical software packages contain OLS regression procedures. Software packages and their respective OLS procedures are listed in Table 01. Refer to the respective user's manual for the details in using these procedures.

61-11 - Table 01

Major OLS Statistical Software

Statistical Package	OLS Procedures
BMDP (1981)	P1R
SPSS (1986)	REGRESSION
SAS (1985	REG, GLM
IMSL (1982)	RLONE, RLMUL
SYSTAT (1987)	MGLH

61.12 - Nonlinear Least Squares (NLLS) Regression

For most stem profile equations, use NLLS regression procedures. Many of the more complex models, including segmented models, are nonlinear relative to some or all of the parameters. Avoid complexity whenever possible, but recognize that a tree stem is a complex solid object, and that simpler stem profile models do not describe the tree as well as flexible and complex models do. Recognize that the NLLS procedure is more complex than OLS. Satisfy NLLS requirements, as necessary, such as providing starting values for the coefficients, placing bounds on the coefficient values, and possibly providing first order partial derivatives with respect to each of the coefficients.

Acquire working knowledge of the model parameters before attempting to fit final coefficients. Recognize that due to starting values and the nature of the SSE surface, the NLLS algorithm may converge to a local minimum instead of the desired global minimum of SSE. Be aware that there is no guarantee that the iterative NLLS procedure will converge to the global minimum of SSE; therefore, use one of several methods to establish starting coefficient values for the stem profile model to be fit. For an existing model, use coefficient values found by others for the same model as reasonable starting values, even though they may be for a different species or geographic area. In the absence of other results, use SSE for different values of the coefficients. Systematically vary the coefficients to provide a range of SSE's. Use the statistical software package (SAS), or an equivalent, to perform the systematic varying of coefficients. Plan to calculate SSE for many combinations of coefficients and for the expense of doing so. Use the coefficient values associated with the lowest SSE as starting values in the NLLS regression procedure.

Provide upper and lower bounds for the coefficients to make the NLLS procedure converge in fewer iterations. Use caution in setting bounds on the coefficients since a coefficient estimated in NLLS as one of its pre-set bounds may be an indicator of a restrictive bound limiting the procedure from finding the global minimum.

Provide first-order partial derivatives with respect to each coefficient in the stem profile model depending on the specific procedure used. Employ a procedure using analytical derivatives if one is available since derivative-free algorithms are usually less efficient than algorithms requiring derivatives. However, derivative-free algorithms may be used for models with difficult-to-derive partial derivatives, since errors in deriving the first order partial derivatives impact the NLLS fitting procedure. Examples of the first order partial derivatives for two common stem profile models are given in tables 01 and 02.

61.12 - Table 01

Partial Derivative, General Form of the Ormerod Stem Profile Model

$$\frac{d^2}{D^2} = b_1\left(\frac{h}{H}-1\right)+b_2\left(\frac{h^2}{H^2}-1\right)+b_3\left(a_1-\frac{h}{H}\right)^2 I_1+b_4\left(a_2-\frac{h}{H}\right)^2 I_2$$

where:

$$I_i = 1 \text{ if } h/H \le a_i$$

$$= 0 \text{ if } h/H > a_i, \text{ for } i=1,2$$

if $y = \frac{d^2}{D^2}$ *then the partial derivitives with respect to the six coefficients* b_1, b_2, b_3, b_4, a_1 *and* a_2 *are*

$$\frac{\partial y}{\partial b_1} = \frac{h}{H}-1 \qquad \frac{\partial y}{\partial b_4} = \left(a_2-\frac{h}{H}\right)^2 I_2$$

$$\frac{\partial y}{\partial b_2} = \frac{h^2}{H^2}-1 \qquad \frac{\partial y}{\partial a_1} = 2b_3 I_1\left(a_1-\frac{h}{H}\right)$$

$$\frac{\partial y}{\partial b_3} = \left(a_1-\frac{h}{H}\right)^2 I_1 \qquad \frac{\partial y}{\partial a_2} = 2b_4 I_2\left(a_2-\frac{h}{H}\right)$$

61.12 - Table 02

Partial Derivative, Segmented Model of Max and Burkhart

$$\frac{d^2}{D^2} = b_1^2\left(\frac{H-h}{H-4.5}\right)^{2b_2}$$

if $y = \frac{d^2}{D^2}$, *the partial derivitives with respect to* b_1 *and* b_2 *are*

$$\frac{\partial y}{\partial b_1} = 2b_1\left(\frac{H-h}{H-4.5}\right)^{2b_2}$$

$$\frac{\partial y}{\partial b_2} = 2b_1^2\left(\frac{H-h}{H-4.5}\right)^{2b_2}\ln\left(\frac{H-h}{H-4.5}\right)$$

61.2 - Statistical Procedures

Table 01 lists the NLLS regression procedures available in each of the major statistical packages.

61.2 - Table 01

Major NLLS Statistical Software

Statistical Package	NLLS Procedures
BMDP (1981)	P3R (derivatives required) PAR (derivative-free)
SPSS (1986)	None available at the present, possibly available in future versions.
SAS (1985)	NLIN (derivatives required or derivative-free)
IMSL (1982)	ZXMIN (derivative-free)
SYSTAT (1987)	NONLIN (derivative free)

Refer to the respective user's manual for the details in using these procedures.

62 - Variables Other than D, H, and h in Stem Profile Equations

Account for Most of the variation in the stem profile using D, H, and h. However, incorporate other variables into a stem profile model when effective in increasing the accuracy and precision of predictions made for the model. Consider other variables often used such as measures of stem form, crown measurements, and indicators of site quality and geographic location.

62.1 - Crown and Form Measurements

Because of the importance of the live crown on stem taper, consider crown measurements (especially crown ratio) as an additional variable in stem profile models. For more detailed discussion of using crown measurements in a stem profile model refer to Dell, Burkhart and Walton, Newberry and Burkhart, and Valenti and Cao (sec. 08). Trees with the same D and H may have different forms but most stem profile equations fit an average profile for trees with the same dimensions. Using a form measurement as a variable may improve the fit of the model. Form quotients (that is a ratio of a fixed upper stem diameter to Dbh, such as Girard form class) or H/D ratios are commonly Used form measurements. Refer to Matney and Sullivan, Schlaegel, and Czaplewski and McClure (sec. 08) for previous work done in this area. Improvement in predictive value of a model requiring an upper stem measurement which may be substantial is gained at the cost of measuring that upper stem diameter on trees with which it will be used. Using the H/D ratio as a proxy for form quotient avoids the need for an additional measurement.

62.2 - Geographic Location and Site Quality Indicators

Use geographic location and indicators of site quality when they may be able to explain some additional variation in stem profile. Geographic variation in the relationship between stem profile and volume has been shown to exist; therefore, expect that geographic location may account for some variation. Likewise, site variables, such as ecological habitat, elevation, slope, aspects, and the like because their relationship to stem growth might be useful for incorporation into a model. Refer to Larson for a brief outline of the work that has been completed (sec. 08). Explore these variables, if possible, as additional predictor variables in stem profile models.

62.3 - Additional Variables

Review the several methods of incorporating additional predictor variables into stem profile models. Select a method that preserves the integrability of the equation for efficient volume determination.

62.31 - Form Quotient

To incorporate form quotient into a stem profile model, condition the model to exactly predict the upper stem diameter corresponding to the form quotient. Condition it by mathematically manipulating the equation so that it meets the desired criteria and integrate the model for volume determination if possible.

62.32 - Fitting Classes of Data

Consider incorporating additional variables into a model by dividing the data set into classes based on the variable of interest and then fitting the model to each class of data. Use this approach as an exploratory method in determining possible relationships between variables and stem profile coefficients, then use the two-stage final fitting procedure described in section 62-33. If many classes exist, maintaining the right regression coefficients for each class will be more

burdensome and deciding how to divide the data into valid classes without knowing the underlying relationships is difficult. If done incorrectly, there may be too few classes to adequately describe relationships, or there may be too many classes resulting in an unnecessary workload in the fitting process.

62.33 - Two-Stage Fitting

Use a two-stage fitting approach in most situations requiring incorporation of additional predictor variables. Fit the stem profile equation to each tree in the data set or to small groups of trees with the same dimensions and characteristics. Relate the coefficients obtained in the first stage to the additional variables of interest by using least squares regression techniques such as stepwise or all possible subsets regression procedures in the second stage. After the second stage model forms are determined, fit the stem profile model, with the second stage equations included, to the full data set for final coefficient determination. Use this procedure to maintain the integrability of the stem profile model and to adequately describe the relationships between additional variables and stem profile.